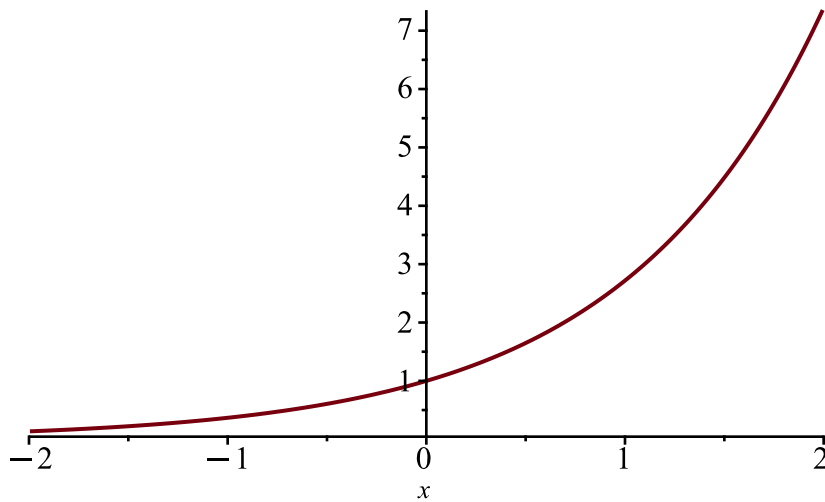


```
> restart
> f := exp(x)
```

$$f := e^x$$

```
> plot(f, x = -2..2)
```



```
> L := 2
```

$$L := 2$$

```
> a[0] := 1/L * int(f, x = -L..L); evalf(%, 4)
```

$$a_0 := -\frac{e^{-2}}{2} + \frac{e^2}{2}$$

$$3.626$$

```
> a[n] := subs(sin(n*Pi) = 0, cos(n*Pi) = (-1)^n, 1/L * int(f*cos(n*Pi/L*x), x = -L..L))
```

$$a_n := \frac{2 e^2 (-1)^n - 2 e^{-2} (-1)^n}{n^2 \pi^2 + 4}$$

```
> b[n] := subs(sin(n*Pi) = 0, cos(n*Pi) = (-1)^n, 1/L * int(f*sin(n*Pi/L*x), x = -L..L))
```

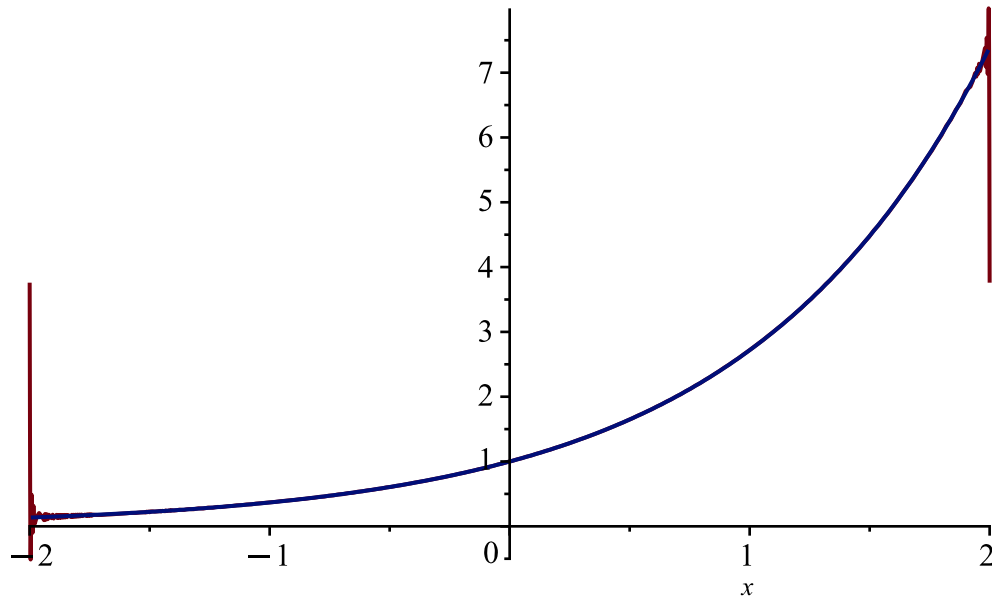
$$b_n := -\frac{e^2 \pi (-1)^n n - e^{-2} \pi (-1)^n n}{n^2 \pi^2 + 4}$$

```
> STF_f := F = a[0]/2 + Sum(a[n]*cos(n*Pi/L*x) + b[n]*sin(n*Pi/L*x), n = 1..infinity)
```

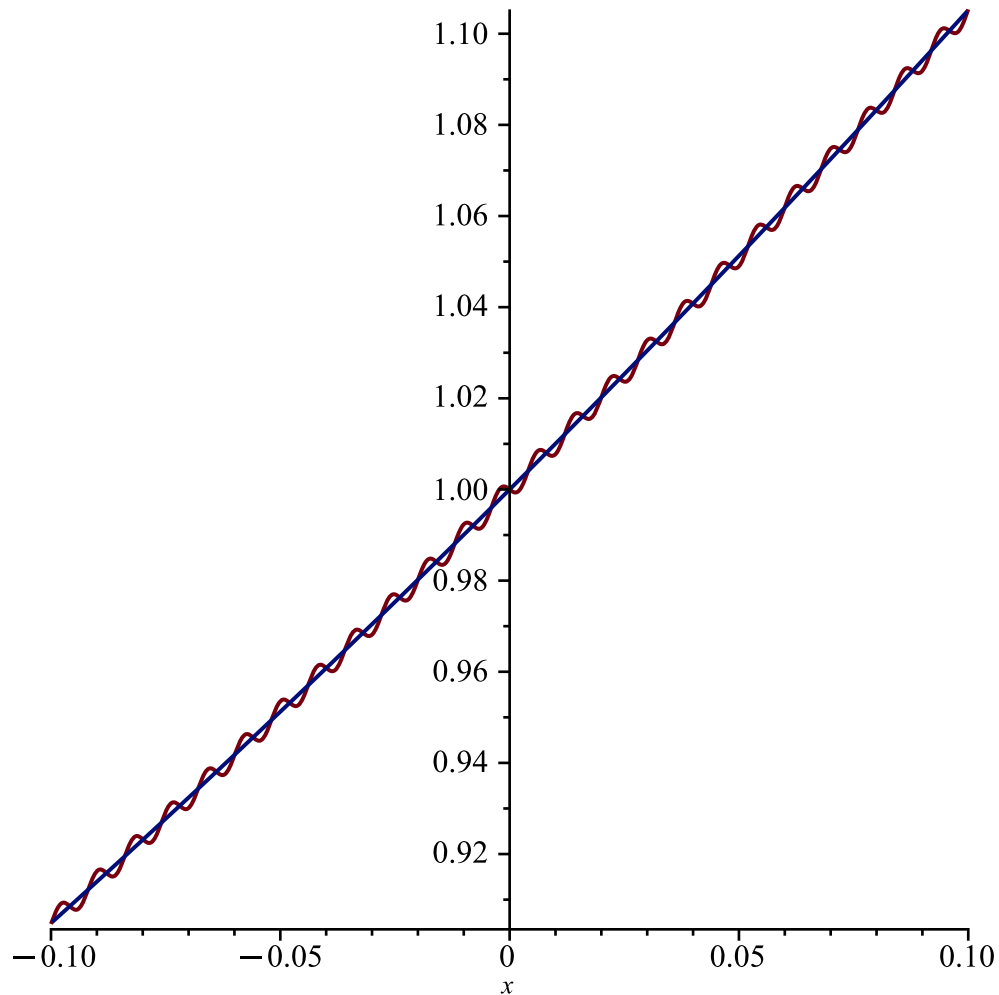
$$STF_f := F = -\frac{e^{-2}}{4} + \frac{e^2}{4} + \sum_{n=1}^{\infty} \left(\frac{(2 e^2 (-1)^n - 2 e^{-2} (-1)^n) \cos\left(\frac{n \pi x}{2}\right)}{n^2 \pi^2 + 4} - \frac{(e^2 \pi (-1)^n n - e^{-2} \pi (-1)^n n) \sin\left(\frac{n \pi x}{2}\right)}{n^2 \pi^2 + 4} \right)$$

```
> STF500 := F[500] =  $\frac{a[0]}{2} + \text{sum}\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1 \dots 500\right) :$ 
```

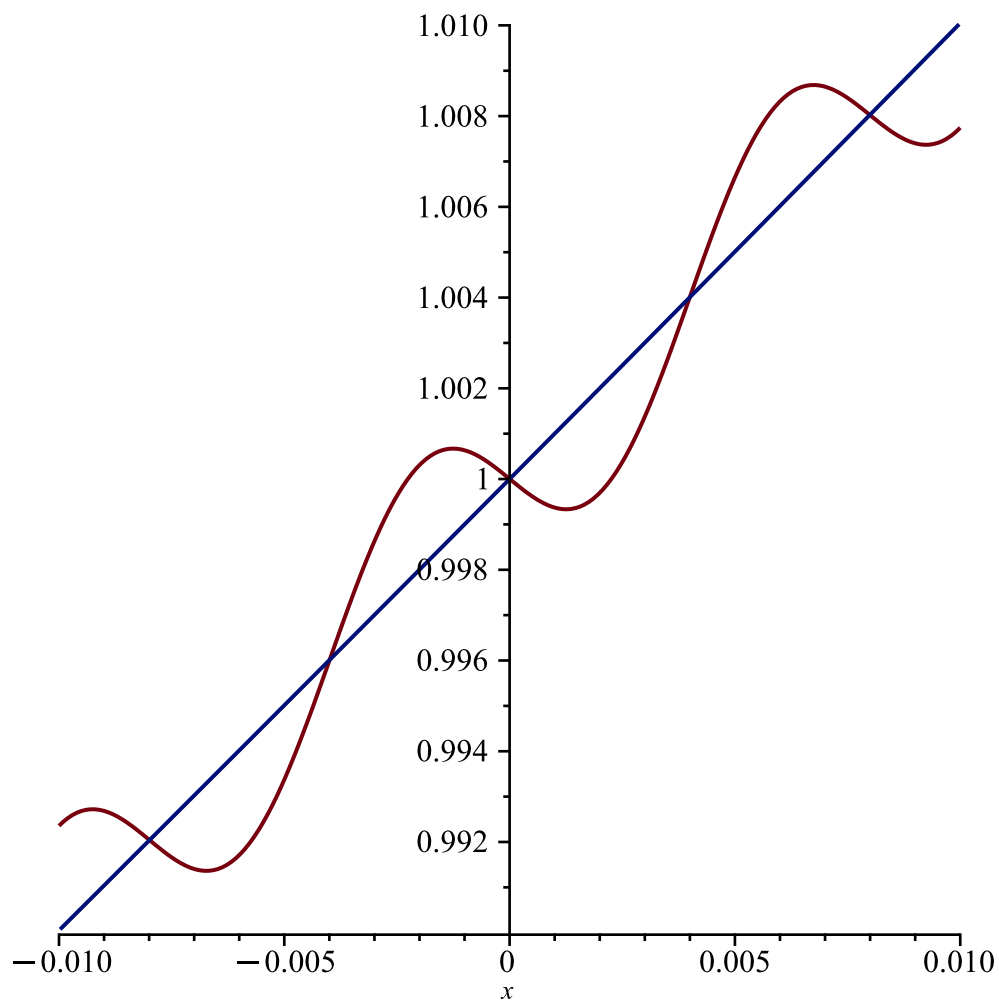
```
> plot( {f, rhs(STF500)}, x = -L..L)
```



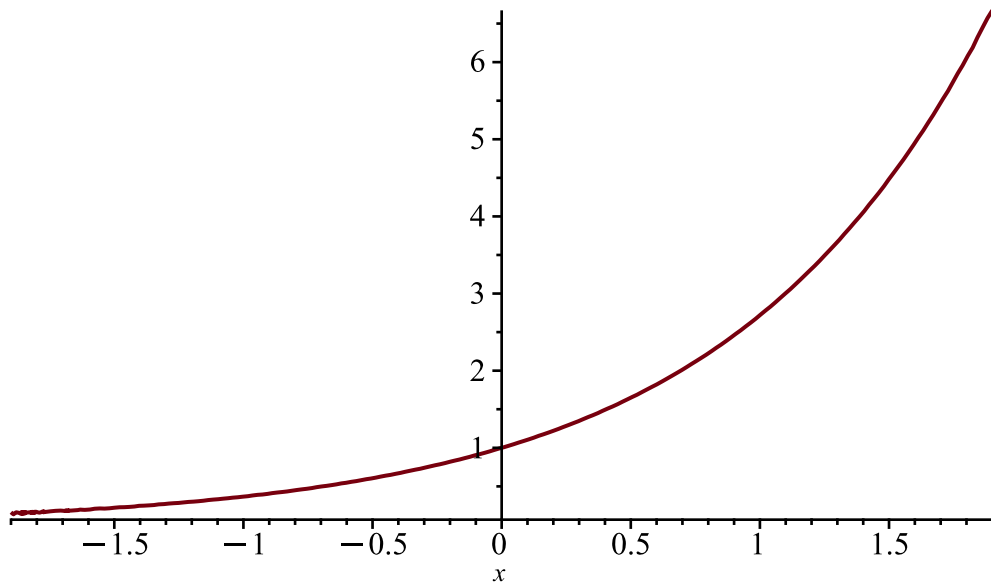
```
> plot( {f, rhs(STF500)}, x = -0.1..0.1)
```



```
> plot( {f, rhs(STF500)}, x = -0.01..0.01)
```

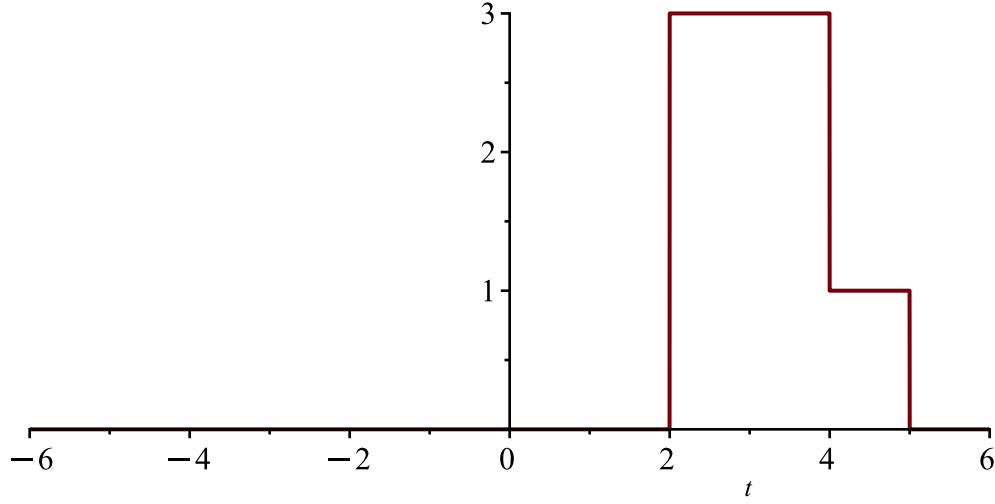


```
> STF1000 := F[1000] =  $\frac{a[0]}{2} + \text{sum}\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1 \dots 1000\right) :$ 
> plot(rhs(STF1000), x = -1.9 .. 1.9)
```



> restart

> $g := 3 \cdot \text{Heaviside}(t - 2) - 2 \cdot \text{Heaviside}(t - 4) - \text{Heaviside}(t - 5); \text{plot}(g, t = -6..6)$
 $g := 3 \text{ Heaviside}(t - 2) - 2 \text{ Heaviside}(t - 4) - \text{Heaviside}(t - 5)$



> $L := 6$

$L := 6$

(7)

> $a[0] := \frac{1}{L} \cdot \text{int}(g, t = -L..L)$

$a_0 := \frac{7}{6}$

(8)

> $a[n] := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot \text{int}\left(g \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

$$a_n := \frac{18 \sin\left(\frac{2 n \pi}{3}\right) - 18 \sin\left(\frac{n \pi}{3}\right)}{6 n \pi} + \frac{-18 \sin\left(\frac{2 n \pi}{3}\right) + 18 \sin\left(\frac{n \pi}{3}\right)}{6 n \pi} \\ + \frac{6 \sin\left(\frac{5 n \pi}{6}\right) - 18 \sin\left(\frac{n \pi}{3}\right) + 12 \sin\left(\frac{2 n \pi}{3}\right)}{6 n \pi}$$

(9)

> $b[n] := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot \text{int}\left(g \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

$$b_n := \frac{-18 \cos\left(\frac{2 n \pi}{3}\right) + 18 \cos\left(\frac{n \pi}{3}\right)}{6 n \pi} + \frac{18 \cos\left(\frac{2 n \pi}{3}\right) - 18 \cos\left(\frac{n \pi}{3}\right)}{6 n \pi} \\ + \frac{-6 \cos\left(\frac{5 n \pi}{6}\right) + 18 \cos\left(\frac{n \pi}{3}\right) - 12 \cos\left(\frac{2 n \pi}{3}\right)}{6 n \pi}$$

(10)

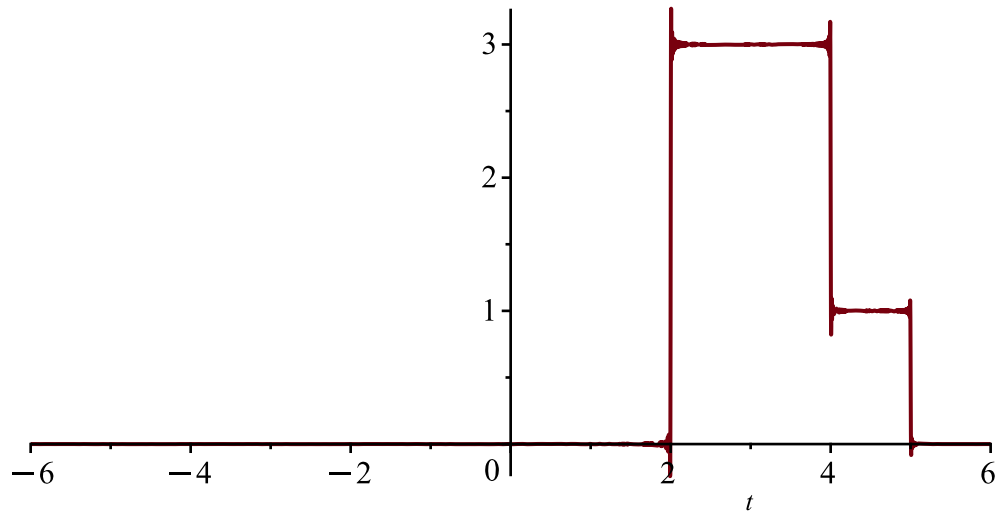
> $\text{STF_g} := G = \frac{a[0]}{2} + \text{Sum}\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), n = 1..infinity\right)$

$$\begin{aligned}
 STF_g := G = & \frac{7}{12} + \sum_{n=1}^{\infty} \left(\left(\frac{18 \sin\left(\frac{2n\pi}{3}\right) - 18 \sin\left(\frac{n\pi}{3}\right)}{6n\pi} \right. \right. \\
 & + \frac{-18 \sin\left(\frac{2n\pi}{3}\right) + 18 \sin\left(\frac{n\pi}{3}\right)}{6n\pi} \\
 & + \frac{6 \sin\left(\frac{5n\pi}{6}\right) - 18 \sin\left(\frac{n\pi}{3}\right) + 12 \sin\left(\frac{2n\pi}{3}\right)}{6n\pi} \left. \right) \cos\left(\frac{n\pi t}{6}\right) \\
 & + \left(\frac{-18 \cos\left(\frac{2n\pi}{3}\right) + 18 \cos\left(\frac{n\pi}{3}\right)}{6n\pi} + \frac{18 \cos\left(\frac{2n\pi}{3}\right) - 18 \cos\left(\frac{n\pi}{3}\right)}{6n\pi} \right. \\
 & \left. + \frac{-6 \cos\left(\frac{5n\pi}{6}\right) + 18 \cos\left(\frac{n\pi}{3}\right) - 12 \cos\left(\frac{2n\pi}{3}\right)}{6n\pi} \right) \sin\left(\frac{n\pi t}{6}\right) \Bigg)
 \end{aligned} \tag{11}$$

```

> STF_g := G =  $\frac{a[0]}{2} + \text{sum}\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot t\right), n = 1 .. 1000\right) :$ 
> plot(rhs(STF_g), t = -L..L)

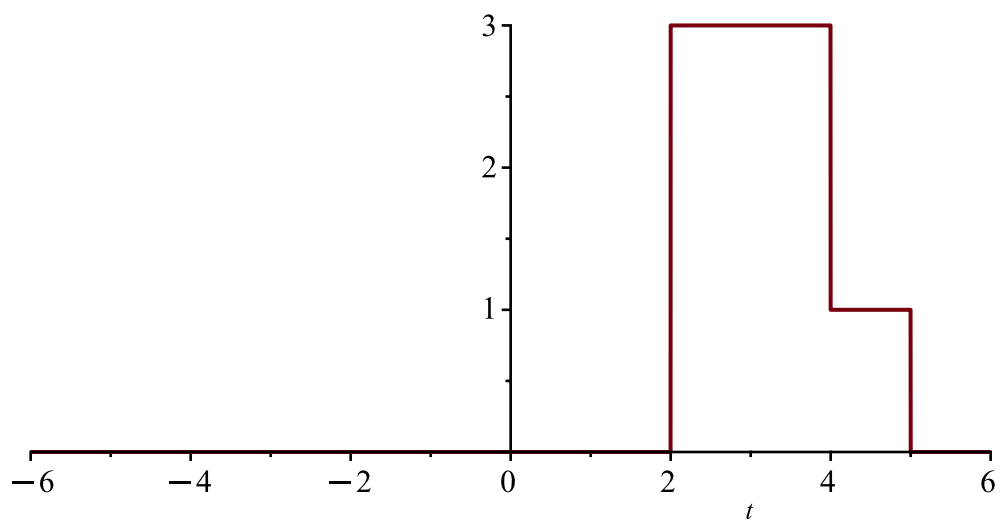
```



```

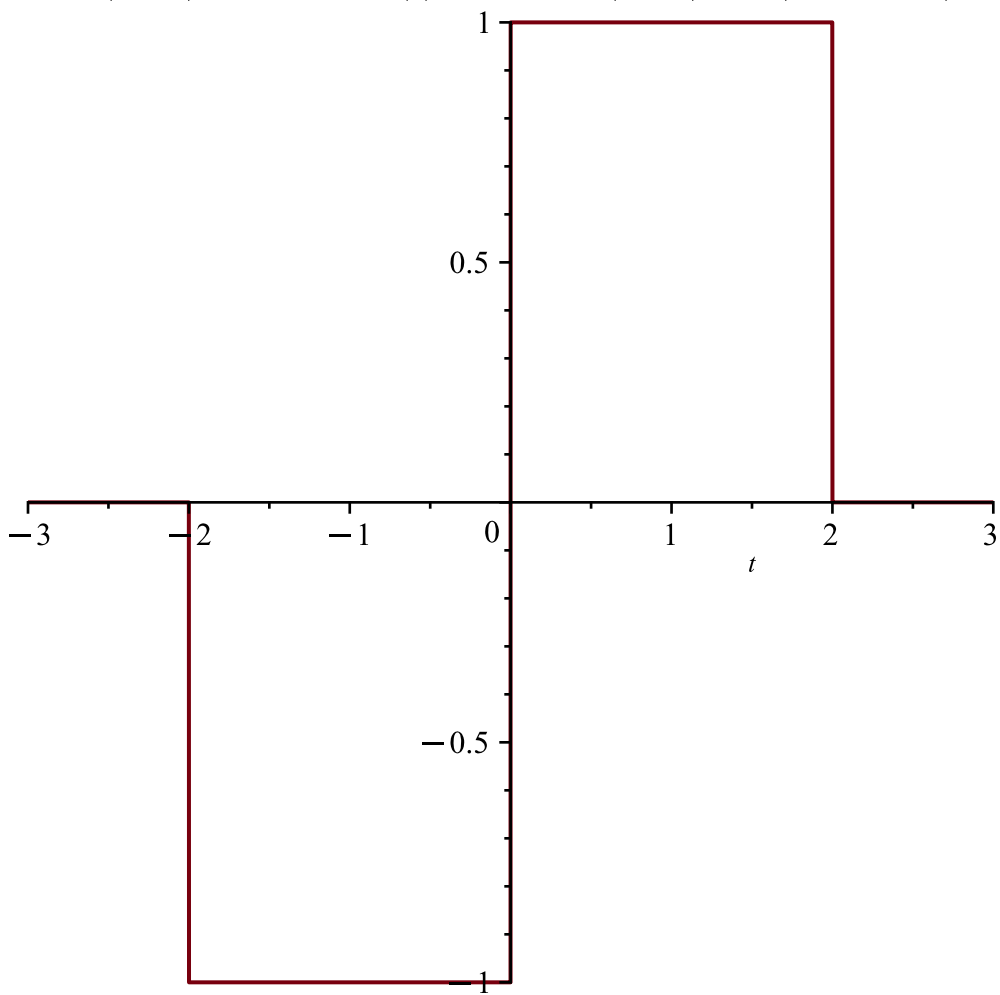
> plot(g, t = -6..6)

```



```
> restart
```

```
> h := -Heaviside(t + 2) + 2*Heaviside(t) - Heaviside(t - 2) : plot(h, t = -3 .. 3)
```



```
> L := 3
```

$L := 3$

(12)

```
> a[0] := 1/L * int(h, t = -L..L)
```

(13)

$$a_0 := 0 \quad (13)$$

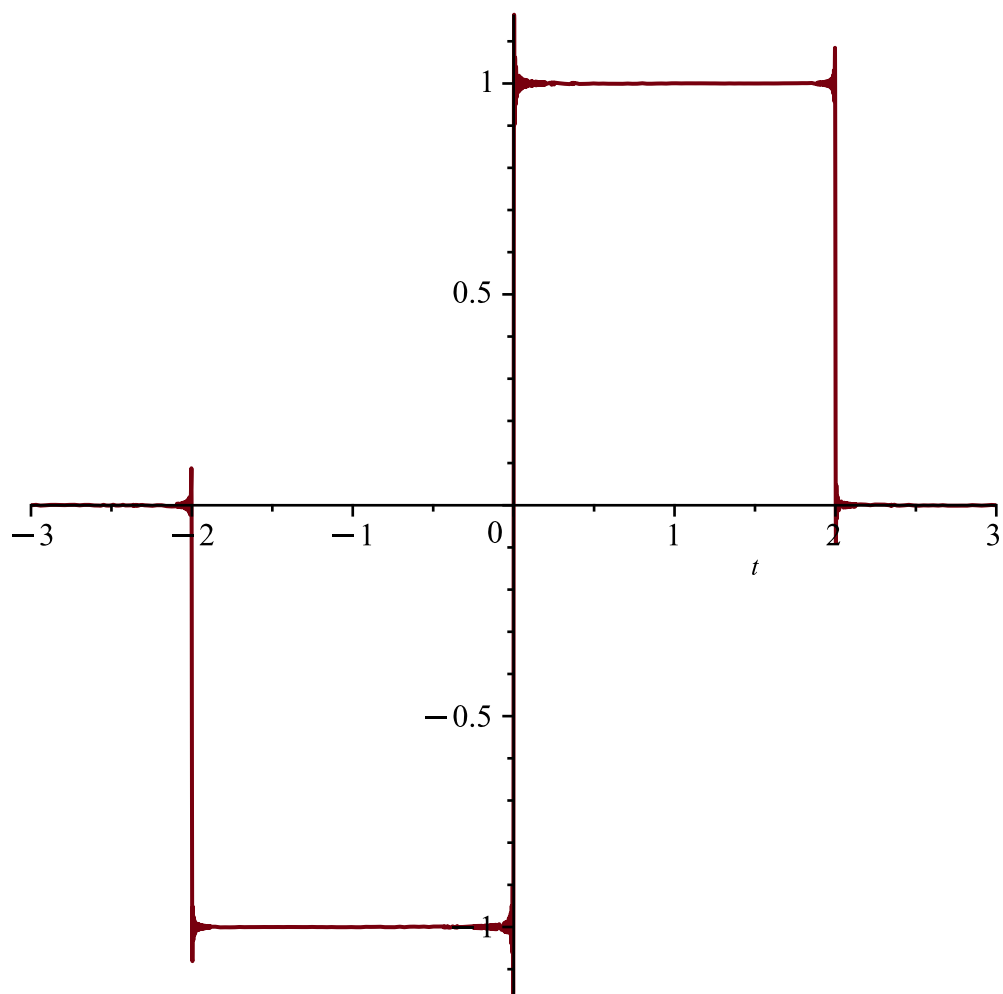
$$\begin{aligned} &> a[n] := \frac{1}{L} \cdot \text{int} \left(h \cdot \cos \left(\frac{n \cdot \text{Pi}}{L} \cdot t \right), t = -L .. L \right) \\ & \quad \quad \quad a_n := 0 \end{aligned} \quad (14)$$

$$\begin{aligned} &> b[n] := \frac{1}{L} \cdot \text{int} \left(h \cdot \sin \left(\frac{n \cdot \text{Pi}}{L} \cdot t \right), t = -L .. L \right) \\ & \quad \quad \quad b_n := \frac{3 - 3 \cos \left(\frac{2 n \pi}{3} \right)}{3 n \pi} + \frac{-3 + 3 \cos \left(\frac{2 n \pi}{3} \right)}{3 n \pi} + \frac{6 - 6 \cos \left(\frac{2 n \pi}{3} \right)}{3 n \pi} \end{aligned} \quad (15)$$

$$\begin{aligned} &> STF_h := H = \text{Sum} \left(b[n] \cdot \sin \left(\frac{n \cdot \text{Pi}}{L} \cdot t \right), n = 1 .. \text{infinity} \right) \\ & STF_h := H = \sum_{n=1}^{\infty} \left(\frac{3 - 3 \cos \left(\frac{2 n \pi}{3} \right)}{3 n \pi} + \frac{-3 + 3 \cos \left(\frac{2 n \pi}{3} \right)}{3 n \pi} \right. \\ & \quad \quad \left. + \frac{6 - 6 \cos \left(\frac{2 n \pi}{3} \right)}{3 n \pi} \right) \sin \left(\frac{n \pi t}{3} \right) \end{aligned} \quad (16)$$

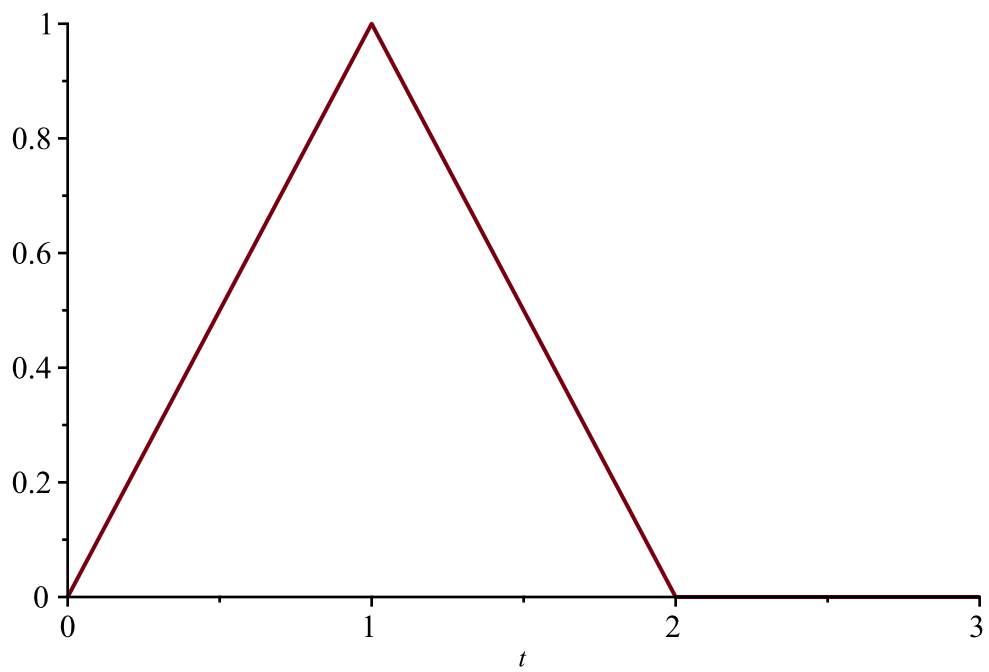
$$> STF1000 := \text{sum} \left(b[n] \cdot \sin \left(\frac{n \cdot \text{Pi}}{L} \cdot t \right), n = 1 .. 1000 \right) :$$

$$> \text{plot}(STF1000, t = -L .. L)$$

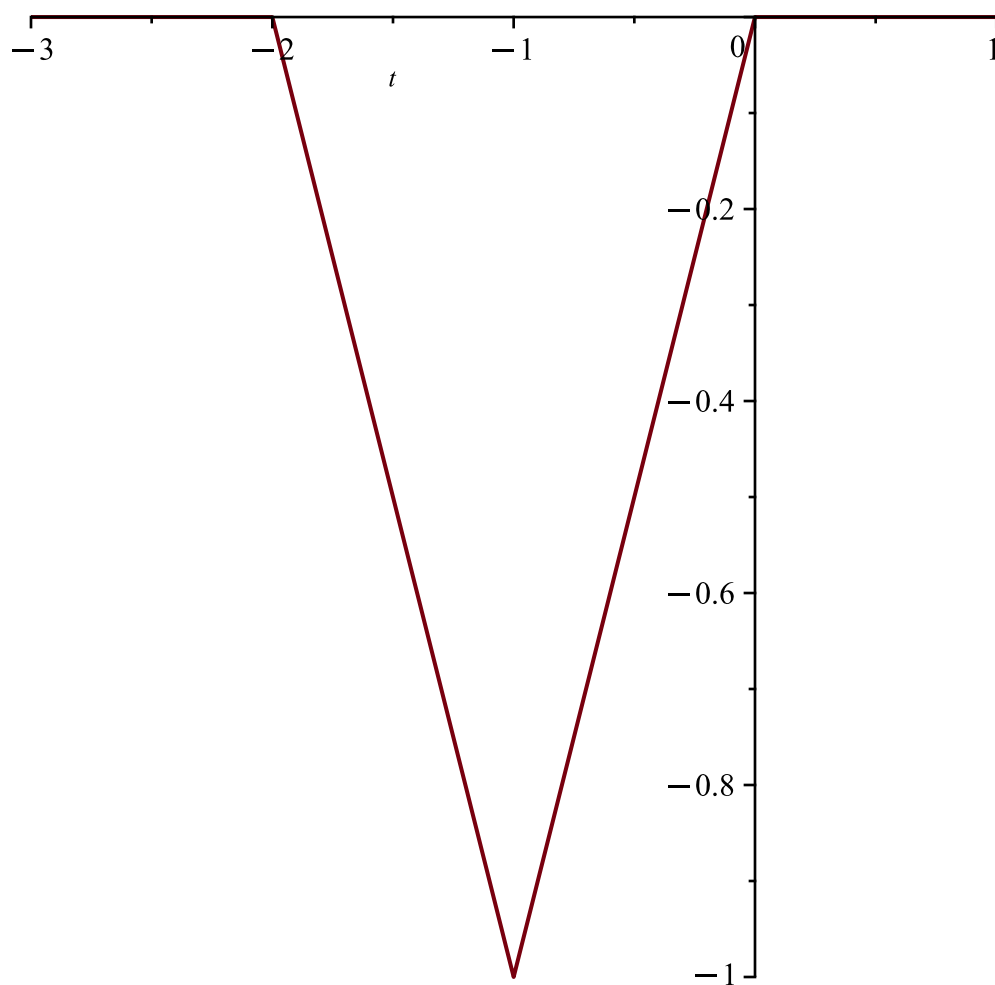


```
> restart
```

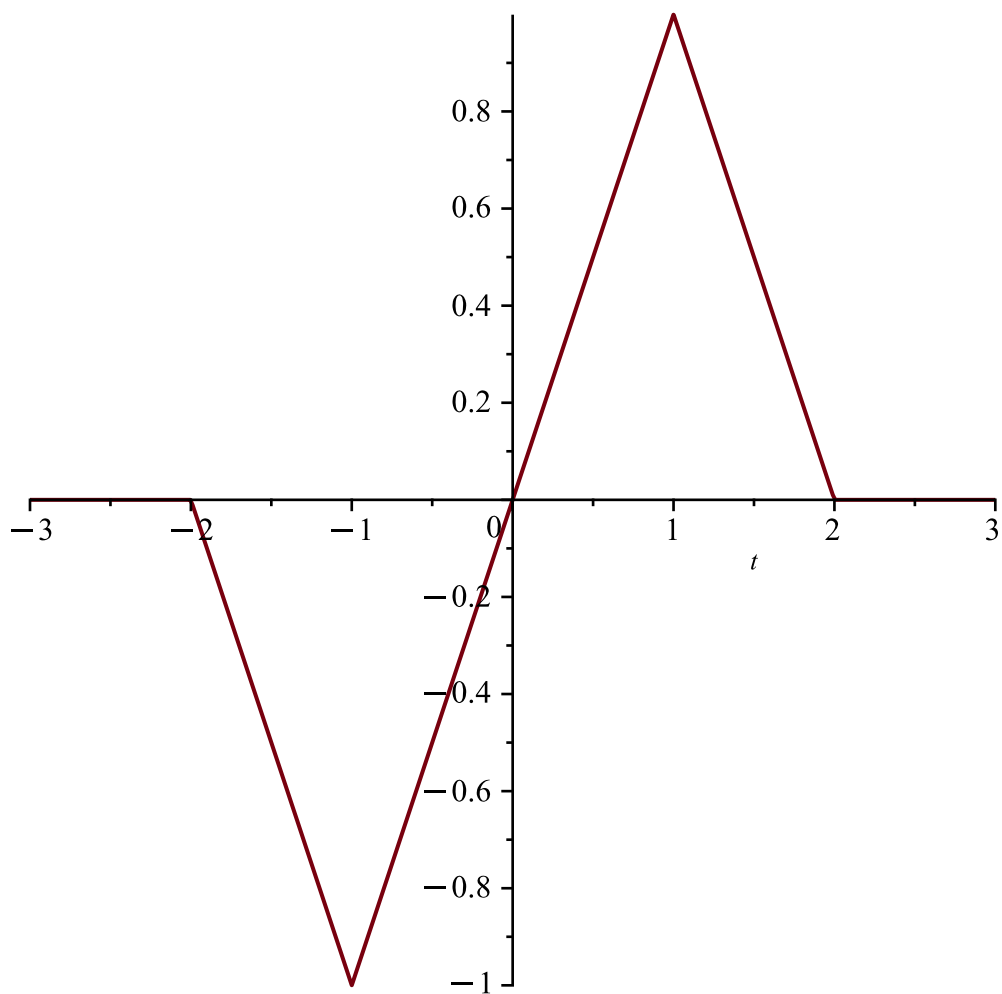
```
> f := (t)·Heaviside(t) - 2(t - 1)·Heaviside(t - 1) + (t - 2)·Heaviside(t - 2) : plot(f, t = 0 .. 3)
```



```
> g := -(t + 2) · Heaviside(t + 2) + 2(t + 1) · Heaviside(t + 1) - t · Heaviside(t) : plot(g, t = -3 .. 1)
```



```
> plot(f + g, t = -3 .. 3)
```



```
> L := 2
```

```
L := 2
```

(17)

```
> a[0] = 1/L * int((f + g), t = -L..L)
```

```
a_0 = 0
```

(18)

```
> a[n] := simplify(1/L * int((f + g) * cos(n * Pi / L * t), t = -L..L))
```

```
a_n := 0
```

(19)

```
> b[n] := simplify(1/L * int((f + g) * sin(n * Pi / L * t), t = -L..L))
```

$$b_n := \frac{8 \sin\left(\frac{n \pi}{2}\right) - 4 \sin(n \pi)}{n^2 \pi^2}$$

(20)

```
>
```