# La Matriz Exponencial

McLaurin $e^{at} = 1 + at + \frac{a^2t^2}{2!} + \frac{a^3t^3}{3!} + \dots + \frac{a^kt^k}{k!} + \dots \infty$

$$e^{At} = I + At + A^2\frac{t^2}{2!} + A^3\frac{t^3}{3!} + \dots + A^k\frac{t^k}{k!} + \dots$$

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toda matriz cuadrada $A$ satisface su propia ecuación característica

$$\det(A - \lambda I) = 0$$

$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ $\quad \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$ Ec. Caract.

$$(2-\lambda)(4-\lambda) - 3 = 0$$

$$\lambda^2 - 6\lambda + 8 - 3 = 0$$

E.C. $\lambda^2 - 6\lambda + 5 = 0$

$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix}$

$$A^2 - 6A + 5I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix} - 6\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7-12+5 & 18-18+0 \\ 6-6+0 & 19-24+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_{n\times n} \quad A^n + a_1 A^{n-1} + a_2 A^{n-2} + \dots a_{n-2} A^2 + a_{n-1} A + a_n I = 0$$

$$A^n = -a_n I - a_{n-1} A - a_{n-2} A^2 - \dots \dots - a_1 A^{n-1}$$

$$A^{n+1} = -a_n A - a_{n-1} A^2 - a_{n-2} A^3 - \dots - a_1 A^n$$

$$A^{n+1} = b_n I + b_{n-1} A + b_{n-2} A^2 + \dots + b_1 A^{n-1}$$

$$A^{n+2} = b_n A + b_{n-1} A^2 + b_{n-2} A^3 + \dots + b_1 A^n$$

$$A^{n+2} = c_n I + c_{n-1} A + c_{n-2} A^2 + \dots + c_1 A^{n-1}$$

$$e^{At} = B_0(t) I + B_1(t) A + \dots \dots + B_{n-1}(t) A^{n-1}$$

$$A_{n\times n}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda - 1)(\lambda - 5) = 0$$

$$e^{At} = \beta_0(t) I + \beta_1(t) A$$

$$e^{\lambda_i t} = \beta_0(t) + \lambda_i \beta_1(t)$$

$$e^{t} = \beta_0(t) + (1)\beta_1(t)$$

$$e^{5t} = \beta_0(t) + 5\beta_1(t)$$

$$-e^{t} = -\beta_0(t) - \beta_1(t)$$

$$(e^{5t} - e^{t}) = 4\beta_1(t) \rightarrow \beta_1(t) = \frac{e^{5t} - e^{t}}{4}$$

$$\beta_0(t) = e^{t} - \beta_1(t)$$

$$\beta_0(t) = e^{t} - \left( \frac{e^{5t} - e^{t}}{4} \right)$$

$$\beta_0(t) = \frac{5e^{t} - e^{5t}}{4}$$

$$e^{At} = \left( \frac{5e^{t} - e^{5t}}{4} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left( \frac{e^{5t} - e^{t}}{4} \right) \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 5-2 & -3 \\ -1 & 5-4 \end{bmatrix} \frac{e^{t}}{4} + \begin{bmatrix} -1+2 & 3 \\ 1 & -1+4 \end{bmatrix} \frac{e^{5t}}{4}$$

$$\boxed{e^{At} = \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^{t}}{4} + \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \frac{e^{5t}}{4}}$$

$$\frac{de^{At}}{dt} = \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^{t}}{4} + \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \frac{5e^{5t}}{4}$$

$$= \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^{t}}{4} + \begin{bmatrix} 5 & 15 \\ 5 & 15 \end{bmatrix} \frac{e^{5t}}{4}$$

$$Ae^{At} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^{t}}{4} + \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \frac{e^{5t}}{4}$$

$$= \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^{t}}{4} + \begin{bmatrix} 5 & 15 \\ 5 & 15 \end{bmatrix} \frac{e^{5t}}{4}$$

$$\frac{d}{dt}\bar{x} = A\bar{x} + \bar{b}(t) \qquad \frac{d}{dt}\bar{x} = A\bar{x}$$

$$\bar{x} = e^{At}\bar{x}(0)$$

$$\bar{x} = e^{At}\bar{x}(0) + \int_0^t e^{A(t-\tau)}\,\bar{b}(\tau)\,d\tau.$$