

EDO(n) LCC NH.

Método del Operador Diferencial.

$$\left. \begin{array}{l} y' \\ y \\ \frac{dy}{dx} \\ \boxed{D_x y} \end{array} \right\}$$

derivada
de "y"
respecto a "x"

$$Dy \quad D^{-1}y \rightarrow D^{-1}(Dy) = y$$

$$D^{-1} \Rightarrow \text{antiderivada} \quad D^0y = y$$

$$D(Dy) = D^2y$$

$$(D+a)y = Dy + ay$$

$f(t)$	D.F.	
1	D	$(D-a)[e^{at}]$
t	D^2	
t^n	D^{n+1}	
e^{at}	$(D-a)$	$D[e^{at}] - ae^{at}$
te^{at}	$(D-a)^2$	$ae^{at} - ae^{at}$
$t^n e^{at}$	$(D-a)^{n+1}$	$= 0.$
$\cos(bt)$ $\sin(bt)$	$(D^2 + b^2)$	
$e^{at} \cos(bt)$ $e^{at} \sin(bt)$	$((D-a)^2 + b^2)$	
$t e^{at} \cos(bt)$	$((D-a)^2 + b^2)^2$	

$$(D-a)^2 [te^{at}] = 0$$

$$(D-a)(D-a)[te^{at}] = 0$$

$$(D-a) [ate^{at} + e^{at} - ate^{at}] = 0$$

$$(D-a)[e^{at}] = 0$$

$$ae^{at} - ae^{at} = 0.$$

$$0 \equiv 0$$

$$(\mathcal{D}^2 + b^2) [\cos(bt) + \text{sen}(bt)] = 0$$

$$\mathcal{D}^2 [\cos(bt) + \text{sen}(bt)] + b^2 \cos(bt) + b^2 \text{sen}(bt) = 0$$

$$\mathcal{D} [-b \text{sen}(bt) + b \cos(bt)] + b^2 \cos(bt) + b^2 \text{sen}(bt) = 0$$

$$-b^2 \cancel{\cos(bt)} - b^2 \cancel{\text{sen}(bt)} + b^2 \cancel{\cos(bt)} + b^2 \cancel{\text{sen}(bt)} = 0$$

$$0 = 0$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$(D^2 - 5D + 6)y = 0$$

$$(D-2)(D-3)y = 0$$

$$\rightarrow y = c_1 e^{2x} + c_2 e^{3x}$$

$$(D-2)(D-3)[c_1 e^{2x} + c_2 e^{3x}] = 0$$

$$(D-2)[\cancel{2c_1} e^{2x} + \cancel{3c_2} e^{3x} - \cancel{3c_1} e^{2x} - \cancel{3c_2} e^{3x}] = 0$$

$$(D-2)[-c_1 e^{2x}] = 0$$

$$-2\cancel{c_1} e^{2x} + 2\cancel{c_1} e^{2x} = 0$$

$$0 \equiv 0$$

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0$$

Caso $(D^2 - 6D + 9)y = 0$

II $(D - 3)^2 y = 0$

$$y_g = C_1 e^{3x} + C_2 x e^{3x}$$

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$$

$$(D^2 + 2D + 2)y = 0$$

Caso III $((D^2 + 2D + 1) - 1 + 2)y = 0$

$$((D + 1)^2 + 1^2)y = 0$$

$$y = C_1 e^{-x} \cos(x) + C_2 e^{-x} \operatorname{sen}(x)$$

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 8y = 5e^{2x} + x^2$$

$$(D^2 - 6D + 8)y = 0$$

$$(D-2)(D-4)y = 0$$

$$y = c_1 e^{2x} + c_2 e^{4x}$$

~~Do(2) cc NH~~

$$(D-2)(D-4)y = 5e^{2x} + x^2 + (0)x + (0)$$

$$(D-2)(D-4)(D-2) \underset{A}{D} \underset{A}{D} \underset{A}{D} y = 0$$

$$y = c_1 e^{2x} + c_2 e^{4x} + c_3 x e^{2x} + \underset{4}{c_4} + \underset{8}{c_5} x + \underset{6}{c_6} x^2$$

$$y = Ax e^{2x} + B + Dx + Ex^2$$

$\neq 0$

$$\frac{dy}{dx} = 2Ax e^{2x} + Ae^{2x} + (0) + D + 2Ex$$

$$\frac{d^2 y}{dx^2} = 4Ax e^{2x} + 4Ae^{2x} + (0) + 2E$$

$$[4Ax e^{2x} + 4Ae^{2x} + 2E] - 12Ax e^{2x} - 6Ae^{2x} - 6D - 12Ex + 8Ax e^{2x} + 8B + 8Dx + 8Ex^2 = 5e^{2x} + x^2$$

$$(4-12+8)Ax e^{2x} + (4-6)Ae^{2x} + (2E-6D+8B) + (-12E+8D)x + (8E)x^2 = 5e^{2x} + x^2$$

$$\begin{aligned} -2A &= 5 & A &= -\frac{5}{2} \\ 2E-6D+8B &= 0 \\ -12E+8D &= 0 & D &= \frac{12}{8} \\ 8E &= 1 & E &= \frac{1}{8} \end{aligned}$$

$$y = c_1 e^{2x} + c_2 e^{4x} + \underbrace{-\frac{5}{2} x e^{2x}}_{y_p/h_1} + \underbrace{\left(\frac{12}{64x^2}\right) + \frac{12}{64} x + \frac{1}{8} x^2}_{y_p/h_2}$$

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = 4 \cos(2x)$$

$$(\mathcal{D}^2 + \mathcal{D} + 1)y = 0$$

$$\left(\left(\mathcal{D} + \frac{1}{2}\right) - \frac{1}{4} + 1\right)y = 0$$

$$\left(\left(\mathcal{D} + \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\right)y = 0$$

$$y_{g/h} = c_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{-\frac{1}{2}x} \sin\left(\frac{\sqrt{3}}{2}x\right)$$

$$\left(\left(\mathcal{D} + \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\right)y = 4 \cos(2x)$$

$$\left(\left(\mathcal{D} + \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\right) (\mathcal{D}^2 + 4)_A y = 0$$

$$y_{p/a} = A \cos(2x) + B \sin(2x)$$

$$\frac{dy}{dx} = -2A \sin(2x) + 2B \cos(2x)$$

$$\frac{d^2 y}{dx^2} = -4A \cos(2x) - 4B \sin(2x)$$

$$(\mathcal{D}^2 + \mathcal{D} + 1)y = 4 \cos(2x)$$

$$(-4A + A) \cos(2x) + 2B \cos(2x) + (-4B + B) \sin(2x) +$$

$$\cancel{y} - 2A \sin(2x)$$

$$(-3A + 2B) \cos(2x) + (-3B - 2A) \sin(2x)$$

$$-3A + 2B = 4$$

$$-2A - 3B = 0$$

$$\hline -6A + 4B = 8$$

$$+6A + 9B = 0$$

$$\hline 0 \quad 13B = 8 \quad B = \frac{8}{13}$$

$$-2A = 3\left(\frac{8}{13}\right)$$

$$A = -\frac{12}{13}$$

$$y = c_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{-\frac{1}{2}x} \sin\left(\frac{\sqrt{3}}{2}x\right) - \frac{12}{13} \cos(2x) + \frac{8}{13} \sin(2x)$$