> $restart$

> $f := \exp(2\,x)$

$$f := \mathrm{e}^{2\,x} \tag{1}$$

> $L := 2$

$$L := 2 \tag{2}$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L);\ evalf(\%, 3)$

$$a_0 := -\frac{\mathrm{e}^{-4}}{4} + \frac{\mathrm{e}^{4}}{4}$$

$$13.6 \tag{3}$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$a_n := \frac{4\,\mathrm{e}^{4}\,(-1)^n - 4\,\mathrm{e}^{-4}\,(-1)^n}{n^2\,\pi^2 + 16} \tag{4}$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

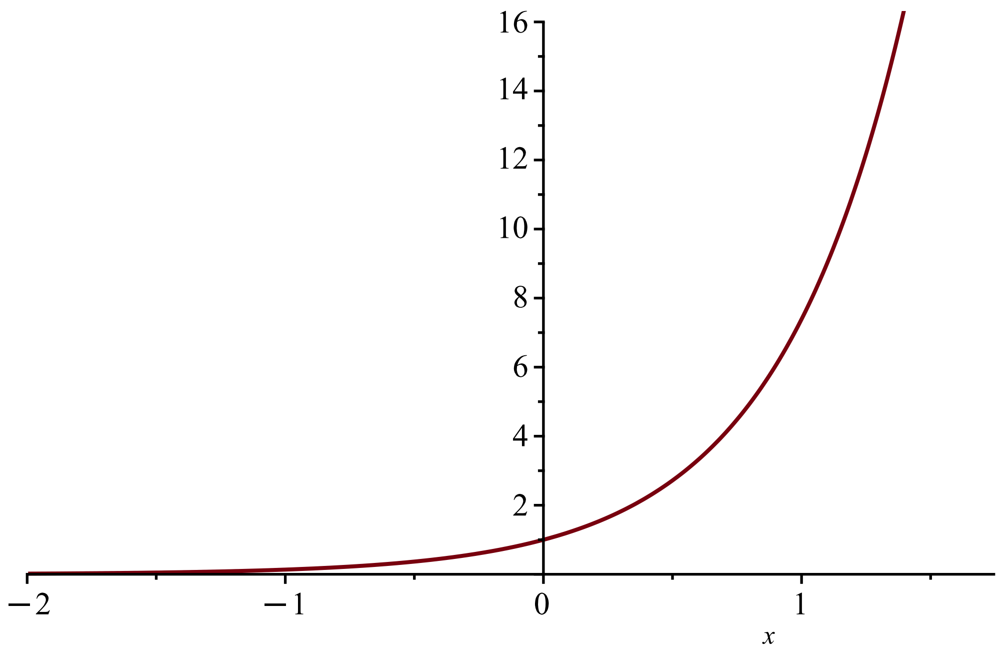
$$b_n := \frac{-\mathrm{e}^{4}\,(-1)^n\,\pi\,n + \mathrm{e}^{-4}\,(-1)^n\,\pi\,n}{n^2\,\pi^2 + 16} \tag{5}$$

> $STF := \frac{a[0]}{2} + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..\mathrm{infinity}\right)$

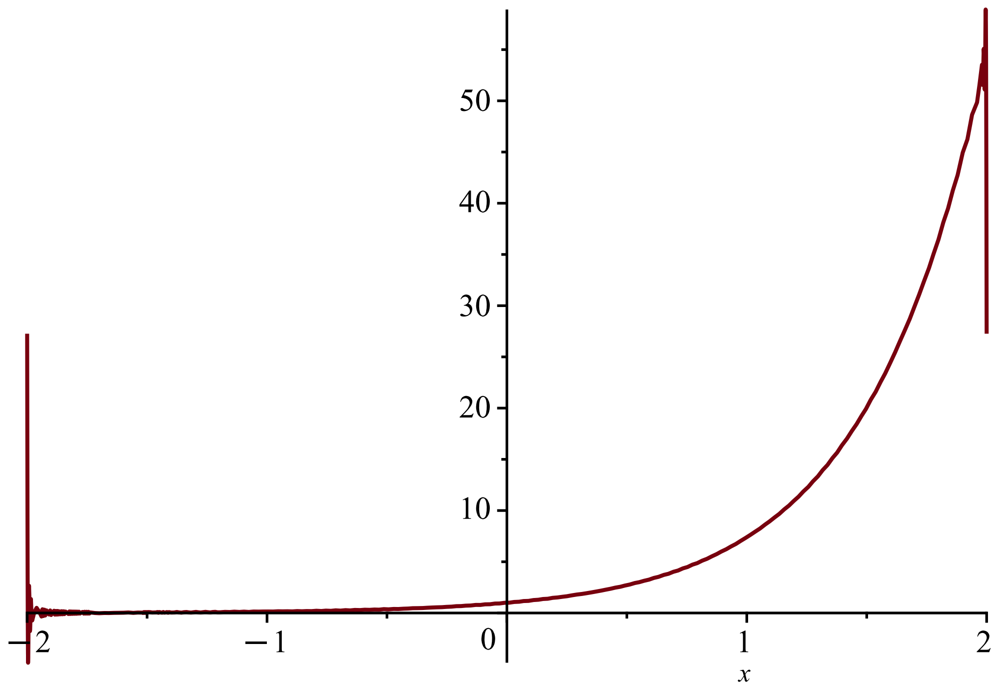
$$STF := -\frac{\mathrm{e}^{-4}}{8} + \frac{\mathrm{e}^{4}}{8} + \sum_{n=1}^{\infty} \left( \frac{\left(4\,\mathrm{e}^{4}\,(-1)^n - 4\,\mathrm{e}^{-4}\,(-1)^n\right) \cos\left(\frac{n\,\pi\,x}{2}\right)}{n^2\,\pi^2 + 16} + \frac{\left(-\mathrm{e}^{4}\,(-1)^n\,\pi\,n + \mathrm{e}^{-4}\,(-1)^n\,\pi\,n\right) \sin\left(\frac{n\,\pi\,x}{2}\right)}{n^2\,\pi^2 + 16} \right) \tag{6}$$

> $STF500 := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..500\right):$

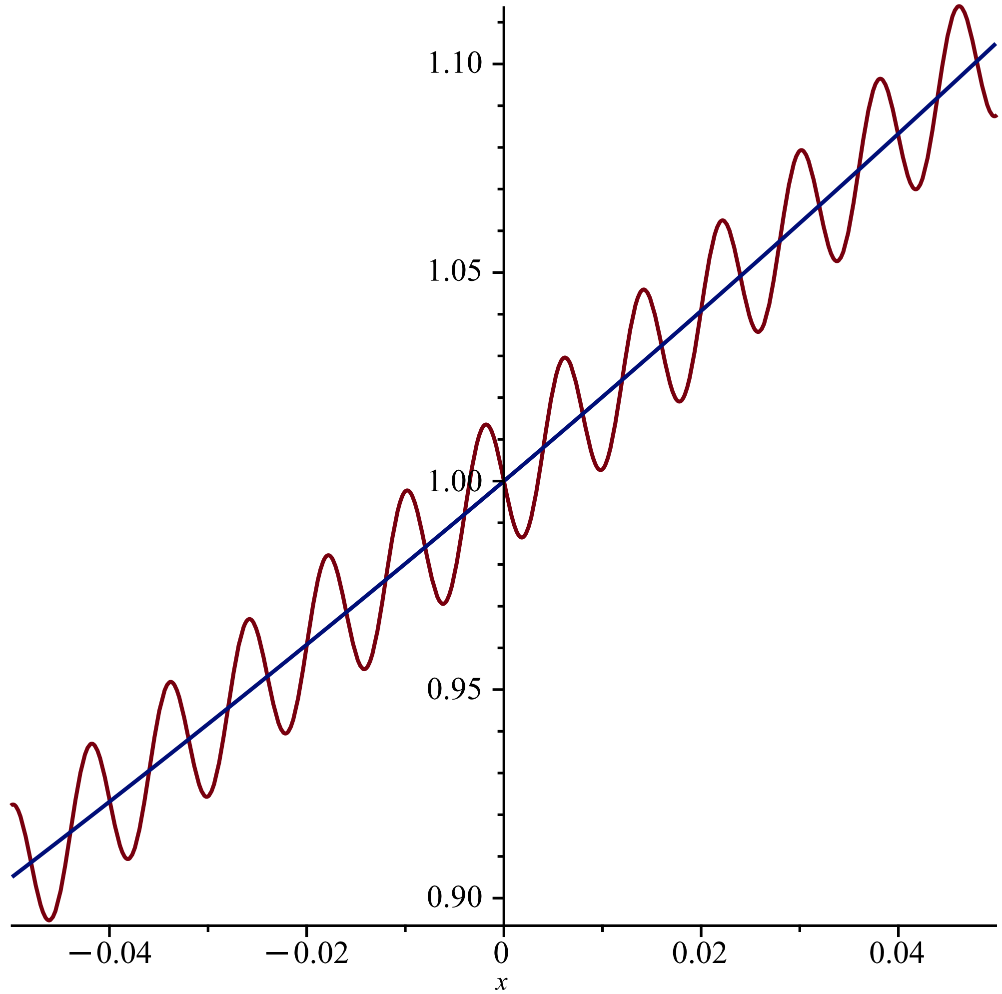
> $plot(f, x = -L..L)$




> $plot(STF500, x = -L..L)$

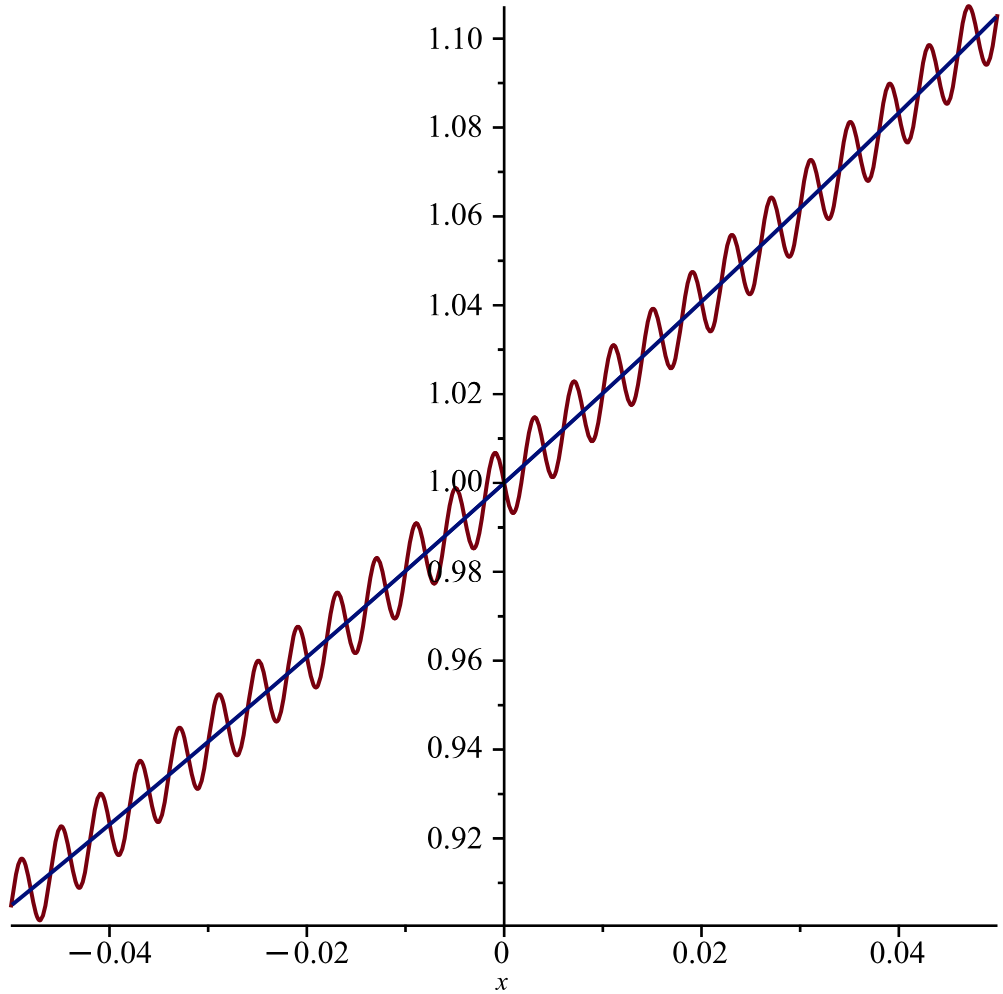



> $plot(\{f, STF500\}, x = -0.05..0.05)$




> $STF1000 := \frac{a[0]}{2} + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1\,..1000\right):$

> $plot(\{f, STF1000\}, x = -0.05\,..0.05)$




> *restart*

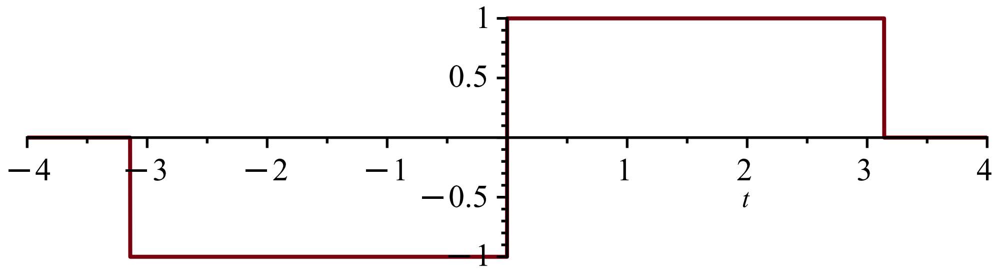
> $g := -\text{Heaviside}(t + \text{Pi}) + 2\cdot\text{Heaviside}(t) - \text{Heaviside}(t - \text{Pi})$

$$g := -\text{Heaviside}(t + \pi) + 2\,\text{Heaviside}(t) - \text{Heaviside}(t - \pi) \qquad \textbf{(7)}$$

> $L := 4$

$$L := 4 \qquad \textbf{(8)}$$

> $plot(g, t = -L..L, scaling = CONSTRAINED)$




> $a[0] := \frac{1}{L} \cdot int(g, t = -L..L)$

$$a_0 := 0 \tag{9}$$

> $a[n] := \frac{1}{L} \cdot int\left(g \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$a_n := 0 \tag{10}$$

> $b[n] := \frac{1}{L} \cdot int\left(g \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

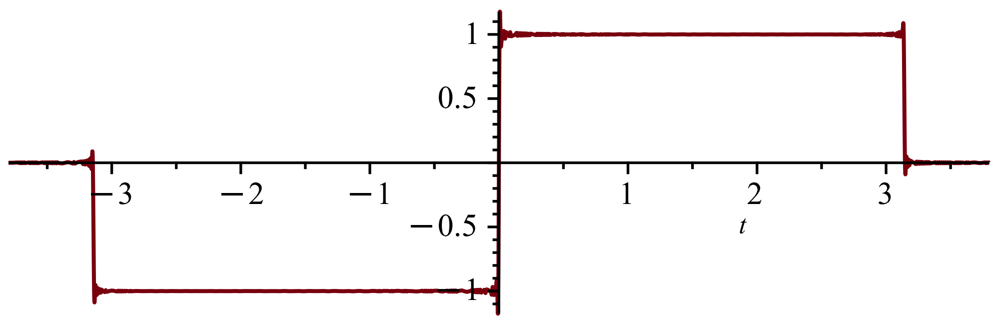
$$b_n := \frac{4 - 4\cos\left(\frac{n\,\pi^2}{4}\right)}{4\,n\,\pi} + \frac{-4 + 4\cos\left(\frac{n\,\pi^2}{4}\right)}{4\,n\,\pi} + \frac{8 - 8\cos\left(\frac{n\,\pi^2}{4}\right)}{4\,n\,\pi} \tag{11}$$

> $STFg := Sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..\mathrm{infinity}\right)$

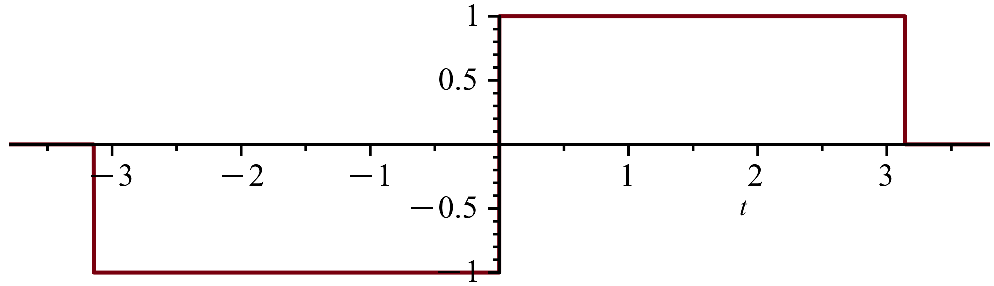
$$STFg := \sum_{n=1}^{\infty} \left(\frac{4 - 4\cos\left(\frac{n\,\pi^2}{4}\right)}{4\,n\,\pi} + \frac{-4 + 4\cos\left(\frac{n\,\pi^2}{4}\right)}{4\,n\,\pi} + \frac{8 - 8\cos\left(\frac{n\,\pi^2}{4}\right)}{4\,n\,\pi}\right) \sin\left(\frac{n\,\pi\,t}{4}\right) \tag{12}$$

> $STFg500 := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..500\right) :$

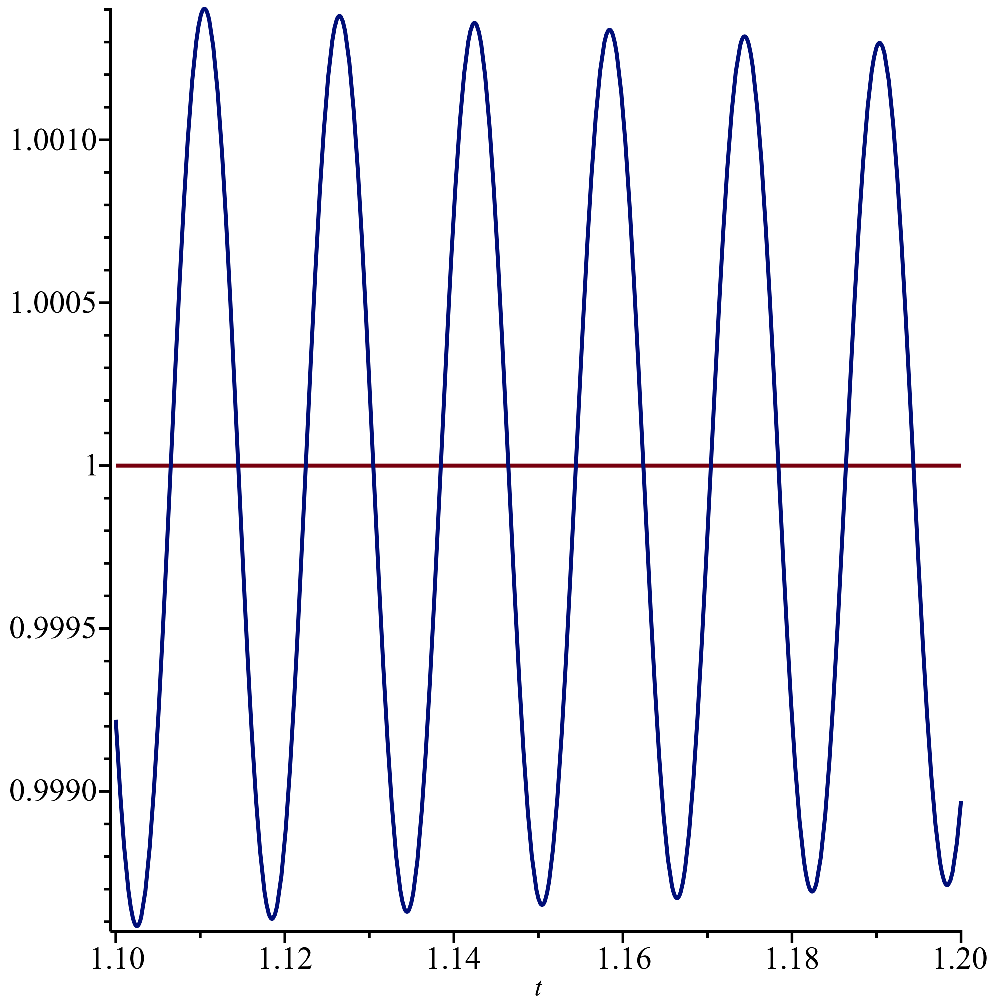
> $plot(STFg500, t = -3.8..3.8, scaling = CONSTRAINED)$



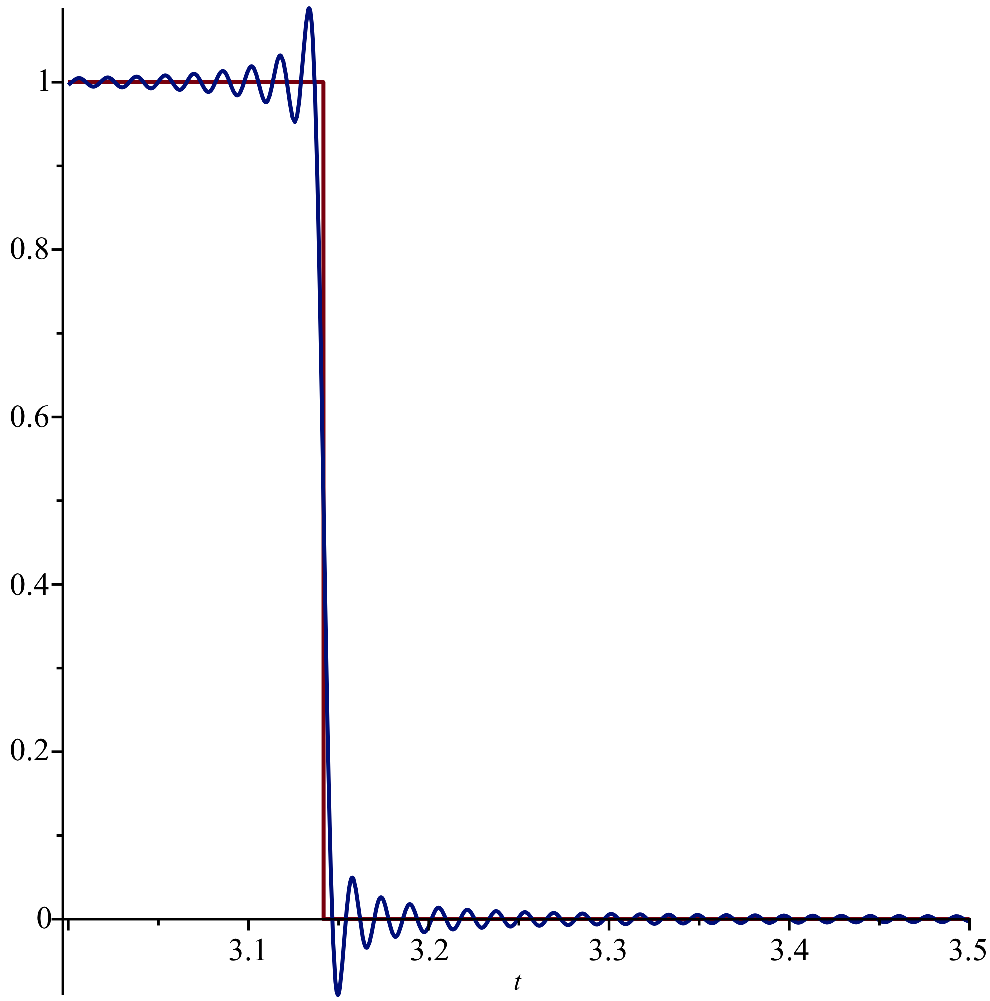

> $plot(g, t = -3.8 .. 3.8, scaling = CONSTRAINED)$




> $plot(\{g, STFg500\}, t = 1.1\,..1.2)$




> $plot(\{g, STFg500\}, t = 3 ..3.5)$




>

Método por Variables Separables

> *restart*

> *Ecua* := *diff*(z(x, y), x$2) + *diff*(z(x, y), x$2, y) − *diff*(z(x, y), y$2) = 0

$$Ecua := \frac{\partial^2}{\partial x^2}\, z(x,y) + \frac{\partial^3}{\partial x^2 \partial y}\, z(x,y) - \frac{\partial^2}{\partial y^2}\, z(x,y) = 0 \tag{13}$$

> *Sol* := *pdsolve*(*Ecua*)

$$Sol := z(x,y) = f_1(x)\, f_2(y) \ \mathbf{where} \left[ \left\{ \frac{d^2}{dx^2} f_1(x) = \_c_1 f_1(x),\ \frac{d^2}{dy^2} f_2(y) = \left( f_2(y) + \frac{d}{dy} f_2(y) \right) \_c_1 \right\} \right] \tag{14}$$

> *with*(*PDEtools*) :

> *SolGral* := *build*(*Sol*)

$$SolGral := z(x,y) = c_3\, e^{\frac{y\,\_c_1}{2}}\, e^{\frac{y\sqrt{\_c_1^2 + 4\,\_c_1}}{2}}\, c_1\, e^{\sqrt{\_c_1}\, x} + \frac{c_3\, e^{\frac{y\,\_c_1}{2}}\, e^{\frac{y\sqrt{\_c_1^2 + 4\,\_c_1}}{2}}\, c_2}{e^{\sqrt{\_c_1}\, x}} \tag{15}$$

$$+ c_4 e^{\frac{y\_c_1}{2}} e^{-\frac{y\sqrt{\_c_1^2+4\_c_1}}{2}} c_1 e^{\sqrt{\_c_1}\,x} + \frac{c_4 e^{\frac{y\_c_1}{2}} e^{-\frac{y\sqrt{\_c_1^2+4\_c_1}}{2}} c_2}{e^{\sqrt{\_c_1}\,x}}$$

> *Comprobar* := *simplify*(*eval*(*subs*(*z*(*x*, *y*) = *rhs*(*SolGral*), *Ecua*)))

$$Comprobar := 0 = 0 \tag{16}$$

> *Ecua*

$$\frac{\partial^2}{\partial x^2} z(x,y) + \frac{\partial^3}{\partial x^2 \partial y} z(x,y) - \frac{\partial^2}{\partial y^2} z(x,y) = 0 \tag{17}$$

> *SolDos* := *z*(*x*, *y*) = *F*(*x*)·*G*(*y*)

$$SolDos := z(x,y) = F(x)\,G(y) \tag{18}$$

> *EcuaDos* := *eval*(*subs*(*z*(*x*, *y*) = *rhs*(*SolDos*), *Ecua*))

$$EcuaDos := \left(\frac{d^2}{dx^2} F(x)\right) G(y) + \left(\frac{d^2}{dx^2} F(x)\right)\left(\frac{d}{dy} G(y)\right) - F(x)\left(\frac{d^2}{dy^2} G(y)\right) = 0 \tag{19}$$

> *EcuaTres* := *lhs*(*EcuaDos*) + $F(x)\left(\frac{d^2}{dy^2} G(y)\right)$ = *rhs*(*EcuaDos*) + $F(x)\left(\frac{d^2}{dy^2} G(y)\right)$

$$EcuaTres := \left(\frac{d^2}{dx^2} F(x)\right) G(y) + \left(\frac{d^2}{dx^2} F(x)\right)\left(\frac{d}{dy} G(y)\right) = F(x)\left(\frac{d^2}{dy^2} G(y)\right) \tag{20}$$

> *EcuaCuatro* := *simplify*$\left(\frac{lhs(EcuaTres)}{F(x)\cdot(G(y) + diff(G(y),y))} = \frac{rhs(EcuaTres)}{F(x)\cdot(G(y) + diff(G(y),y))}\right)$

$$EcuaCuatro := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d^2}{dy^2} G(y)}{G(y) + \frac{d}{dy} G(y)} \tag{21}$$

para alpha igual a cero

> *EcuaCeroX* := *lhs*(*EcuaCuatro*) = 0

$$EcuaCeroX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \tag{22}$$

> *EcuaCeroY* := *rhs*(*EcuaCuatro*) = 0

$$EcuaCeroY := \frac{\frac{d^2}{dy^2} G(y)}{G(y) + \frac{d}{dy} G(y)} = 0 \tag{23}$$

> *SolGralCeroX* := *dsolve*(*EcuaCeroX*)

$$SolGralCeroX := F(x) = c_1 x + c_2 \tag{24}$$

> *SolGralCeroY* := *dsolve*(*EcuaCeroY*)

$$SolGralCeroY := G(y) = c_1 y + c_2 \tag{25}$$

> $SolGralCero := z(x,y) = subs\left(c_1 = c_{11}, c_2 = c_{21}, rhs(SolGralCeroX)\right) \cdot subs\left(c_1 = c_{21}, c_2 = c_{22}, rhs(SolGralCeroY)\right)$

$$SolGralCero := z(x,y) = \left(c_{11}\,x + c_{21}\right)\,\left(c_{21}\,y + c_{22}\right) \quad (26)$$

> $Ecua$

$$\frac{\partial^2}{\partial x^2}\,z(x,y) + \frac{\partial^3}{\partial x^2\,\partial y}\,z(x,y) - \frac{\partial^2}{\partial y^2}\,z(x,y) = 0 \quad (27)$$

> $ComprobarCero := eval(subs(z(x,y) = rhs(SolGralCero), Ecua))$

$$ComprobarCero := 0 = 0 \quad (28)$$

para alpha positiva

> $EcuaPosX := lhs(EcuaCuatro) = \beta^2$

$$EcuaPosX := \frac{\frac{d^2}{dx^2}\,F(x)}{F(x)} = \beta^2 \quad (29)$$

> $EcuaPosY := rhs(EcuaCuatro) = \beta^2$

$$EcuaPosY := \frac{\frac{d^2}{dy^2}\,G(y)}{G(y) + \frac{d}{dy}\,G(y)} = \beta^2 \quad (30)$$

> $SolGralPosX := dsolve(EcuaPosX)$

$$SolGralPosX := F(x) = c_1\,e^{-\beta x} + c_2\,e^{\beta x} \quad (31)$$

> $SolGralPosY := dsolve(EcuaPosY)$

$$SolGralPosY := G(y) = c_1\,e^{\frac{\left(\beta+\sqrt{\beta^2+4}\right)\beta y}{2}} + c_2\,e^{-\frac{\left(-\beta+\sqrt{\beta^2+4}\right)\beta y}{2}} \quad (32)$$

> $SolGralPos := z(x,y) = subs\left(c_1 = c_{11}, c_2 = c_{21}, rhs(SolGralPosX)\right) \cdot subs\left(c_1 = c_{21}, c_2 = c_{22}, rhs(SolGralPosY)\right)$

$$SolGralPos := z(x,y) = \left(c_{11}\,e^{-\beta x} + c_{21}\,e^{\beta x}\right)\left(c_{21}\,e^{\frac{\left(\beta+\sqrt{\beta^2+4}\right)\beta y}{2}} + c_{22}\,e^{-\frac{\left(-\beta+\sqrt{\beta^2+4}\right)\beta y}{2}}\right) \quad (33)$$

> $ComprobarPos := simplify(eval(subs(z(x,y) = rhs(SolGralPos), Ecua)))$

$$ComprobarPos := 0 = 0 \quad (34)$$

>

para alpha negativa

> $EcuaNegX := lhs(EcuaCuatro) = -\beta^2$

$$EcuaNegX := \frac{\frac{d^2}{dx^2}\,F(x)}{F(x)} = -\beta^2 \quad (35)$$

> $EcuaNegY := rhs(EcuaCuatro) = -\beta^2$

$$EcuaNegY := \frac{\frac{d^2}{dy^2} G(y)}{G(y) + \frac{d}{dy} G(y)} = -\beta^2 \tag{36}$$

> $SolGralNegX := dsolve(EcuaNegX)$

$$SolGralNegX := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \tag{37}$$

> $SolGralNegY := dsolve(EcuaNegY)$

$$SolGralNegY := G(y) = c_1 e^{\frac{\left(-\beta + \sqrt{\beta^2 - 4}\right)\beta y}{2}} + c_2 e^{-\frac{\left(\beta + \sqrt{\beta^2 - 4}\right)\beta y}{2}} \tag{38}$$

> $SolGralNeg := z(x, y) = subs(c_1 = c_{11}, c_2 = c_{21}, rhs(SolGralNegX)) \cdot subs(c_1 = c_{21}, c_2 = c_{22}, rhs(SolGralNegY))$

$$SolGralNeg := z(x, y) = \left(c_{11} \sin(\beta x) + c_{21} \cos(\beta x)\right) \left(c_{21} e^{\frac{\left(-\beta + \sqrt{\beta^2 - 4}\right)\beta y}{2}} + c_{22} e^{-\frac{\left(\beta + \sqrt{\beta^2 - 4}\right)\beta y}{2}}\right) \tag{39}$$

> $ComprobarNeg := simplify(eval(subs(z(x, y) = rhs(SolGralNeg), Ecua)))$

$$ComprobarNeg := 0 = 0 \tag{40}$$

> restart

>

>