> restart

>

Problema de cuerda de guitarra

> $Ecua := diff(y(x,t), t\$2) = c^2 \cdot diff(y(x,t), x\$2)$

$$Ecua := \frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t)\right) \quad (1)$$

> $EcuaUno := subs(c^2 = 1, Ecua)$

$$EcuaUno := \frac{\partial^2}{\partial t^2} y(x,t) = \frac{\partial^2}{\partial x^2} y(x,t) \quad (2)$$

> $EcuaDos := simplify(eval(subs(y(x,t) = F(x) \cdot G(t), EcuaUno)))$

$$EcuaDos := F(x) \left(\frac{d^2}{dt^2} G(t)\right) = \left(\frac{d^2}{dx^2} F(x)\right) G(t) \quad (3)$$

> $EcuaSeparada := \frac{EcuaDos}{F(x) \cdot G(t)}$

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \quad (4)$$

> $EcuaX := rhs(EcuaSeparada) = \text{alpha}$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \quad (5)$$

> $EcuaT := lhs(EcuaSeparada) = \text{alpha}$

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (6)$$

> $CondFront := y(0,t) = 0, y(1,t) = 0 : CondFront[1]; CondFront[2]$

$$y(0,t) = 0$$
$$y(1,t) = 0 \quad (7)$$

> $EcuaCeroX := subs(\text{alpha} = 0, EcuaX)$

$$EcuaCeroX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \quad (8)$$

> $SolCeroX := dsolve(EcuaCeroX)$

$$SolCeroX := F(x) = c_1 x + c_2 \quad (9)$$

> $ComprobarUno := subs(x = 0, rhs(SolCeroX) = 0)$

$$ComprobarUno := c_2 = 0 \quad (10)$$

> $ComprobarDos := subs(x = 1, c_2 = 0, rhs(SolCeroX) = 0)$

$$ComprobarDos := c_1 = 0 \quad (11)$$

Se descarta la solucion para alpha = 0

> $EcuaPosX := subs\left(\text{alpha} = \beta^2, EcuaX\right)$

$$EcuaPosX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \tag{12}$$

> $SolPosX := dsolve(EcuaPosX)$

$$SolPosX := F(x) = c_1 e^{-\beta x} + c_2 e^{\beta x} \tag{13}$$

> $SisPos := eval(subs(x = 0, rhs(SolPosX) = 0)), subs(x = 1, rhs(SolPosX) = 0) : SisPos[1];$
$SisPos[2]$

$$c_1 + c_2 = 0$$
$$c_1 e^{-\beta} + c_2 e^{\beta} = 0 \tag{14}$$

> $ParaPos := solve(\{SisPos\})$

$$ParaPos := \left\{\beta = \beta, c_1 = 0, c_2 = 0\right\}, \left\{\beta = 0, c_1 = -c_2, c_2 = c_2\right\} \tag{15}$$

Se descarta la solución para alpha = beta^2

> $EcuaNegX := subs\left(\text{alpha} = -\beta^2, EcuaX\right)$

$$EcuaNegX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \tag{16}$$

> $SolNegX := dsolve(EcuaNegX)$

$$SolNegX := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \tag{17}$$

> $ComprobarCinco := simplify(eval(subs(x = 0, rhs(SolNegX) = 0)))$

$$ComprobarCinco := c_2 = 0 \tag{18}$$

> $SolNegX := F(x) = subs\left(\text{beta} = n \cdot \text{Pi}, c_1 \cdot \sin(\beta \cdot x)\right)$

$$SolNegX := F(x) = c_1 \sin(n \pi x) \tag{19}$$

> $EcuaNegT := subs\left(\text{alpha} = -(n \cdot \text{Pi})^2, EcuaT\right)$

$$EcuaNegT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -n^2 \pi^2 \tag{20}$$

> $SolNegT := dsolve(EcuaNegT)$

$$SolNegT := G(t) = c_1 \sin(n \pi t) + c_2 \cos(n \pi t) \tag{21}$$

> $SolGralNeg := y(x, t) = subs\left(c_1 = 1, rhs(SolNegX)\right) \cdot rhs(SolNegT)$

$$SolGralNeg := y(x, t) = \sin(n \pi x) \left(c_1 \sin(n \pi t) + c_2 \cos(n \pi t)\right) \tag{22}$$

> $SolGral := y(x, t) = \sin(n \cdot \text{Pi} \cdot x) \cdot (b \cdot \cos(n \cdot \text{Pi} \cdot t) + a \cdot \sin(n \cdot \text{Pi} \cdot t))$

$$SolGral := y(x, t) = \sin(n \pi x) \left(b \cos(n \pi t) + a \sin(n \pi t)\right) \tag{23}$$

> $SolGralFourier := Sum(\sin(n \cdot \text{Pi} \cdot x) \cdot (b[n] \cdot \cos(n \cdot \text{Pi} \cdot t) + a[n] \cdot \sin(n \cdot \text{Pi} \cdot t)), n = 1 ..\text{infinity})$

**(24)**

$$SolGralFourier := \sum_{n=1}^{\infty} \sin(n\,\pi\,x)\,\left(b_n \cos(n\,\pi\,t) + a_n \sin(n\,\pi\,t)\right) \tag{24}$$

> $CondIni := y(0,0) = \frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)} \cdot x, y(1,0) = \frac{2}{100} - \frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)} \cdot x : CondIni[1]; CondIni[2]$

$$y(0,0) = \frac{x}{50}$$

$$y(1,0) = \frac{1}{50} - \frac{x}{50} \tag{25}$$

> $CondIniDos := subs(t=0, diff(y(x,t),t)) = 0$

$$CondIniDos := diff(y(x,0),0) = 0 \tag{26}$$

> $SolGralTotal := eval(subs(t=0, SolGralFourier))$

$$SolGralTotal := \sum_{n=1}^{\infty} \sin(n\,\pi\,x)\,b_n \tag{27}$$

> $b[n] := subs\left(\sin(n\cdot\mathrm{Pi}) = 0, \cos(n\cdot\mathrm{Pi}) = (-1)^n, simplify\left(\frac{1}{\left(\frac{5}{10}\right)} \cdot int\left(\left(\frac{\frac{5}{1000}}{\frac{5}{10}} \cdot x \cdot \sin(n\cdot\mathrm{Pi} \cdot x\right), x = 0..\frac{5}{10}\right)\right) + \frac{1}{\left(\frac{5}{10}\right)} \cdot int\left(\left(-\frac{\left(\frac{5}{1000}\right)}{\frac{5}{10}} \cdot x + \frac{1}{100}\right) \cdot \sin(n\cdot\mathrm{Pi}\cdot x), x = \frac{5}{10}..1\right)\right)\right)$

$$b_n := \frac{\sin\left(\frac{n\,\pi}{2}\right)}{25\,n^2\,\pi^2} \tag{28}$$

> $a[n] := 0$

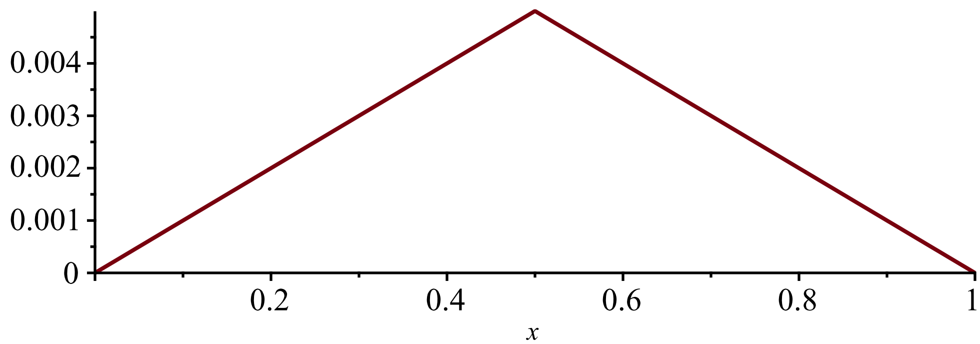
$$a_n := 0 \tag{29}$$

> $SolGralFourier$

$$\sum_{n=1}^{\infty} \frac{\sin(n\,\pi\,x)\,\sin\left(\frac{n\,\pi}{2}\right)\cos(n\,\pi\,t)}{25\,n^2\,\pi^2} \tag{30}$$

> $SolPart500 := y(x,t) = sum(\sin(n\cdot\mathrm{Pi}\cdot x)\cdot(b[n]\cdot\cos(n\cdot\mathrm{Pi}\cdot t) + a[n]\cdot\sin(n\cdot\mathrm{Pi}\cdot t)), n = 1..500) :$

> $plot(subs(t=0, rhs(SolPart500)), x = 0..1)$

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> with(plots) :
> animate(rhs(SolPart500), x = 0 ..1, t = 0 ..4, frames = 150, view = [0 ..1, −0.01 ..0.01])
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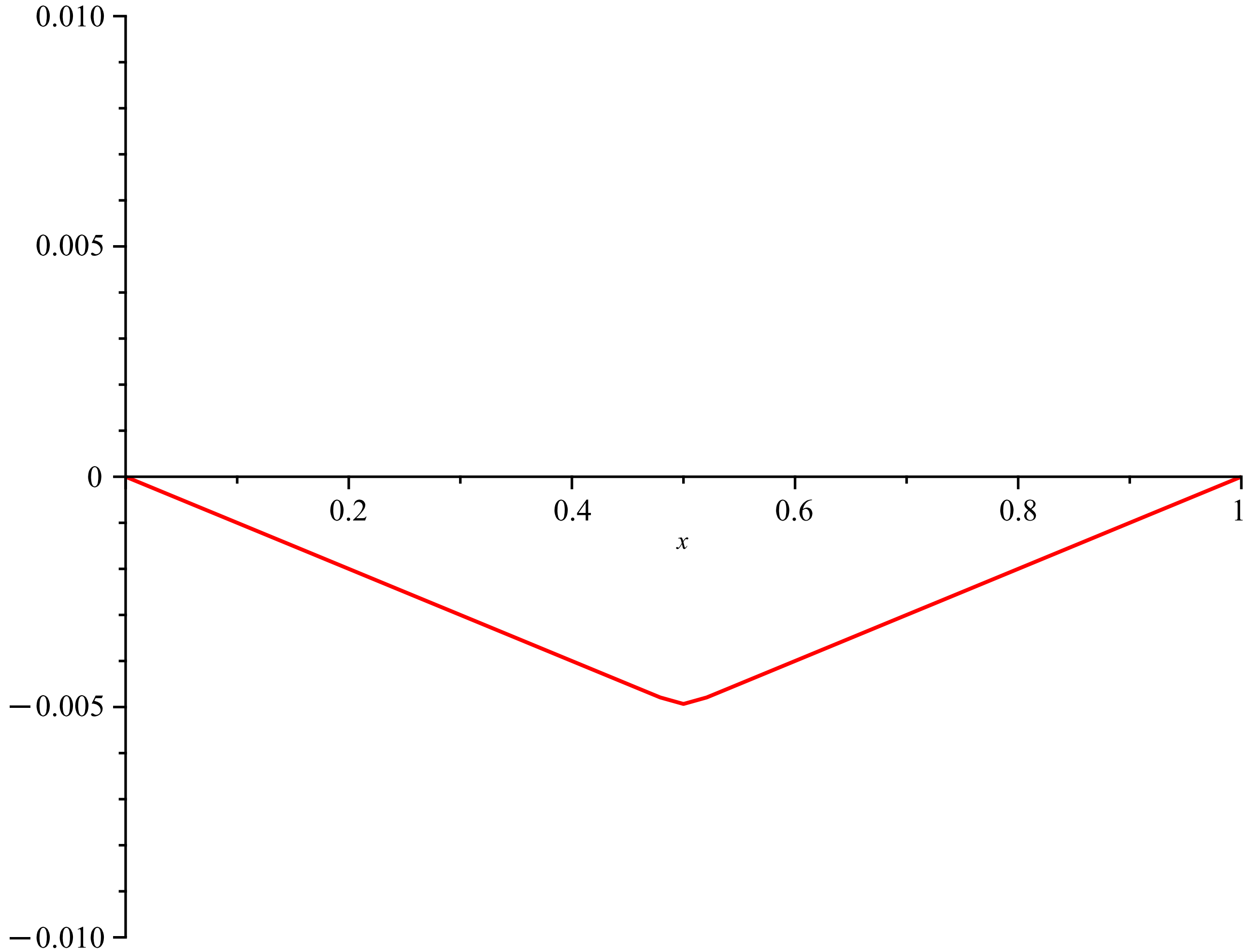


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