ECUACIONES DIFERENCIALES

incógnita

$$F\left(x, y(x), \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots\right) = 0 \qquad y(x)$$

ECUACIÓN

SOLUCIÓN

Variable independiente

al menos una de las derivadas de la incógnita.

Una ED es una expresión matemática que tiene forma de ecuación $\left[F(\ )=0\right]$ y que contiene al menos una de las derivadas de una función desconocida llamada incógnita $y(x)$ que depende de al menos una variable independiente (x)

Ecuación

$\xrightarrow[1]{}$ $\dfrac{dy}{dx} = 0$ $\quad y(x) \quad x$

$\xrightarrow[2]{}$ $y'(x) = 0$

$\dot{y}_x = 0$

$\xrightarrow[2,3]{}$ $D_x y = 0$

$y(x) = C_1 \rightarrow \dfrac{dy}{dx} = 0$

$[0] = 0 \longrightarrow 0 \equiv 0$

es una solución

$y = 5 \quad y = \sqrt{3}$

$y = C_1$ solución general

$\left.\begin{array}{l} y = 5 \\ y = \sqrt{3} \end{array}\right\}$ soluciones particulares

$\boxed{\dfrac{dy}{dx} = 0}$

ED

$\text{EDO} \longrightarrow \text{SG}$

$\text{EDO} \not\longleftarrow \text{SG}$

$$y = c_1 e^x \qquad \frac{dy}{dx} = c_1\left[e^x\right]$$

solución
general

$$c_1 = \frac{y}{e^x} \qquad c_1 = \frac{\frac{dy}{dx}}{e^x}$$

$$\frac{y}{e^x} = \frac{\frac{dy}{dx}}{e^x} \rightarrow \frac{dy}{dx} = y$$

$$y = c_1 e^x \longleftrightarrow \boxed{\frac{dy}{dx} - y = 0}$$

SG

EDO

$$y = C_1 \cos(x) + C_2 \operatorname{sen}(x)$$

SG

EDO

$$\frac{dy}{dx} = -C_1 \operatorname{sen}(x) + C_2 \cos(x)$$

$$\frac{d^2y}{dx^2} = -C_1 \cos(x) - C_2 \operatorname{sen}(x)$$

$$\frac{d^2y}{dx^2} = -\left(C_1 \cos(x) + C_2 \operatorname{sen}(x)\right)$$

$$\frac{d^2y}{dx^2} = -y \rightarrow \boxed{\frac{d^2y}{dx^2} + y = 0}$$

$$y = C_1 x^2 + C_2 x + C_3 \qquad \text{orden} = 3$$

$$\frac{dy}{dx} = 2C_1 x + C_2 + (0)$$

$$\frac{d^2y}{dx^2} = 2C_1 + (0) + (0)$$

$$\boxed{\frac{d^3y}{dx^3} = 0}$$

$$y = c_1 e^x + c_2 x e^x + c_3 x^2 + c_4 x + c_5$$

$$\frac{dy}{dx} = c_1 e^x + c_2\left(xe^x + e^x\right) + 2c_3 x + c_4$$
$$= (c_1 + c_2)e^x + c_2 x e^x + 2c_3 x + c_4$$

$$\frac{d^2y}{dx^2} = (c_1 + c_2)e^x + c_2\left(xe^x + e^x\right) + 2c_3$$
$$= (c_1 + 2c_2)e^x + c_2 x e^x + 2c_3$$

$$\frac{d^3y}{dx^3} = (c_1 + 2c_2)e^x + c_2\left(xe^x + e^x\right) + (0)$$
$$= (c_1 + 3c_2)e^x + c_2 x e^x$$

$$\frac{d^4y}{dx^4} = (c_1 + 3c_2)e^x + c_2\left(xe^x + e^x\right)$$

$$\longrightarrow \frac{d^4y}{dx^4} = (c_1 + 4c_2)e^x + c_2 x e^x$$

$$\frac{d^5y}{dx^5} = (c_1 + 4c_2)e^x + c_2\left(xe^x + e^x\right)$$

$$\longrightarrow \frac{d^5y}{dx^5} = (c_1 + 5c_2)e^x + c_2\left(xe^x\right)$$

$$\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4} = c_2 e^x + (0)$$

$$c_2 = \frac{\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4}}{e^x}$$

$$\frac{d^5y}{dx^5} = c_1 + 5\left(\frac{\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4}}{e^x}\right)e^x + \left(\frac{\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4}}{e^x}\right)xe^x$$

$$c_1 = \frac{d^5y}{dx^5} - 5\left(\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4}\right) - x\left(\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4}\right)$$

$$c_1 = \frac{d^5y}{dx^4}\left(1 - 5 - x\right) + \frac{d^4y}{dx^4}\left(5 + x\right)$$

$$(D-1)^2 D^3 y = 0 \qquad \left(D^2 - 2D + 1\right)D^3 y = 0$$

$$\frac{d^5y}{dx^5} - 2\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} = 0$$

$$\left(D^2 - 2D + 1\right)y = 0$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$y = c_1 e^x + c_2 x e^x$$

$$\frac{dy}{dx} = c_1 e^x + c_2 (x e^x + e^x)$$

$$= (c_1 + c_2) e^x + c_2 x e^x$$

$$\frac{d^2y}{dx^2} = (c_1 + c_2) e^x + c_2 (x e^x + e^x)$$

$$= (c_1 + 2c_2) e^x + c_2 x e^x$$

$$\frac{d^2y}{dx^2} \Leftrightarrow (c_1 + 2c_2) e^x + c_2 x e^x$$

$$+$$

$$-2\frac{dy}{dx} \Rightarrow (-2c_1 - 2c_2) e^x - 2c_2 x e^x$$

$$+$$

$$y \Leftrightarrow c_1 e^x + c_2 x e^x$$

$$= \quad ((0) + 0) e^x + (0) x e^x$$

$$0$$

propiedad 1 EDO

"orden" de una ecuación diferencial ordinaria al "orden" de la derivada de mayor "orden"

$$\frac{d^4y}{dx^4} + 5\frac{d^2y}{dx^2} - 6y = 0 \quad \text{orden} = 4$$

$$y_g = c_1 y_1 + c_2 y_2 + c_3 y_3 + c_4 y_4$$

$$\left.\begin{array}{l} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array}\right\} \text{soluciones particulares "fundamentales"}$$

$$P(D) = \begin{cases} x^n & n = 0, 1, 2, \ldots\ldots \\ e^{ax} & a \in \mathbb{R} \\ \begin{cases} \cos(bx) \\ \mathrm{sen}(bx) \end{cases} & b \in \mathbb{R} \end{cases}$$

$\times$ $\quad \alpha(x) \quad \dfrac{1}{x^2} \quad \tan(bx)$

EDO

LIN — 1 SG. (única)

∞ SP.

NL.

1 SG

— ∞ SP

1 SG

∞ SP

∄ SS.

Tareas (1–10) – 20%

Series (4) – 20%

Ex. Parc. (2) – 60%

PROM SEM 100%

APROB EP. → EXENTOS

EF ⇒ PS – 50%
EF – 50%