TEMA 2.- EDO(n)LCCH.

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \qquad y_p = e^{mx}$$

E(A)C. $\quad m^2 + a_1 m + a_2 = 0 \qquad \left\{ \begin{array}{l} m_1 \\ m_2 \end{array} \right.$ raíces

CASO I.- raíces reales y distintas $\quad m_1 m_2 \in \mathbb{R} \quad m_1 \neq m_2$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x} \qquad W(e^{m_1 x}, e^{m_2 x}) \neq 0$$

CASO II.- raíces reales e iguales $\quad m_1 m_2 \in \mathbb{R} \quad m_1 = m_2$

CASO III.- raíces complejas o imaginarias $\quad m_1 m_2 \in \mathbb{C}$

$x \in \mathbb{R}$
$y \in \mathbb{R}$

$$y_g = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x} \qquad \begin{array}{l} m_1 = a+bi \\ m_2 = a-bi \end{array} \quad m_1 \neq m_2$$

$$e^{\pi i} = -1 \qquad \begin{array}{l} a \in \mathbb{R} \\ b \in \mathbb{R}^+ \end{array}$$

funciones polares

$$r e^{\theta i} = r\cos(\theta) + [r\,\mathrm{sen}(\theta)]i$$
$$r e^{-\theta i} = r\cos(\theta) - [r\,\mathrm{sen}(\theta)]i$$

$$e^{\theta i} = \cos(\theta) + i \cdot \mathrm{sen}(\theta)$$
$$e^{-\theta i} = \cos(\theta) - i \cdot \mathrm{sen}(\theta)$$

$$y(x) = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$
$$= C_1 e^{ax} e^{bxi} + C_2 e^{ax} e^{-bxi}$$
$$= e^{ax}\left(C_1 e^{bxi} + C_2 e^{-bxi}\right)$$
$$y(x) = e^{ax}\left(C_1\left[\cos(bx) + i\,\mathrm{sen}(bx)\right] + C_2\left[\cos(bx) - i\,\mathrm{sen}(bx)\right]\right)$$
$$= e^{ax}\left((C_1 + C_2)\cos(bx) + (C_1 i - C_2 i)\,\mathrm{sen}(bx)\right)$$
$$= e^{ax}\left(C_{10}\cos(bx) + C_{20}\,\mathrm{sen}(bx)\right)$$

$$y(x) = C_{10} e^{ax}\cos(bx) + C_{20} e^{ax}\,\mathrm{sen}(bx) \qquad \begin{array}{l} m_1 = a+bi \\ m_2 = a-bi \end{array}$$

SOL GRAL

CASO II: raíces reales e iguales $m_1 = m_2$

$m^2 + a_1 m + a_2 = 0$ $m_1 \neq m_2$

$(m - m_1)(m - m_2) = 0$ $EC = 0$

$\frac{d}{dm}(m^2 + a_1 m + a_2) = 0$

$2m + a_1 = 0$

$\frac{d}{dm}\left[(m - m_1)\cdot(m - m_2)\right] = 0$ $\frac{d}{dm}(EC) \neq 0$

$(m - m_1)\cdot(1) + (m - m_2)\cdot(1) = 0$

$m_1 = m_2$

$m^2 + a_1 m + a_2 = 0$ $EC = 0$

$(m - m_1)^2 = 0$

$\frac{d}{dm}$

$2m + a_1 = 0$ $\frac{d}{dm} EC = 0$

$2(m - m_1) = 0$

$\frac{d}{dm}$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2$$

$$y(x) = c_1 e^{m_1 x} + c_2\,(?)$$

$$y(x) = e^{mx} \xrightarrow{m=m_1} e^{m_1 x}$$

$$\frac{d}{dm}$$

$$y(x) = xe^{mx} \xrightarrow{m=m_1} xe^{m_1 x}$$

$$y(x) = c_1 e^{m_1 x} + c_2 x e^{m_1 x}$$

$$\frac{dy}{dx} = m_1 c_1 e^{m_1 x} + c_2\left(m_1 x e^{m_1 x} + e^{m_1 x}\right)$$

$$= (m_1 c_1 + c_2)e^{m_1 x} + c_2 m_1 x e^{m_1 x}$$

$$\frac{d^2y}{dx^2} = m_1 (m_1 c_1 + c_2) e^{m_1 x} + c_2 m_1 \left(m_1 x e^{m_1 x} + e^{m_1 x}\right)$$

$$= (m_1^2 c_1 + m_1 c_2 + c_2 m_1) e^{m_1 x} + c_2 m_1^2 x e^{m_1 x}$$

$$y'' \Rightarrow \left(\cancel{m_1^2 c_1} + 2c_2 m_1\right) e^{m_1 x} + c_2 m_1^2 x e^{m_1 x}$$

$$a_1 y' \Rightarrow \left(a_1(\cancel{m_1} c_1 + c_2) e^{m_1 x} + a_1 (c_2 m_1 x e^{m_1 x})\right).$$

$$a_2 y \Rightarrow \cancel{a_2 c_1} e^{m_1 x} + a_2 c_2 x e^{m_1 x}$$

$$\left(m_1^2 c_1 + a_1 m_1 c_1 + a_2 c_1\right) e^{m_1 x} + \left(2c_2 m_1 + a_1 c_2\right) e^{m_1 x} +$$

$$\left(c_2 m_1^2 + a_1 c_2 m_1 + a_2 c_2\right) x e^{m_1 x}$$

EDO(n)LCC NH.

$$y'' + a_1 y' + a_2 y = Q(x)$$

$$y_{g/n-h} = y_{g/H} + y_{P/Q}.$$