OBTENER LA MATRIZ EXPONENCIAL

$$y = e^{at} \qquad \frac{dy}{dt} = ae^{at}$$

$$\bar{y} = e^{At} \qquad \frac{d}{dt}\bar{y} = Ae^{At}$$

$$e^{at} = 1 + at + \frac{a^2}{2!}t^2 + \frac{a^3}{3!}t^3 + \ldots + \frac{a^n}{n!}t^n + \ldots \quad \infty$$

$$e^{A} = I + At + \frac{A^2}{2!}t^2 + \frac{A^3}{3!}t^3 + \ldots + \frac{A^n}{n!}t^n + \ldots \quad \infty$$

Teorema Hamilton-Cayle

toda matriz cuadrada A satisface su propia ecuación característica.

$$A \longrightarrow \lambda^n + b_1\lambda^{n-1} + b_2\lambda^{n-2} + \ldots + b_{n-1}\lambda + b_n(1) = 0$$

$$A^n + b_1 A^{n-1} + b_2 A^{n-2} + \ldots + b_{n-1}A + b_n I = [0]$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \det(A - \lambda I) = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\cdot(4-\lambda) - 3 = 0$$

$$\lambda^2 - 6\lambda + 8 - 3 = 0$$

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$$\lambda^2 - 6\lambda + 5 = 0$$

$$A^2 - 6A + 5I = [0]$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} - 6\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 6+12 \\ 2+4 & 3+16 \end{bmatrix} - \begin{bmatrix} 12 & 18 \\ 6 & 24 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7-12+5 & 18-18+0 \\ 6-6+0 & 19-24+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$e^{At} = I + At + A^2\frac{t^2}{2!} + A^3\frac{t^3}{3!} + \dots + A^k\frac{t^k}{k!} + \dots$$

$$A^n + b_1 A^{n-1} + b_2 A^{n-2} + \dots + b_{n-1}A + b_n I = 0$$

$$A^n = -b_n I - b_{n-1}A - \dots - b_2 A^{n-2} - b_1 A^{n-1}$$

$$e^{At} = I + At + A^2\frac{t^2}{2!} + \dots + A^n\frac{t^n}{n!} + A^{n+1}\frac{t^{n+1}}{(n+1)!} + \dots$$

$$e^{At} = \underbrace{I + At + A^2\frac{t^2}{2!} + \dots}_{n} + \left(\underbrace{-b_n I - b_{n-1}A - \dots - b_1 A^{n-1}}_{n}\right)\frac{t^n}{n!} + A^{n+1}\frac{t^{n+1}}{(n+1)!} + \dots$$

$$A^{n+1} = -b_n A - b_{n-1}A^2 - \dots - b_2 A^{n-1} - b_1 A^n$$

$$A^{n+1} = \underbrace{-C_n I - C_{n-1}A - \dots - C_2 A^{n-2} - C_1 A^{n-1}}_{n}$$

$$e^{At} = \underbrace{B_0(t)I + B_1(t)A + B_2(t)A^2 + \dots + B_n(t)A^{n-1}}_{n}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda - 5)(\lambda - 1) = 0 \qquad \lambda_1 = 1, \quad \lambda_2 = 5$$

$$e^{At} = \beta_0(t) I + \beta_1(t) A.$$

$$e^{t} = \beta_0(t) + \beta_1(t)$$

$$e^{5t} = \beta_0(t) + 5\beta_1(t)$$

$$e^{5t} - e^{t} = (0) + 4\beta_1(t) \qquad \beta_1(t) = \frac{e^{5t} - e^{t}}{4}$$

$$e^{t} = \beta_0(t) + \frac{e^{5t} - e^{t}}{4}$$

$$\beta_0(t) = e^{t} - \frac{e^{5t} - e^{t}}{4}$$

$$\beta_0(t) = \frac{5e^{t} - e^{5t}}{4}$$

$$e^{At} = \left(\frac{5e^{t} - e^{5t}}{4}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{e^{5t} - e^{t}}{4}\right)\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} -1+2 & 3 \\ 1 & -1+4 \end{bmatrix}\frac{e^{5t}}{4} + \begin{bmatrix} 5-2 & -3 \\ -1 & 5-4 \end{bmatrix}\frac{e^{t}}{4}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t}$$

$$e^{A(0)} = \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}$$

$$e^{A(0)} = \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad e^{At} \qquad \bar{X}_0 = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

Sol Part.

$$x(t) = e^{At} \cdot \bar{X}_0$$

$$\frac{dx_1}{dt} = -x_2$$

$$\frac{dx_2}{dt} = x_1$$

$$x_1 = c_1 \cos(t) - c_2 \operatorname{sen}(t)$$

$$x_2 = c_1 \operatorname{sen}(t) + c_2 \cos(t)$$