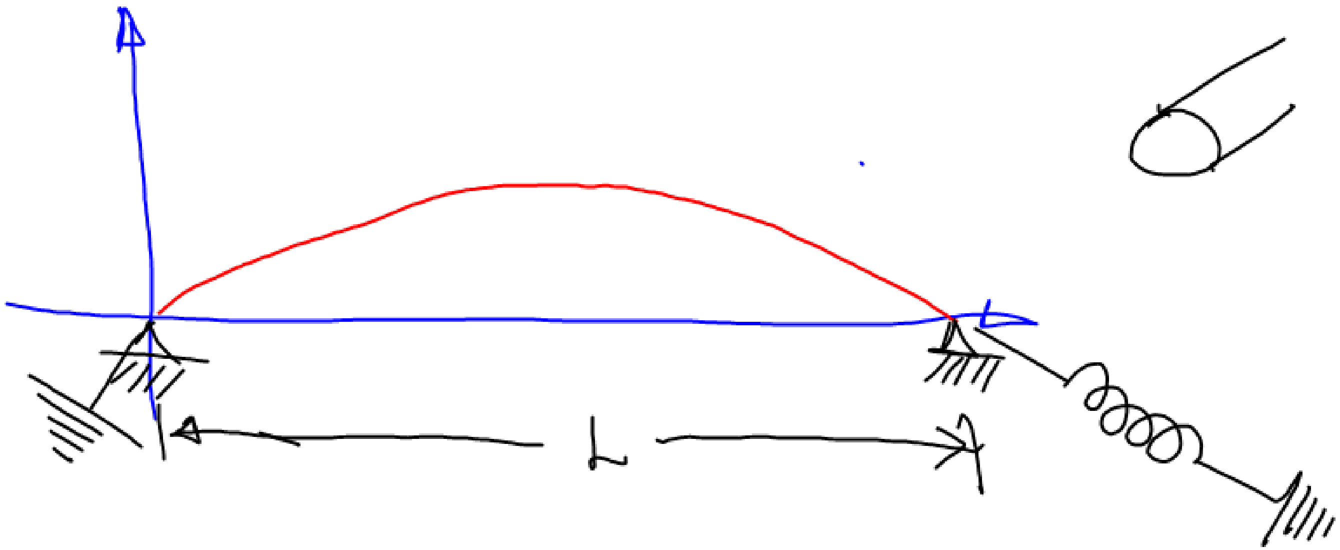
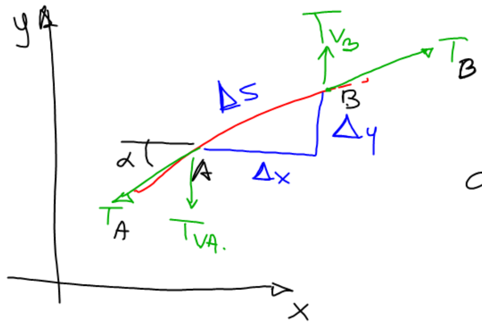
Ejemplo: una cuerda de guitarra





$$F_r = ma$$

$$u(x,t)$$

$$a = \frac{d^2u}{dt^2}$$

$$m = \rho \cdot \Delta s$$

$$F_r = \rho \cdot \Delta s \cdot \frac{d^2u}{dt^2}$$

$$F_r = T_{V_B} - T_{V_A}$$

$$T_{VA} = T\,\mathrm{sen}(\alpha) \doteq T\frac{\Delta u}{\Delta x}$$

$$T_{VB} = T\frac{\Delta u}{\Delta x} + \frac{\partial}{\partial x}\left(T\frac{\partial u}{\partial x}\right)\Delta x$$

$$= T\frac{\Delta u}{\Delta x} + T\frac{\partial^2 u}{\partial x^2}\Delta x$$

$$T_R = T_{VB} - T_{VA}$$

$$T_R = \left(\cancel{T\frac{\Delta u}{\Delta x}} + T\frac{\partial^2 u}{\partial x^2}\Delta x\right) - \cancel{T\frac{\Delta u}{\Delta x}}$$

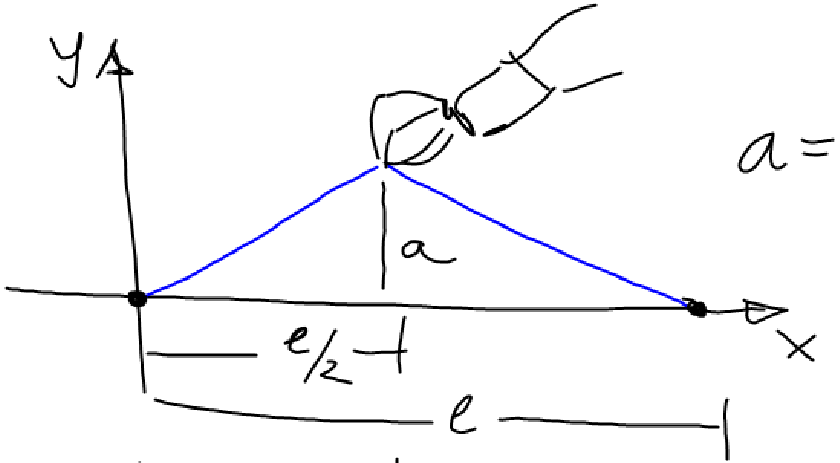
$$T_R = T\frac{\partial^2 u}{\partial x^2}\Delta x$$

$$T\frac{\partial^2 u}{\partial x^2}\Delta x = \rho\,\Delta s\,\frac{\partial^2 u}{\partial t^2}$$

$$\lim_{\Delta s \to 0} \Delta s = \Delta x$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2} \qquad c^2 = \frac{\rho}{T}$$

$$\boxed{\frac{\partial^2 u(x,t)}{\partial x^2} = c^2\frac{\partial^2 u(x,t)}{\partial t^2}}$$



$a = \frac{1}{100} \qquad \ell = 1$

condiciones de frontera $\begin{cases} y(0,t) = 0 \\ y(\ell,t) = 0 \end{cases}$

Condicion inicial $\begin{cases} y(x,0) = \begin{cases} \frac{2a}{\ell}x \ ; & 0 \le x \le \ell/2 \\ 2a - \frac{2a}{\ell}x \ ; & \ell/2 \le x \le \ell \end{cases} \\ \frac{\partial y}{\partial t}(x,0) = 0 \end{cases}$

$$y(x,t) = \sum_{n=1}^{\infty} \left(\operatorname{sen}(n\pi x)\right)\left(b_n \cos(cn\pi t) + a_n \operatorname{sen}(cn\pi t)\right)$$

$$y(x,0) = \sum_{n=1}^{\infty} \left(\operatorname{sen}(n\pi x)\, b_n\right)$$

$$F(x)_{\alpha=0} = C_1 x + C_2$$

$$CF \begin{cases} y(0,t) = 0 \\ y(1,t) = 0 \end{cases}$$

$$F(0) \Rightarrow C_1 \cdot (0) + C_2 = 0 \qquad \boxed{C_2 = 0}$$

$$F(1) \Rightarrow C_1 \cdot (1) = 0 \qquad \boxed{C_1 = 0}$$

$$F(x)_{\alpha=0} = 0$$

$$\text{Sol Pos X} = F(x) = C_1 e^{-\frac{\beta}{c}x} + C_2 e^{\frac{\beta}{c}x}$$

$$y(0,t) = 0 \qquad F(0) = C_1(1) + C_2(1) = 0$$

$$F(x) = C_1 e^{-\frac{\beta}{c}x} - C_1 e^{\frac{\beta}{c}x} \qquad C_1 = -C_2$$

$$= \frac{C_1}{e^{\frac{\beta}{c}x}} - C_1 e^{\frac{\beta}{c}x}$$

$$y(1,0) \Rightarrow F(1) = \frac{C_1}{e^{\frac{\beta}{c}}} - C_1 e^{\frac{\beta}{c}} = 0$$

$$\frac{C_1}{e^{\frac{\beta}{c}}} = C_1 e^{\frac{\beta}{c}}$$

$$C_1 = C_1 \left(e^{\frac{\beta}{c}}\right)^2$$

$$\alpha > 0$$

$$F(x)_{\alpha<0} = C_1 \cos\left(\frac{\beta}{c}x\right) + C_2 \operatorname{sen}\left(\frac{\beta}{c}x\right)$$

$$F(0) = C_1 \cos(0) + C_2 (0) = 0 \quad \boxed{C_1 = 0}$$

$$F(x) = C_2 \operatorname{sen}\left(\frac{\beta}{c}x\right)$$

$$F(1) = C_2 \operatorname{sen}\left(\frac{\beta}{c}\right) = 0 \quad \text{si } \beta = cn\pi$$

$$\alpha = -c^2 n^2 \pi^2 \quad C_2 \neq 0$$

$$F(x_1) = C_2 \operatorname{sen}\left(\frac{cn\pi}{e}x\right)$$

$$G(t) = C_1 \operatorname{sen}\left(\frac{cn\pi}{e}t\right) + C_2 \cos\left(\frac{cn\pi}{e}t\right).$$