convolución

$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t).$$

EJEMPLO $\quad f(t) * g(t) = \int_0^t f(z)\cdot g(t-z)\,dz.$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \mathcal{L}^{-1}\left\{\left(\frac{s}{s^2+4}\cdot\frac{1}{s^2+4}\right)\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\cdot\frac{2}{s^2+4}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} = \cos(2t)$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} = \operatorname{sen}(2t)$$

$$= \frac{1}{2}\left(\cos(2t) * \operatorname{sen}(2t)\right)$$

$$= \frac{1}{2}\int_0^t \cos(2z)\cdot\operatorname{sen}\left(2(t-z)\right)dz$$

$$= \frac{1}{2}\int_0^t \cos(2z)\left[\operatorname{sen}(2t)\cos(2z) - \operatorname{sen}(2z)\cdot\cos(2t)\right]dz$$

$$= \frac{1}{2}\left(\operatorname{sen}(2t)\int_0^t \cos^2(2z)\,dz - \cos(2t)\int_0^t \cos(2z)\operatorname{sen}(2z)\,dz\right)$$

$$= \frac{1}{2}\left(\operatorname{sen}(2t)\left(\frac{\cos(2t)\operatorname{sen}(2t)}{4} + \frac{t}{2}\right) - \cos(2t)\left(\frac{1}{4} - \frac{\cos^2(2t)}{4}\right)\right)$$

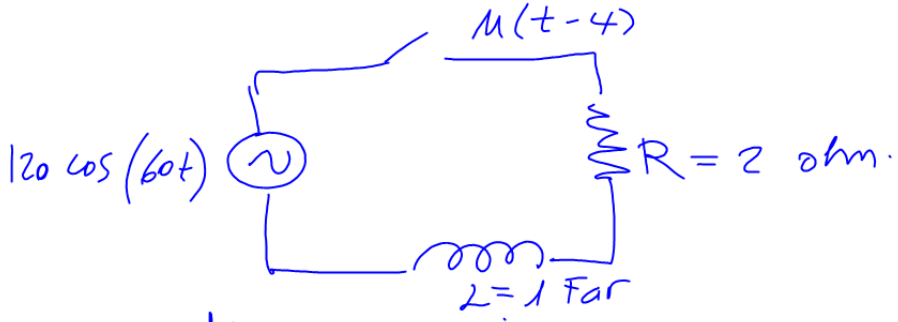
$$= \frac{1}{2}\left(\frac{\cos(2t)\operatorname{sen}^2(2t)}{4} + \frac{t}{2}\operatorname{sen}(2t)\right) + \frac{\cos^3(2t)}{4} - \frac{1}{4}\cos(2t))$$

$$= \frac{1}{2}\left(\frac{\cos(2t)}{4}\underbrace{\left(\operatorname{sen}^2(2t) + \cos^2(2t)\right)}_{1} + \frac{t}{2}\operatorname{sen}(2t) - \frac{1}{4}\cos(2t)\right)$$

$$= \frac{1}{2}\left(\cancel{\frac{\cos(2t)}{4}} + \frac{t\operatorname{sen}(2t)}{2} - \cancel{\frac{1}{4}\cos(2t)}\right)$$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \frac{1}{4}\,t\operatorname{sen}(2t)}$$

24 septiembre 2024.gwb - 2/2 - 24 sep. 2024 12:28:40



$$L\frac{di}{dt} + Ri = u(t-4)\,120\cos(60t) \qquad i(0)=0$$

$$\frac{di}{dt} + 2i = u(t-4)\,120\cos(60t)$$

$$\mathcal{L}\left\{\frac{di}{dt} + 2i\right\} = \mathcal{L}\left\{u(t-4)\,120\cos(60t)\right\}$$

$$\mathcal{L}\left\{\frac{di}{dt}\right\} + 2\mathcal{L}\{i\} = 120\left(\frac{e^{-4s}}{s^2+3600}\right)$$

$$\left(s\mathcal{L}\{i\} - i(0)\right) + 2\mathcal{L}\{i\} = \frac{120e^{-4s}}{s^2+3600}$$

$$(s+2)\mathcal{L}\{i\} = \frac{120e^{-4s}}{s^2+3600}$$

$$\mathcal{L}\{i\} = \frac{120e^{-4s}}{(s+2)(s^2+3600)}$$