> $restart$

> $Ecua := diff(y(x,t), t\$2) = c^2 \cdot diff(y(x,t), x\$2)$

$$Ecua := \frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t)\right) \tag{1}$$

> $EcuaUno := subs(c^2 = 1, Ecua)$

$$EcuaUno := \frac{\partial^2}{\partial t^2} y(x,t) = \frac{\partial^2}{\partial x^2} y(x,t) \tag{2}$$

> $EcuaDos := eval(subs(y(x,t) = F(x) \cdot G(t), EcuaUno))$

$$EcuaDos := F(x)\left(\frac{d^2}{dt^2} G(t)\right) = \left(\frac{d^2}{dx^2} F(x)\right) G(t) \tag{3}$$

> $EcuaSeparada := \frac{EcuaDos}{F(x) \cdot G(t)}$

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \tag{4}$$

> $EcuaT := lhs(EcuaSeparada) = \mathrm{alpha}$

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \tag{5}$$

> $EcuaX := rhs(EcuaSeparada) = \mathrm{alpha}$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \tag{6}$$

> $CondFront := y(0,t) = 0, y(1,t) = 0 : CondFront[1]; CondFront[2]$

$$y(0,t) = 0$$

$$y(1,t) = 0 \tag{7}$$

> $CondIni := y(0,0) = \frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)} \cdot x, y(1,0) = \left(\frac{2}{100}\right) - \frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)} \cdot x : CondIni[1]; CondIni[2]$

$$y(0,0) = \frac{x}{50}$$

$$y(1,0) = \frac{1}{50} - \frac{x}{50} \tag{8}$$

> $ComprobarUno := subs(x = 0, CondIni[1])$

$$ComprobarUno := y(0,0) = 0 \tag{9}$$

> $ComprobarDos := subs(x = 1, CondIni[2])$

$$ComprobarDos := y(1,0) = 0 \tag{10}$$

> $CondIniDos := subs(t = 0, diff(y(x,t), t)) = 0$

$$CondIniDos := diff(y(x,0), 0) = 0 \tag{11}$$

> $EcuaX$

$$\frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \tag{12}$$

> $EcuaT$

$$\frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \tag{13}$$

> $EcuaCeroX := subs(\mathrm{alpha} = 0, EcuaX)$

$$EcuaCeroX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \tag{14}$$

> $SolCeroX := dsolve(EcuaCeroX)$

$$SolCeroX := F(x) = c_1 x + c_2 \tag{15}$$

> $ComprobarUno := subs(x = 0, rhs(SolCeroX) = 0)$

$$ComprobarUno := c_2 = 0 \tag{16}$$

> $ComprobarDos := subs(x = 1, c_2 = 0, rhs(SolCeroX) = 0)$

$$ComprobarDos := c_1 = 0 \tag{17}$$

Por lo tanto, la solución general para alpha igual a cero no cumple las condiciones de frontera

> $EcuaPosX := subs(\mathrm{alpha} = \beta^2, EcuaX)$

$$EcuaPosX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \tag{18}$$

> $SolPosX := dsolve(EcuaPosX)$

$$SolPosX := F(x) = c_1 e^{\beta x} + c_2 e^{-\beta x} \tag{19}$$

> $SistemaPos := eval(subs(x = 0, rhs(SolPosX) = 0)), subs(x = 1, rhs(SolPosX) = 0) :$
> $SistemaPos[1]; SistemaPos[2]$

$$c_1 + c_2 = 0$$

$$c_1 e^{\beta} + c_2 e^{-\beta} = 0 \tag{20}$$

> $ParaPos := solve(\{SistemaPos\}, \{c_1, c_2\})$

$$ParaPos := \{c_1 = 0, c_2 = 0\} \tag{21}$$

Por lo tanto, la solución general para alpha positiva no cumple las condiciones de frontera

> $EcuaNegX := subs(\mathrm{alpha} = -\beta^2, EcuaX)$

$$EcuaNegX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \tag{22}$$

> $SolNegX := dsolve(EcuaNegX)$

(23)

$$SolNegX := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \quad \textbf{(23)}$$

> $ComprobarCinco := simplify(subs(x = 0, rhs(SolNegX) = 0))$

$$ComprobarCinco := c_2 = 0 \quad \textbf{(24)}$$

> $ComprobarSeis := subs\left(x = 1, c_2 = 0, \mathrm{beta} = nPi, rhs(SolNegX) = 0\right)$

$$ComprobarSeis := c_1 \sin(nPi) = 0 \quad \textbf{(25)}$$

> $SolNegX := F(x) = subs\left(\mathrm{beta} = n \cdot \mathrm{Pi}, c_1 \cdot \sin(\beta x)\right)$

$$SolNegX := F(x) = c_1 \sin(n \pi x) \quad \textbf{(26)}$$

> $EcuaNegT := subs\left(\mathrm{alpha} = -(n \cdot \mathrm{Pi})^2, EcuaT\right)$

$$EcuaNegT := \frac{\frac{\mathrm{d}^2}{\mathrm{d}t^2} G(t)}{G(t)} = -n^2 \pi^2 \quad \textbf{(27)}$$

> $SolNegT := dsolve(EcuaNegT)$

$$SolNegT := G(t) = c_1 \sin(n \pi t) + c_2 \cos(n \pi t) \quad \textbf{(28)}$$

> $SolGralNeg := y(x, t) = subs\left(c_1 = 1, rhs(SolNegX)\right) \cdot rhs(SolNegT)$

$$SolGralNeg := y(x, t) = \sin(n \pi x) \left(c_1 \sin(n \pi t) + c_2 \cos(n \pi t)\right) \quad \textbf{(29)}$$

> $SolGral := y(x, t) = \sin(n \cdot \mathrm{Pi} \cdot x) \cdot (b \cdot \cos(n \cdot \mathrm{Pi} \cdot t) + a \cdot \sin(n \cdot \mathrm{Pi} \cdot t))$

$$SolGral := y(x, t) = \sin(n \pi x) \left(b \cos(n \pi t) + a \sin(n \pi t)\right) \quad \textbf{(30)}$$

> $SolGralFourier := y(x, t) = Sum(\sin(n \cdot \mathrm{Pi} \cdot x) \cdot (b[n] \cdot \cos(n \cdot \mathrm{Pi} \cdot t) + a[n] \cdot \sin(n \cdot \mathrm{Pi} \cdot t)), n = 1 \,..\mathrm{infinity})$

$$SolGralFourier := y(x, t) = \sum_{n=1}^{\infty} \sin(n \pi x) \left(b_n \cos(n \pi t) + a_n \sin(n \pi t)\right) \quad \textbf{(31)}$$

> $SolGralTotal := eval(subs(t = 0, SolGralFourier))$

$$SolGralTotal := y(x, 0) = \sum_{n=1}^{\infty} \sin(n \pi x) \, b_n \quad \textbf{(32)}$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1) \cdot \cdot n, simplify\left(\left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(\frac{\frac{5}{1000}}{\frac{5}{10}} \cdot x \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = 0 \,.. \frac{5}{10}\right)\right) + \left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(-\frac{\left(\frac{5}{1000}\right)}{\frac{5}{10}} \cdot x + \frac{1}{100}\right) \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = \frac{5}{10} \,..1\right)\right)\right)$

$$\textbf{(33)}$$

$$b_n := \frac{\sin\left(\frac{n\pi}{2}\right)}{25\, n^2\, \pi^2} \tag{33}$$

> $eval(rhs(subs(t=0, diff(SolGralFourier, t))) = 0)$

$$\sum_{n=1}^{\infty} \sin(n\,\pi\,x)\; a_n\, n\,\pi = 0 \tag{34}$$

> $a[n] := 0$

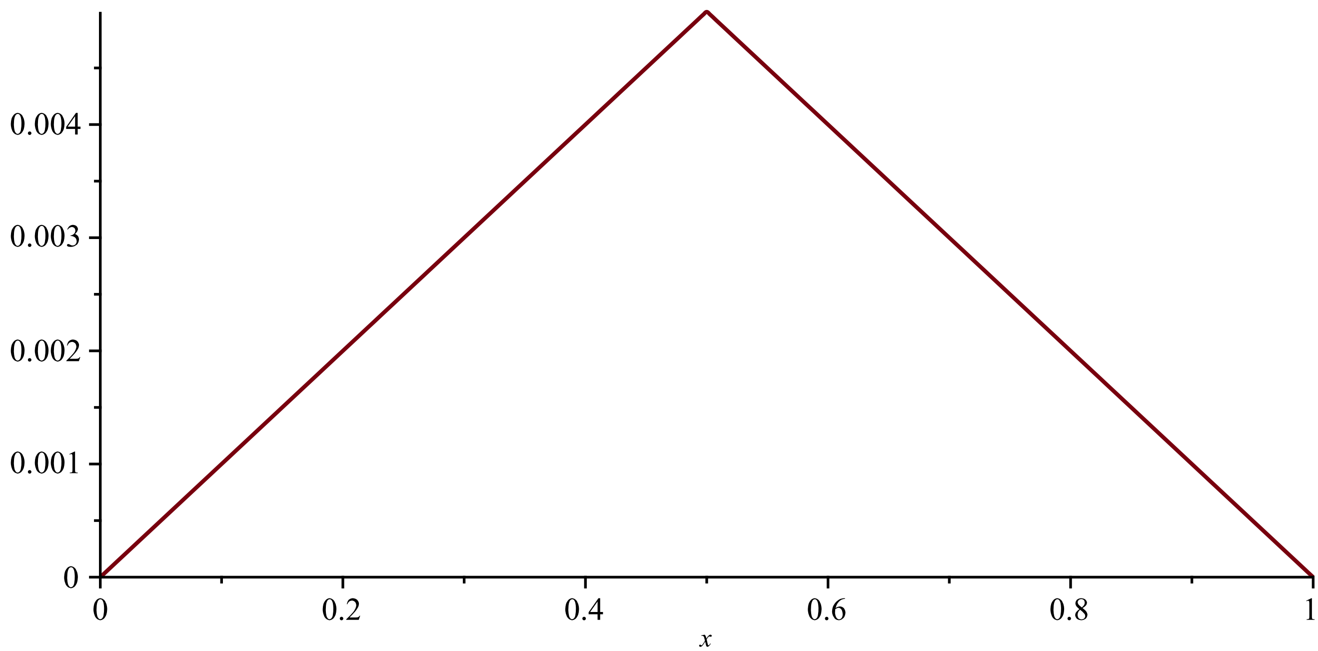
$$a_n := 0 \tag{35}$$

> $SolGralFourier$

$$y(x, t) = \sum_{n=1}^{\infty} \frac{\sin(n\,\pi\,x)\, \sin\left(\frac{n\pi}{2}\right) \cos(n\,\pi\,t)}{25\, n^2\, \pi^2} \tag{36}$$

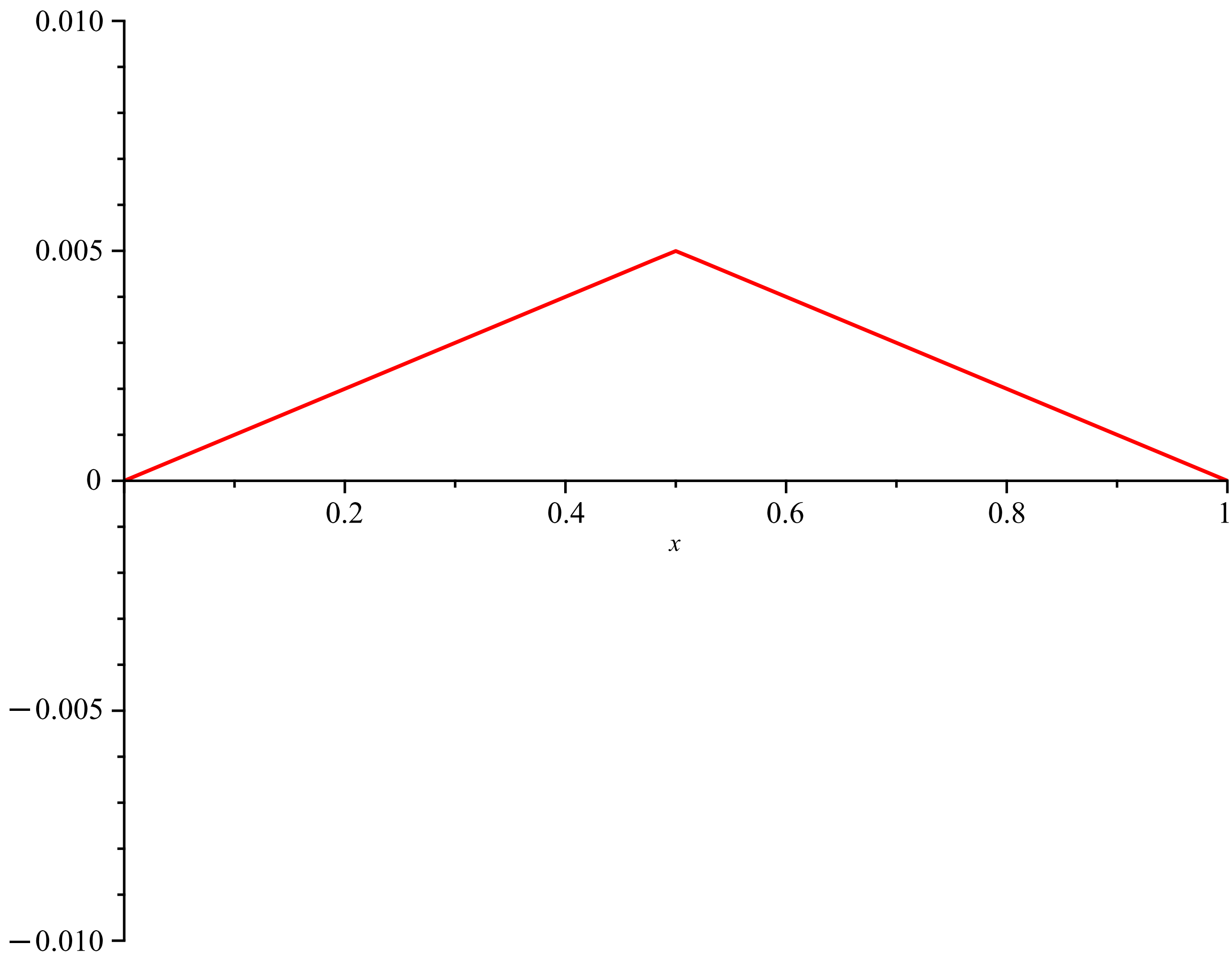
> $SolPart500 := y(x, t) = sum\left(\frac{\sin(n\,\pi\,x)\, \sin\left(\frac{n\pi}{2}\right) \cos(n\,\pi\,t)}{25\, n^2\, \pi^2}, n = 1\,..500\right) :$

> $plot(subs(t=0, rhs(SolPart500)), x = 0\,..1)$



> $with(plots) :$

> $animate(rhs(SolPart500), x = 0\,..1, t = 0\,..4, frames = 150, view = [0\,..1, -0.01\,..0.01])$


0.010
0.005
0
−0.005
−0.010
0.2
0.4
0.6
0.8
1
x

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