

> restart

> $EcuaOriginal := x^2 \cdot y'' + x \cdot y' + \left(x^2 - \frac{1}{4}\right) \cdot y = x^{\left(\frac{3}{2}\right)}$

$$EcuaOriginal := x^2 \left(\frac{d^2}{dx^2} y(x) \right) + x \left(\frac{d}{dx} y(x) \right) + \left(x^2 - \frac{1}{4} \right) y(x) = x^{3/2} \quad (1)$$

> $EcuaNormal := \text{expand}\left(\frac{EcuaOriginal}{x^2}\right)$

$$EcuaNormal := \frac{d^2}{dx^2} y(x) + y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{4x^2} = \frac{1}{\sqrt{x}} \quad (2)$$

> $yy[1] := x^{\left(-\frac{1}{2}\right)} \cdot \cos(x); yy[2] := x^{\left(-\frac{1}{2}\right)} \cdot \sin(x)$

$$yy_1 := \frac{\cos(x)}{\sqrt{x}}$$

$$yy_2 := \frac{\sin(x)}{\sqrt{x}} \quad (3)$$

> $EcuaHom := lhs(EcuaNormal) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{4x^2} = 0 \quad (4)$$

> $ComprobarUno := \text{simplify}(\text{eval}(\text{subs}(y(x) = yy[1], EcuaHom)))$

$$ComprobarUno := 0 = 0 \quad (5)$$

> $ComprobarDos := \text{simplify}(\text{eval}(\text{subs}(y(x) = yy[2], EcuaHom)))$

$$ComprobarDos := 0 = 0 \quad (6)$$

> $EcuacionOriginalNoHom := lhs(EcuaOriginal) = x^{\left(\frac{3}{2}\right)}$

$$EcuacionOriginalNoHom := x^2 \left(\frac{d^2}{dx^2} y(x) \right) + x \left(\frac{d}{dx} y(x) \right) + \left(x^2 - \frac{1}{4} \right) y(x) = x^{3/2} \quad (7)$$

> $EcuaNoHomNormal := \text{expand}\left(\frac{EcuacionOriginalNoHom}{x^2}\right)$

$$EcuaNoHomNormal := \frac{d^2}{dx^2} y(x) + y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{4x^2} = \frac{1}{\sqrt{x}} \quad (8)$$

> $Q := rhs(EcuaNoHomNormal)$

$$Q := \frac{1}{\sqrt{x}} \quad (9)$$

> with(linalg):

> $WW := \text{wronskian}([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \frac{\cos(x)}{\sqrt{x}} & \frac{\sin(x)}{\sqrt{x}} \\ -\frac{\cos(x)}{2x^{3/2}} - \frac{\sin(x)}{\sqrt{x}} & -\frac{\sin(x)}{2x^{3/2}} + \frac{\cos(x)}{\sqrt{x}} \end{bmatrix} \quad (10)$$

> $BB := \text{array}([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{1}{\sqrt{x}} \end{bmatrix} \quad (11)$$

> $\text{ParaVar} := \text{simplify}(\text{linsolve}(WW, BB))$

$$\text{ParaVar} := \begin{bmatrix} -\sin(x) & \cos(x) \end{bmatrix} \quad (12)$$

> $\text{Aprima} := \text{ParaVar}[1]; \text{Bprima} := \text{ParaVar}[2]$

$$\text{Aprima} := -\sin(x)$$

$$\text{Bprima} := \cos(x) \quad (13)$$

> $\text{SolGral} := y(x) = \text{simplify}((\text{int}(\text{Aprima}, x) + _C1) \cdot yy[1] + (\text{int}(\text{Bprima}, x) + _C2) \cdot yy[2])$

$$\text{SolGral} := y(x) = \frac{1 + \cos(x) _C1 + \sin(x) _C2}{\sqrt{x}} \quad (14)$$

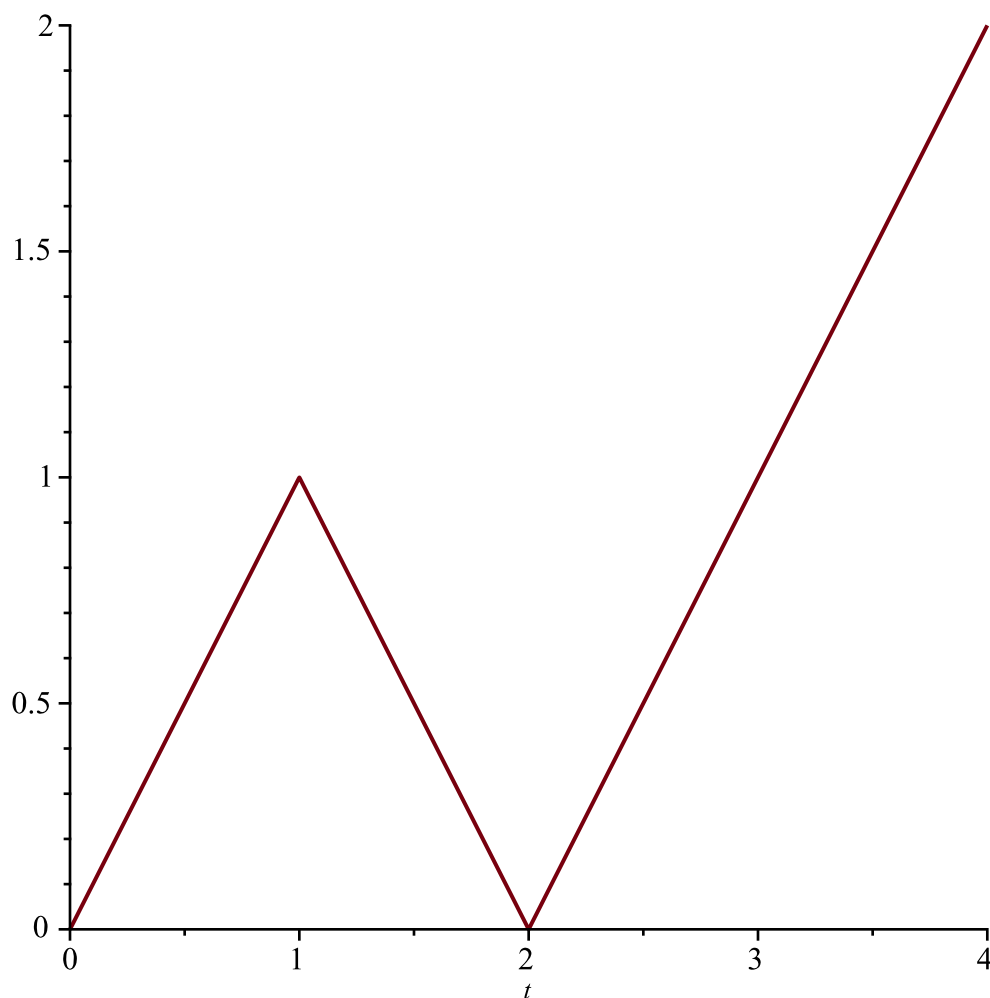
> $\text{ComprobarTres} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{EcuacionOriginalNoHom})))$

$$\text{ComprobarTres} := x^{3/2} = x^{3/2} \quad (15)$$

> restart

> $f := t \cdot \text{Heaviside}(t) - 2 \cdot (t - 1) \cdot \text{Heaviside}(t - 1) + 2 \cdot (t - 2) \cdot \text{Heaviside}(t - 2); \text{plot}(f, t = 0 \dots 4)$

$$f := t \text{Heaviside}(t) - 2 (t - 1) \text{Heaviside}(t - 1) + 2 (t - 2) \text{Heaviside}(t - 2)$$



> restart

> Ecua := diff(x(t), t\$2) + 2*x(t) = 4*Dirac(t - 2*Pi)

$$Ecua := \frac{d^2}{dt^2} x(t) + 2 x(t) = 4 \operatorname{Dirac}(t - 2 \pi) \quad (16)$$

> CondIni := x(0) = 3, D(x)(0) = 0

$$CondIni := x(0) = 3, D(x)(0) = 0 \quad (17)$$

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTL := s^2 \mathcal{L}(x(t), t, s) - 3 s + 2 \mathcal{L}(x(t), t, s) = 4 e^{-2 s \pi} \quad (18)$$

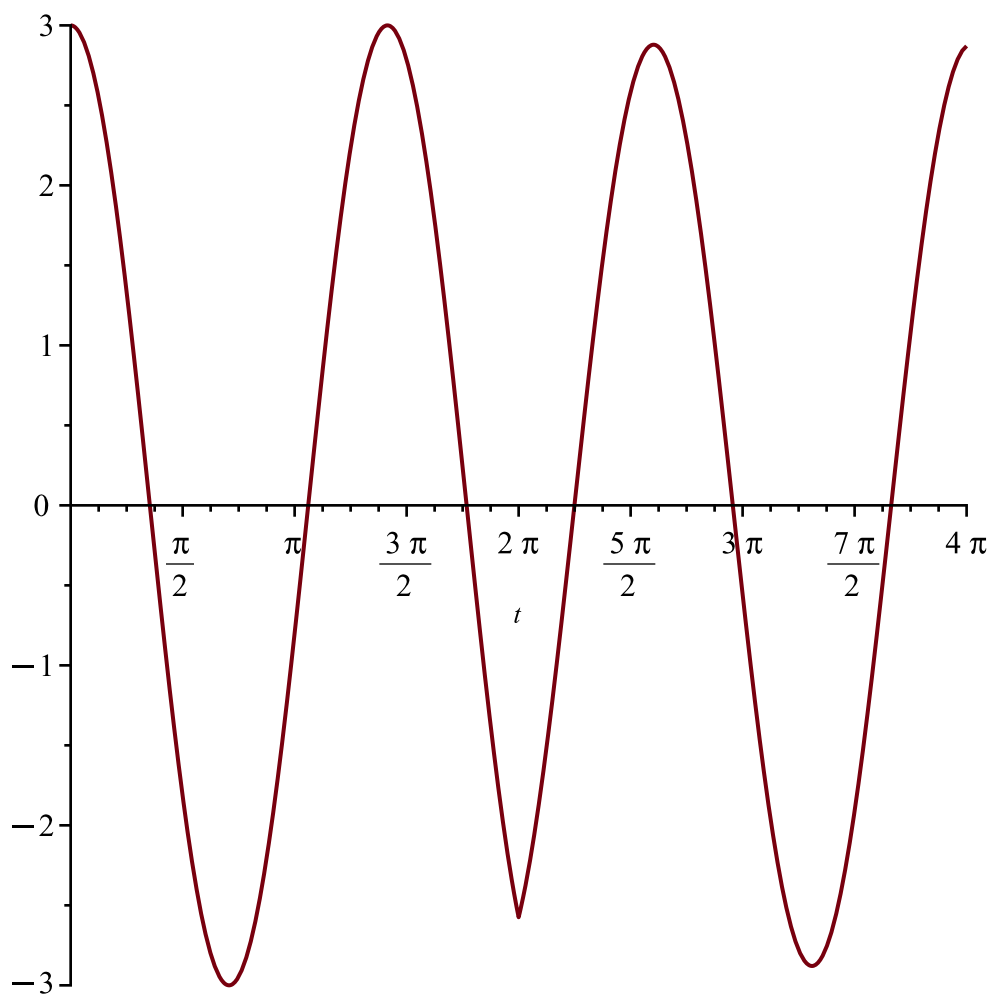
> SolPartTL := isolate(EcuaTL, laplace(x(t), t, s))

$$SolPartTL := \mathcal{L}(x(t), t, s) = \frac{4 e^{-2 s \pi} + 3 s}{s^2 + 2} \quad (19)$$

> SolPart := invlaplace(SolPartTL, s, t)

$$SolPart := x(t) = 2 \operatorname{Heaviside}(t - 2 \pi) \sin(\sqrt{2} (t - 2 \pi)) \sqrt{2} + 3 \cos(\sqrt{2} t) \quad (20)$$

> plot(rhs(SolPart), t = 0 .. 4*Pi)



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> Comprobar := simplify(eval(subs(x(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))
Comprobar := 0 = 0 (21)
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> restart
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> Sistema := diff(x[1](t), t) = x[2](t), diff(x[2](t), t) = -9·x[1](t) + sin(2 t) : Sistema[1];
Sistema[2]
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$$\frac{d}{dt} x_1(t) = x_2(t)$$

$$\frac{d}{dt} x_2(t) = -9 x_1(t) + \sin(2 t) \quad (22)$$

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> Xcero := array([0, 0])
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$$Xcero := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (23)$$

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> BB := array([0, sin(2·t)])
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$$BB := \begin{bmatrix} 0 & \sin(2 t) \end{bmatrix} \quad (24)$$

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> AA := array([[0, 1], [-9, 0]])
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$$AA := \begin{bmatrix} 0 & 1 \\ -9 & 0 \end{bmatrix} \quad (25)$$

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> with(linalg) :
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> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \cos(3 t) & \frac{\sin(3 t)}{3} \\ -3 \sin(3 t) & \cos(3 t) \end{bmatrix} \quad (26)$$

> $MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} \cos(3 t - 3 \tau) & \frac{\sin(3 t - 3 \tau)}{3} \\ -3 \sin(3 t - 3 \tau) & \cos(3 t - 3 \tau) \end{bmatrix} \quad (27)$$

> $BBtau := map(rcurry(eval, t='tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & \sin(2 \tau) \end{bmatrix} \quad (28)$$

> $ProdTau := evalm(MatExpTau \&* BBtau) : ProdTau[1]; ProdTau[2]$

$$\frac{\sin(3 t - 3 \tau) \sin(2 \tau)}{3} \quad (29)$$

$$\cos(3 t - 3 \tau) \sin(2 \tau)$$

>

> $SolFinalHom := evalm(MatExp \&* Xcero) : x[1](t) = SolFinalHom[1]; x[2](t) = SolFinalHom[2]$

$$x_1(t) = 0$$

$$x_2(t) = 0 \quad (30)$$

> $SolFinalNoHom := map(int, ProdTau, tau = 0 .. t) : x[1](t) = SolFinalNoHom[1]; x[2](t) = SolFinalNoHom[2]$

$$x_1(t) = -\frac{2 \sin(3 t)}{15} + \frac{\sin(2 t)}{5}$$

$$x_2(t) = -\frac{2 \cos(3 t)}{5} + \frac{2 \cos(2 t)}{5} \quad (31)$$

> $EcuaOriginal := diff(x(t), t$2) + 9 \cdot x(t) = \sin(2 t)$

$$EcuaOriginal := \frac{d^2}{dt^2} x(t) + 9 x(t) = \sin(2 t) \quad (32)$$

> $ComprobarOriginal := simplify(eval(subs(x(t) = SolFinalNoHom[1], EcuaOriginal)))$

$$ComprobarOriginal := \sin(2 t) = \sin(2 t) \quad (33)$$

> $ComprobarDos := diff(SolFinalNoHom[1], t) = SolFinalNoHom[2]$

$$ComprobarDos := -\frac{2 \cos(3 t)}{5} + \frac{2 \cos(2 t)}{5} = -\frac{2 \cos(3 t)}{5} + \frac{2 \cos(2 t)}{5} \quad (34)$$

>