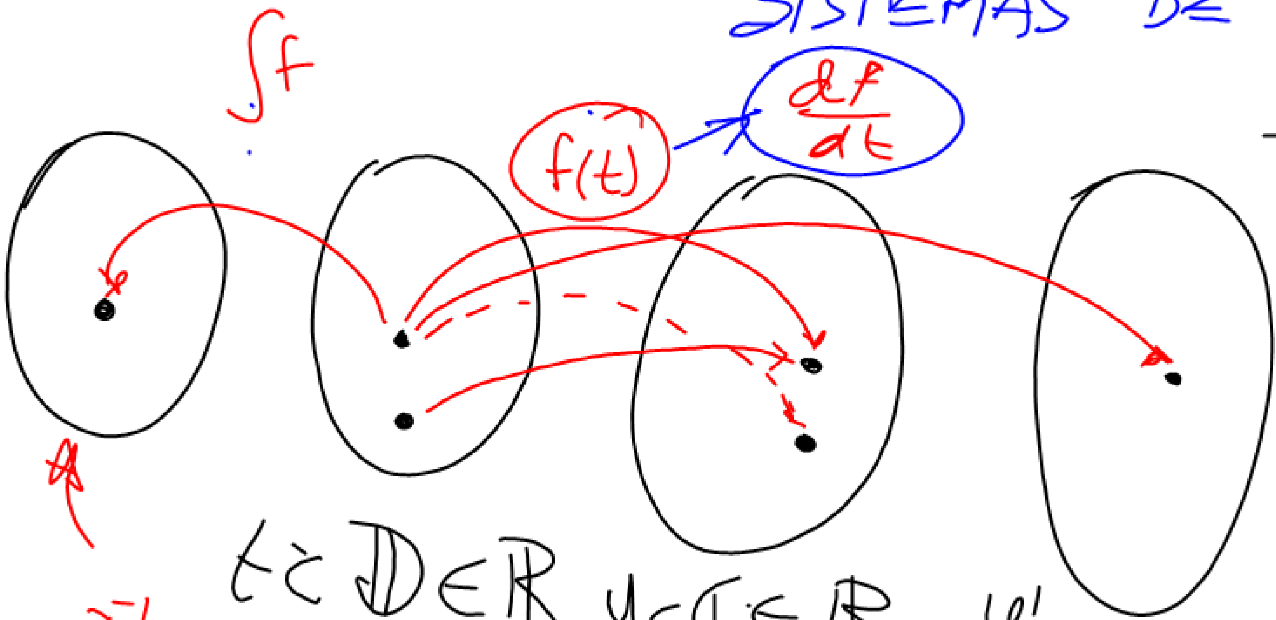
TEMA III.- TRANSFORMADA DE LAPLACE.

SISTEMAS DE EDO(1).

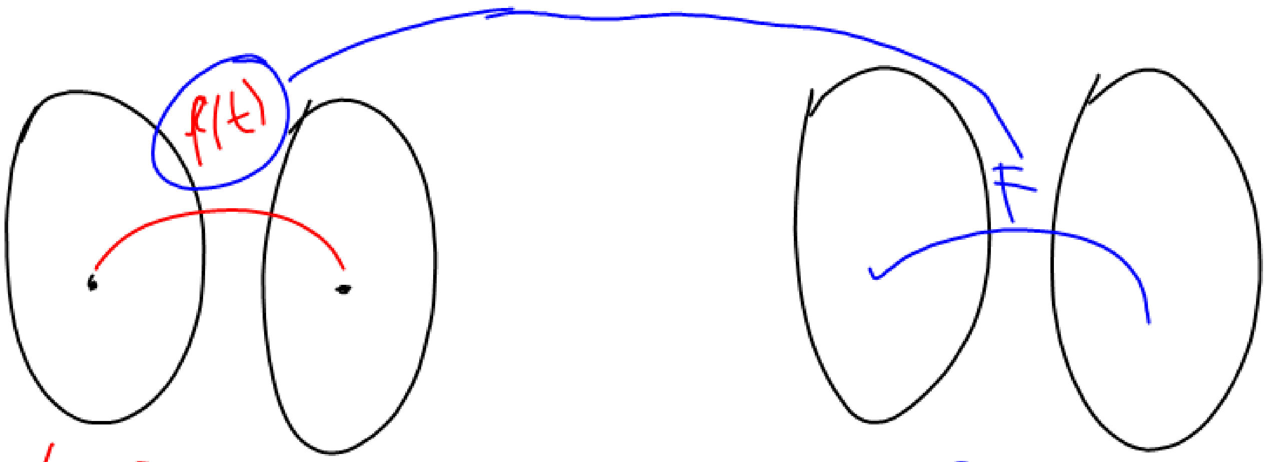


$t \subset \mathbb{D} \in \mathbb{R}$ $\quad y \subset \mathbb{C} \in \mathbb{R}$ $\quad y' \subset \mathbb{C} \in \mathbb{R}$

$\int y \subset \mathbb{C} \in \mathbb{R}$

$\mathbb{D}$	$\mathbb{C}$	C	$¢$
t	t^2	$2t$	$t^3/3$
0	0	0	0
1	1	2	1/3
2	4	4	8/3
3	9	6	9
4	16	8	64/3
5			

$$\mathcal{L}\{f(t)\} = F(s)$$



$t \in \mathbb{R}$ $\quad f \in \mathbb{R}$ $\quad s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

$$af(t) + bg(t) \longrightarrow aF(s) + bG(s)$$

$$f'(t) \xrightarrow{\checkmark} sF(s)$$

$$\int f(t) \xleftarrow{\neq} \frac{F(s)}{s}$$

$$f''(t) \xrightarrow{\checkmark} s^2F(s)$$

$$T\left\{ f(t) \right\} = \int_{-\infty}^{\infty} f(t)\, N(t,s)\, dt \Rightarrow F(s)$$

operador

argumento.

núcleo

RESULTADO

$$\underset{\text{Laplace}}{N}(t,s) = \begin{cases} t \geq 0 \ ; \ e^{-st} \\ t < 0 \ ; \ 0 \end{cases}$$

$$\boxed{\mathcal{L}\left\{ f(t) \right\} = \int_{0}^{\infty} f(t)\, e^{-st}\, dt.}$$

TRANSFORMADA DE LAPLACE.

$$f(t) = k_1$$

$$\mathcal{L}\{k_1\} = k_1 \mathcal{L}\{1\} \Rightarrow \int^{\infty} (1) e^{-st} dt$$

$$= \left[\int e^{-st} dt \right]_0^{\infty} \Rightarrow \left[-\frac{1}{s} \int -s e^{-st} dt \right]_0^{\infty}$$

$$= \frac{k_1}{-s} \left[\lim_{t \to \infty} e^{-st} - 1 \right]$$

$$\lim_{t \to \infty} e^{-st} = \lim_{t \to \infty} \frac{1}{e^{st}} \Rightarrow \lim_{b \to \infty} \frac{1}{b} = 0$$

$$\mathcal{L}\{k_1\} = \frac{k_1}{s} \longrightarrow \mathcal{L}\{1\} = \frac{1}{s}$$

$$L\{e^{at}\} = \int_0^\infty e^{at} e^{-st}\,dt$$

$$= \int_0^\infty e^{-(s-a)t}\,dt$$

$$= \frac{1}{-(s-a)} \int_0^\infty -(s-a) e^{-(s-a)t}\,dt$$

$$= \frac{1}{-(s-a)} \cdot \left(-1\right)$$

$$L\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt \Rightarrow F(s)$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\, ds$$

$$\int_{a-i\infty}^{a+i\infty} e^{st} F(s)\, ds = \lim_{b\to\infty} \int_{a-bi}^{a+bi} e^{st} F(s)\, ds.$$

propiedades de TL.

① Lineal

$$\mathcal{L}\{af(t)+bg(t)\}=aF(s)+bG(s)$$

② semejanza

$$\mathcal{L}\{f(t)\}=F(s)$$

$$\mathcal{L}\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\{e^{t}\}=\frac{1}{s-1}$$

$$\mathcal{L}\{e^{-3t}\}=\frac{1}{-3}\mathcal{L}\left\{\frac{s}{-3}\right\}$$

$$=\frac{1}{-3}\left(\frac{1}{\frac{s}{-3}-1}\right)$$

$$=\frac{1}{\cancel{-3}}\left(\frac{\cancel{-3}}{s+3}\right)$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{ad}{cb}$$

$$\mathcal{L}\{e^{-3t}\}=\frac{1}{s+3}$$

3 derivación

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2 \cdot f(0) - sf'(0) - f''(0)$$

4 Deriv. trans

$$\mathcal{L}^{-1}\{F'(s)\} = -t \cdot f(t)$$

$$\mathcal{L}^{-1}\{F''(s)\} = (-1)^2 t^2 f(t)$$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t).$$

⑤ Integral.

$$\mathcal{L}\left\{\int_0^t f(z)\,dz\right\} = \frac{F(s)}{s}$$

⑥ integral

$$\mathcal{L}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \frac{f(t)}{t}$$

7 tardanza

$$\mathcal{L}\{f(t-\tau)\} = e^{-\tau s} F(s)$$

8 desplazamiento

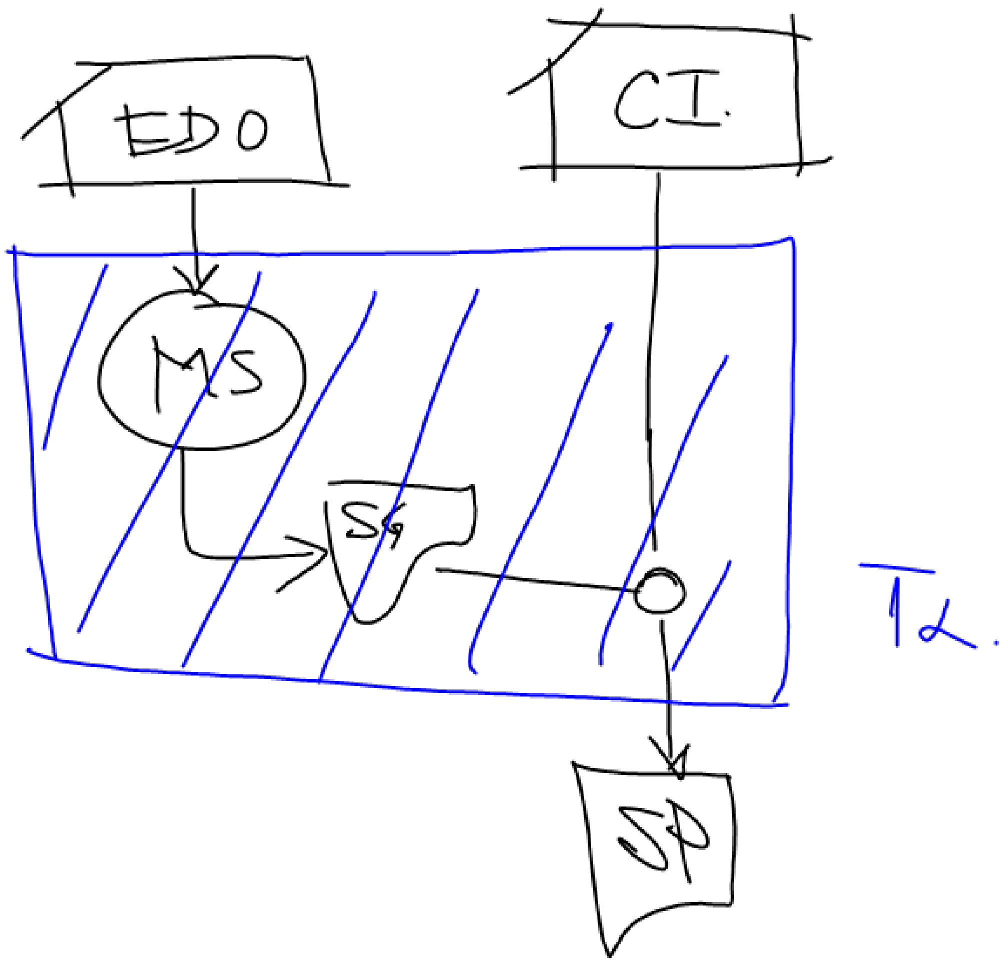
$$\mathcal{L}\{e^{\sigma t} f(t)\} = F(s-\sigma)$$

9 producto

$$\mathcal{L}^{-1}\{F(s)G(s)\} = f(t) * g(t)$$

$$f(t) * g(t) = \int_0^t f(\tau) \cdot g(t-\tau)\, d\tau.$$

convolución.


EDO
CI.
MS
Sg
SP
Tα.

$$\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = 0 \qquad \begin{matrix} y(0) = 1 \\ y'(0) = -2 \end{matrix}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 7\mathcal{L}\left\{\frac{dy}{dt}\right\} + 12\mathcal{L}\{y\} = 0$$

$$\left[s^2\mathcal{L}\{y\} - s(1) - (-2)\right] - 7\left[s\mathcal{L}\{y\} - 1\right] + 12\mathcal{L}\{y\} = 0$$

$$(s^2 - 7s + 12)\cdot\mathcal{L}\{y\} - s + 2 + 7 = 0$$

$$(s^2 - 7s + 12)\mathcal{L}\{y\} = s - 9$$

$$\mathcal{L}\{y\} = \frac{s-9}{(s^2-7s+12)}$$

$$\mathcal{L}\{y\} = \frac{s-9}{(s-3)(s-4)}$$

$$\frac{s-9}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s - 9 = A(s-4) + B(s-3)$$

$$s - 9 = (A+B)s + (-4A - 3B)$$

$$\begin{aligned} A + B &= 1 \\ -4A - 3B &= -9 \\ 4A + 4B &= 4 \\ \hline B &= -5 \end{aligned} \qquad \begin{aligned} A &= 1 - B \\ A &= 6 \end{aligned}$$

$$\mathcal{L}\{y\} = \frac{6}{s-3} - \frac{5}{s-4}$$

$$y = \mathcal{L}^{-1}\{y\} = 6\mathcal{L}^{-1}\left(\frac{1}{s-3}\right) - 5\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$\boxed{y = 6e^{3t} - 5e^{4t}}$$