> *restart*

ECUACIONES DIFERENCIALES
SEMESTRE 2024-2
GRUPO 11
SERIE 1

>

1) Resuelva

> $Ecua := y' = \dfrac{x \cdot y^2 - \cos(x) \cdot \sin(x)}{y \cdot (1 - x^2)}$

$$Ecua := \frac{d}{dx}\, y(x) = \frac{x\, y(x)^2 - \cos(x)\,\sin(x)}{y(x)\,(-x^2 + 1)} \tag{1}$$

> $CondIni := y(0) = 2$

$$CondIni := y(0) = 2 \tag{2}$$

>

> $M := -(x\, y^2 - \cos(x)\,\sin(x))$

$$M := -x\, y^2 + \cos(x)\,\sin(x) \tag{3}$$

> $diff(M, y)$

$$-2\, x\, y \tag{4}$$

> $N := y\,(-x^2 + 1)$

$$N := y\,(-x^2 + 1) \tag{5}$$

> $diff(N, x)$

$$-2\, x\, y \tag{6}$$

ES EXACTA

> $IntMx := int(M, x)$

$$IntMx := \frac{\sin(x)^2}{2} - \frac{y^2\, x^2}{2} \tag{7}$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGral := \frac{\sin(x)^2}{2} - \frac{y^2\, x^2}{2} + \frac{y^2}{2} = \_C1 \tag{8}$$

> $SolGralFinal := \dfrac{\sin(x)^2}{2} - \dfrac{y(x)^2\, x^2}{2} + \dfrac{y(x)^2}{2} = \_C1$

$$SolGralFinal := \frac{\sin(x)^2}{2} - \frac{y(x)^2\, x^2}{2} + \frac{y(x)^2}{2} = \_C1 \tag{9}$$

> $Parametro := simplify(subs(x = 0, y(0) = 2, SolGralFinal))$

$$Parametro := 2 = \_C1 \tag{10}$$

> $SolPartFinal := subs(\_C1 = lhs(Parametro), SolGralFinal) \cdot 2$

$$SolPartFinal := \sin(x)^2 - y(x)^2\, x^2 + y(x)^2 = 4 \tag{11}$$

> $DerSolPartFinal := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPartFinal := \frac{d}{dx}\, y(x) = \frac{-x\, y(x)^2 + \cos(x)\,\sin(x)}{y(x)\,(x^2 - 1)} \tag{12}$$

> *Ecua*

$$\frac{d}{dx}\,y(x) = \frac{x\,y(x)^2 - \cos(x)\,\sin(x)}{y(x)\,(-x^2+1)}$$

(13)

> *Comprobacion := simplify(rhs(DerSolPartFinal) − rhs(Ecua)) = 0*

$$Comprobacion := 0 = 0$$

(14)

> *Condicion := simplify(subs(x = 0, SolPartFinal))*

$$Condicion := y(0)^2 = 4$$

(15)

> *CondicionInicial := y(0) = sqrt(rhs(Condicion))*

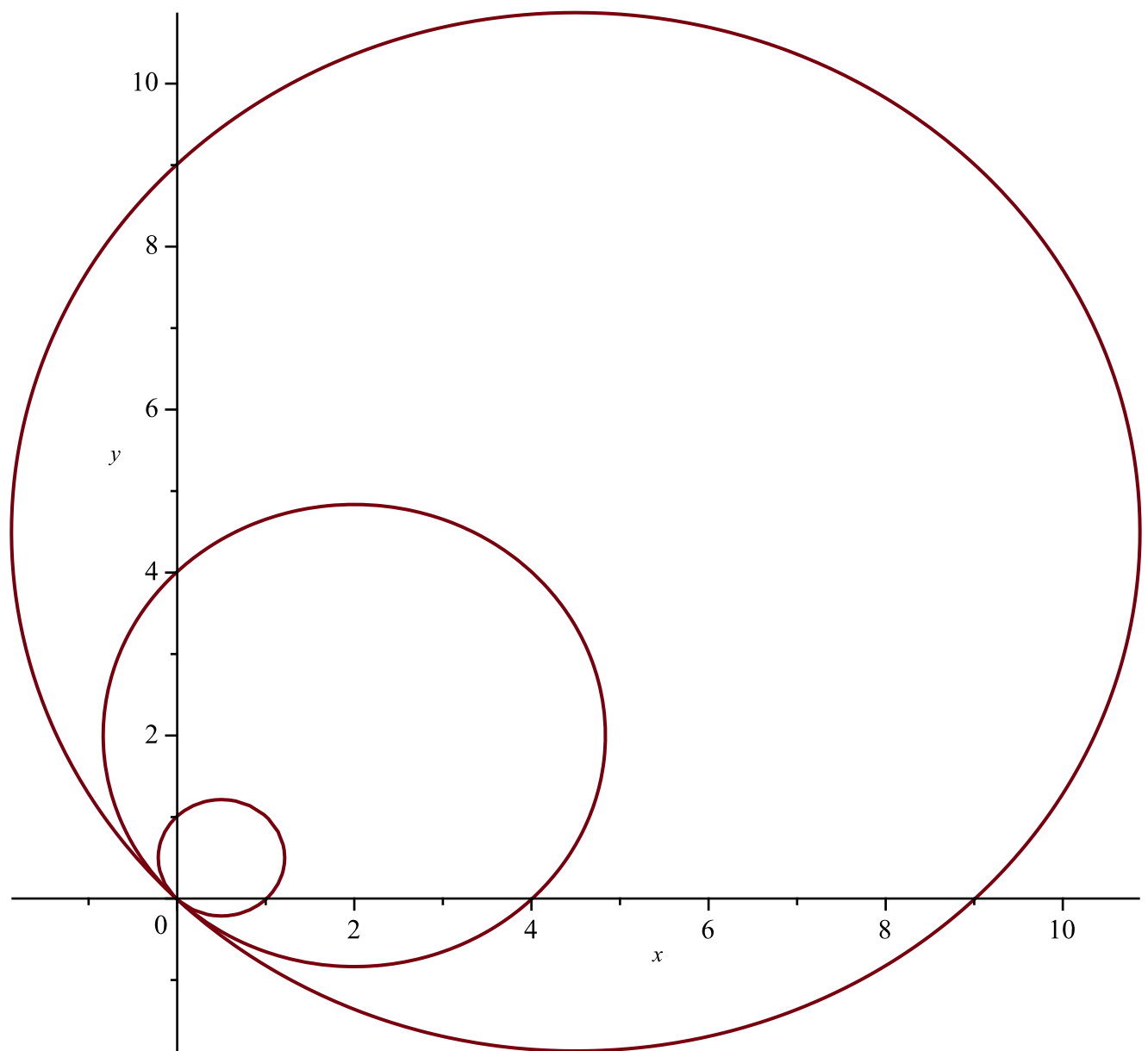$$CondicionInicial := y(0) = 2$$

(16)

> *CondIni*

$$y(0) = 2$$

(17)

> *restart*

_2)

> *with(plots) :*

> $SolGral := \left(y^2 - \_C1^2 \cdot y\right) + \left(x^2 - \_C1^2 \cdot x\right) = \left(\dfrac{\_C1}{10}\right)^2$; *implicitplot({subs(_C1 = 1,*

SolGral), subs(_C1 = 2, SolGral), subs(_C1 = 3, SolGral)})

$$SolGral := -\_C1^2\,x - \_C1^2\,y + x^2 + y^2 = \frac{\_C1^2}{100}$$

> $Solucion := \left(y(x)^2 - \_C1^2 \cdot y \right) + \left(x^2 - \_C1^2 \cdot x \right) = \left(\dfrac{\_C1}{10} \right)^2$

$$Solucion := y(x)^2 - \_C1^2\, y + x^2 - \_C1^2\, x = \dfrac{\_C1^2}{100} \qquad (18)$$

> $simplify\big(isolate\big(rhs\big(isolate(Solucion, \_C1^2) \big) = rhs\big(isolate(diff(Solucion, x), \_C1^2) \big) \big),$
> $\quad diff(y(x), x) \big) \big)$

$$\dfrac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \dfrac{50\, y(x)^2 - 50\left(x + 2\,y + \dfrac{1}{50} \right) x}{(100\, x + 100\, y + 1)\, y(x)} \qquad (19)$$

> $restart$

3) Resuelva por dos métodos

> $Ecua := \left(4 \cdot x \cdot y + y^2 \right) + \left(2 \cdot x^2 + 2 \cdot x \cdot y \right) \cdot y' = 0$

$$\qquad (20)$$

$$Ecua := 4\,x\,y(x) + y(x)^2 + \left(2\,x^2 + 2\,x\,y(x)\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{20}$$

> `with(DEtools)` :

> `odeadvisor(Ecua)`

$$[\,[\,\_homogeneous,\ class\ A\,],\ \_exact,\ \_rational,\ [\,\_Abel,\ 2nd\ type,\ class\ B\,]\,] \tag{21}$$

Por coeficientes homogeneos

> $EcuaDos := simplify(isolate(eval(subs(y(x) = x \cdot u(x), Ecua)), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx}\,u(x) = -\frac{3\,u(x)\,(u(x) + 2)}{2\,(1 + u(x))\,x} \tag{22}$$

> $M := 3\,u\,(u + 2)$

$$M := 3\,u\,(u + 2) \tag{23}$$

> $N := 2\,(1 + u)\,x$

$$N := 2\,(1 + u)\,x \tag{24}$$

> $P := 1;\ Q := 3\,u\,(u + 2);\ R := x;\ S := 2\,(1 + u)$

$$P := 1$$
$$Q := 3\,u\,(u + 2)$$
$$R := x$$
$$S := 2 + 2\,u \tag{25}$$

> $SolGralDos := int\left(\dfrac{P}{R}, x\right) + int\left(\dfrac{S}{Q}, u\right) = \_C1$

$$SolGralDos := \ln(x) + \frac{\ln(u\,(u + 2))}{3} = \_C1 \tag{26}$$

> $SolGralTres := simplify(\exp(lhs(SolGralDos))) = \_C1$

$$SolGralTres := x\,(u\,(u + 2))^{1/3} = \_C1 \tag{27}$$

> $SolGralCuatro := simplify\left(subs\left(u = \dfrac{y(x)}{x}, SolGralTres\right)\right)$

$$SolGralCuatro := x\left(\frac{y(x)\,(y(x) + 2\,x)}{x^2}\right)^{1/3} = \_C1 \tag{28}$$

> $SolFinalUno := expand(lhs(SolGralCuatro)^3) = \_C1$

$$SolFinalUno := x\,y(x)^2 + 2\,x^2\,y(x) = \_C1 \tag{29}$$

> $DerSolFinalUno := isolate(diff(SolFinalUno, x), diff(y(x), x))$

$$DerSolFinalUno := \frac{d}{dx}\,y(x) = \frac{-4\,x\,y(x) - y(x)^2}{2\,x^2 + 2\,x\,y(x)} \tag{30}$$

> $DerEcuaUno := isolate(Ecua, diff(y(x), x))$

$$DerEcuaUno := \frac{d}{dx}\,y(x) = \frac{-4\,x\,y(x) - y(x)^2}{2\,x^2 + 2\,x\,y(x)} \tag{31}$$

> $ComprobarUno := simplify(rhs(DerSolFinalUno) - rhs(DerEcuaUno)) = 0$

$$ComprobarUno := 0 = 0 \tag{32}$$

Método "Exacta"

> *Ecua*

$$4\, x\, y(x) + y(x)^2 + \left(2\, x^2 + 2\, x\, y(x)\right)\left(\frac{d}{dx}\, y(x)\right) = 0 \tag{33}$$

> $MM := 4\, x\, y + y^2$

$$MM := 4\, x\, y + y^2 \tag{34}$$

> $NN := \left(2\, x^2 + 2\, x\, y\right)$

$$NN := 2\, x^2 + 2\, x\, y \tag{35}$$

> $ComprobarExacta := diff(MM, y) = diff(NN, x)$

$$ComprobarExacta := 4\, x + 2\, y = 4\, x + 2\, y \tag{36}$$

> $IntMMx := expand(int(MM, x))$

$$IntMMx := 2\, x^2\, y + x\, y^2 \tag{37}$$

> $SolGralSeis := expand(IntMMx + int((NN - diff(IntMM, y)), y)) = \_C1$

$$SolGralSeis := 4\, x^2\, y + 2\, x\, y^2 = \_C1 \tag{38}$$

> $SolFinalDos := 4\, x^2\, y(x) + 2\, x\, y(x)^2 = \_C1$

$$SolFinalDos := 4\, x^2\, y(x) + 2\, x\, y(x)^2 = \_C1 \tag{39}$$

> $DerSolFinalDos := isolate(diff(SolFinalDos, x), diff(y(x), x))$

$$DerSolFinalDos := \frac{d}{dx}\, y(x) = \frac{-8\, x\, y(x) - 2\, y(x)^2}{4\, x^2 + 4\, x\, y(x)} \tag{40}$$

> $DerEcuaDos := isolate(Ecua, diff(y(x), x))$

$$DerEcuaDos := \frac{d}{dx}\, y(x) = \frac{-4\, x\, y(x) - y(x)^2}{2\, x^2 + 2\, x\, y(x)} \tag{41}$$

> $ComprobarDos := simplify(rhs(DerSolFinalDos) - rhs(DerEcuaDos)) = 0$

$$ComprobarDos := 0 = 0 \tag{42}$$

> $SolFinalUno; \dfrac{lhs(SolFinalDos)}{2} = \_C1$

$$x\, y(x)^2 + 2\, x^2\, y(x) = \_C1$$
$$x\, y(x)^2 + 2\, x^2\, y(x) = \_C1 \tag{43}$$

> *restart*

4)

> $Ecua := \left(x^{-2}\cdot y^{-1} - x^{-1}\right) + \left(x^{-1}\cdot y^{-2} + 1\right)\cdot y' = 0$

$$Ecua := \frac{1}{x^2\, y(x)} - \frac{1}{x} + \left(\frac{1}{x\, y(x)^2} + 1\right)\left(\frac{d}{dx}\, y(x)\right) = 0 \tag{44}$$

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[\_exact, \_rational] \tag{45}$$

> $M := \dfrac{1}{x^2\, y} - \dfrac{1}{x}$

$$M := -\frac{1}{x} + \frac{1}{x^2\, y} \tag{46}$$

> $N := \left(\dfrac{1}{x\,y^2} + 1 \right)$

$$N := \frac{1}{x\,y^2} + 1 \tag{47}$$

> $ComprobarExacta := diff(M, y) = diff(N, x)$

$$ComprobarExacta := -\frac{1}{x^2\,y^2} = -\frac{1}{x^2\,y^2} \tag{48}$$

> $IntNy := int(N, y)$

$$IntNy := y - \frac{1}{x\,y} \tag{49}$$

> $SolGral := IntNy + int((M - diff(IntNy, x)), x) = \_C1$

$$SolGral := y - \frac{1}{x\,y} - \ln(x) = \_C1 \tag{50}$$

> $SolFinal := y(x) - \dfrac{1}{x\,y(x)} - \ln(x) = \_C1$

$$SolFinal := y(x) - \frac{1}{x\,y(x)} - \ln(x) = \_C1 \tag{51}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = \frac{(x\,y(x) - 1)\,y(x)}{x\,(x\,y(x)^2 + 1)} \tag{52}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{(x\,y(x) - 1)\,y(x)}{x\,(x\,y(x)^2 + 1)} \tag{53}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolFinal)) = 0$

$$Comprobar := 0 = 0 \tag{54}$$

> $restart$

5) Resolver el problema de valor inicial

> $Ecua := y' = \dfrac{(2 \cdot x \cdot y - y^2)}{x^2}$

$$Ecua := \frac{d}{dx}\,y(x) = \frac{2\,x\,y(x) - y(x)^2}{x^2} \tag{55}$$

> $CondIni := y(1) = -1$

$$CondIni := y(1) = -1 \tag{56}$$

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_Bernoulli] \tag{57}$$

> $EcuaSeparable := simplify(isolate(eval(subs(y(x) = x \cdot u(x), Ecua)), diff(u(x), x)))$

$$EcuaSeparable := \frac{d}{dx}\,u(x) = -\frac{u(x)\,(u(x) - 1)}{x} \tag{58}$$

> $M := \dfrac{u\,(u-1)}{x}$

$$M := \frac{u\,(u-1)}{x} \tag{59}$$

> $N := 1$

$$N := 1 \tag{60}$$

> $P := \dfrac{1}{x}; Q := u\,(u-1); R := 1; S := 1$

$$P := \frac{1}{x}$$
$$Q := u\,(u-1)$$
$$R := 1$$
$$S := 1 \tag{61}$$

> $SolGral := int\left(\dfrac{P}{R}, x\right) + int\left(\dfrac{S}{Q}, u\right) = \_C1$

$$SolGral := \ln(x) - \ln(u) + \ln(u-1) = \_C1 \tag{62}$$

> $SolGralDos := simplify(\exp(lhs(SolGral)) = \_C1)$

$$SolGralDos := \frac{x\,(u-1)}{u} = \_C1 \tag{63}$$

> $SolGralFinal := simplify\left(subs\left(u = \dfrac{y(x)}{x}, SolGralDos\right)\right)$

$$SolGralFinal := \frac{x\,(y(x)-x)}{y(x)} = \_C1 \tag{64}$$

> $Parametro := subs(x = 1, y(1) = -1, SolGralFinal)$

$$Parametro := 2 = \_C1 \tag{65}$$

> $SolPart := subs(\_C1 = lhs(Parametro), SolGralFinal)$

$$SolPart := \frac{x\,(y(x)-x)}{y(x)} = 2 \tag{66}$$

> $Ecua$

$$\frac{d}{dx}\,y(x) = \frac{2\,x\,y(x) - y(x)^2}{x^2} \tag{67}$$

> $DerSolPart := simplify(isolate(diff(SolPart, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\,y(x) = \frac{y(x)\,(2\,x - y(x))}{x^2} \tag{68}$$

> $ComprobarUno := simplify(rhs(Ecua) - rhs(DerSolPart)) = 0$

$$ComprobarUno := 0 = 0 \tag{69}$$

> $ComprobarDos := isolate(simplify(subs(x = 1, SolPart)), y(1))$

$$ComprobarDos := y(1) = -1 \tag{70}$$

> $CondIni$

$$y(1) = -1 \tag{71}$$

> restart

6) obtener la solución general

> $Ecua := (2 \cdot x \cdot y^2 - y) = -x \cdot y'$

$$Ecua := 2\,x\,y(x)^2 - y(x) = -x\left(\frac{d}{dx}\,y(x)\right) \tag{72}$$

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[[\_homogeneous,\ class\ D],\ \_rational,\ \_Bernoulli] \tag{73}$$

> $intfactor(Ecua)$

$$\frac{1}{y(x)^2} \tag{74}$$

> $FI := \dfrac{1}{y^2}$

$$FI := \frac{1}{y^2} \tag{75}$$

> $M := 2\,x\,y^2 - y$

$$M := 2\,x\,y^2 - y \tag{76}$$

> $N := x$

$$N := x \tag{77}$$

> $MM := expand(FI \cdot M)$

$$MM := 2\,x - \frac{1}{y} \tag{78}$$

> $NN := expand(FI \cdot N)$

$$NN := \frac{x}{y^2} \tag{79}$$

> $ComprobarExacta := diff(MM, y) = diff(NN, x)$

$$ComprobarExacta := \frac{1}{y^2} = \frac{1}{y^2} \tag{80}$$

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{x^2\,y - x}{y} \tag{81}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{x^2\,y - x}{y} = \_C1 \tag{82}$$

> $SolFinal := \dfrac{x^2\,y(x) - x}{y(x)} = \_C1$

$$SolFinal := \frac{x^2\,y(x) - x}{y(x)} = \_C1 \tag{83}$$

> $DerSolFinal := isolate(diff(SolFinal, x), diff(y(x), x))$

$$DerSolFinal := \frac{d}{dx} \, y(x) = \frac{-2 \, x \, y(x)^2 + y(x)}{x} \tag{84}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx} \, y(x) = \frac{-2 \, x \, y(x)^2 + y(x)}{x} \tag{85}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolFinal)) = 0$

$$Comprobar := 0 = 0 \tag{86}$$

> $restart$

7)

> $Ecua := (x \cdot y + y^2 + x^2) - x^2 \cdot y' = 0$

$$Ecua := x \, y(x) + y(x)^2 + x^2 - x^2 \left(\frac{d}{dx} \, y(x) \right) = 0 \tag{87}$$

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[\, [\_homogeneous, class \, A], \_rational, \_Riccati \,] \tag{88}$$

> $EcuaDos := simplify(isolate(eval(subs(y(x) = u(x) \cdot x, Ecua)), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx} \, u(x) = \frac{1 + u(x)^2}{x} \tag{89}$$

> $M := -\left(\frac{1 + u^2}{x} \right)$

$$M := -\frac{u^2 + 1}{x} \tag{90}$$

> $N := 1$

$$N := 1 \tag{91}$$

> $P := -\frac{1}{x}; Q := u^2 + 1; R := 1; S := 1$

$$P := -\frac{1}{x}$$

$$Q := u^2 + 1$$

$$R := 1$$

$$S := 1 \tag{92}$$

> $SolGral := int\left(\frac{P}{R}, x \right) + int\left(\frac{S}{Q}, u \right) = \_C1$

$$SolGral := -\ln(x) + \arctan(u) = \_C1 \tag{93}$$

> $SolFinal := subs\left(u = \frac{y(x)}{x}, SolGral \right)$

$$SolFinal := -\ln(x) + \arctan\left(\frac{y(x)}{x} \right) = \_C1 \tag{94}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$\tag{95}$$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{x\, y(x) + y(x)^2 + x^2}{x^2} \tag{95}$$

> $DerEcua := simplify(isolate(Ecua, diff\,(y(x), x)))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{x\, y(x) + y(x)^2 + x^2}{x^2} \tag{96}$$

> $Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \tag{97}$$

> $restart$

8)

> $Ecua := y' = y \cdot \left(\dfrac{1}{x} - 3 \cdot y \right)$

$$Ecua := \frac{d}{dx}\, y(x) = y(x) \left(\frac{1}{x} - 3\, y(x) \right) \tag{98}$$

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\,[\_homogeneous,\ class\ D],\ \_rational,\ \_Bernoulli\,] \tag{99}$$

> $intfactor(Ecua)$

$$\frac{x}{y(x)^2},\ \frac{1}{y(x)\,(3\,x\,y(x) - 2)} \tag{100}$$

> $FI := \dfrac{x}{y^2}$

$$FI := \frac{x}{y^2} \tag{101}$$

> $M := -y \cdot \left(\dfrac{1}{x} - 3 \cdot y \right)$

$$M := -y \left(\frac{1}{x} - 3\, y \right) \tag{102}$$

> $N := 1$

$$N := 1 \tag{103}$$

> $MM := expand(FI \cdot M)$

$$MM := -\frac{1}{y} + 3\, x \tag{104}$$

> $NN := FI \cdot N$

$$NN := \frac{x}{y^2} \tag{105}$$

> $ComprobarExacta := diff\,(MM, y) = diff\,(NN, x)$

$$ComprobarExacta := \frac{1}{y^2} = \frac{1}{y^2} \tag{106}$$

> $IntNNy := int(NN, y)$

$$\tag{107}$$

$$IntNNy := -\frac{x}{y} \tag{107}$$

> $SolGral := IntNNy + int((MM - diff(IntNNy, x)), x) = \_C1$

$$SolGral := -\frac{x}{y} + \frac{3x^2}{2} = \_C1 \tag{108}$$

> $SolFinal := -\frac{x}{y(x)} + \frac{3x^2}{2} = \_C1$

$$SolFinal := -\frac{x}{y(x)} + \frac{3x^2}{2} = \_C1 \tag{109}$$

> $DerSolFinal := expand(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}y(x) = \frac{y(x)}{x} - 3y(x)^2 \tag{110}$$

> $expand(Ecua)$

$$\frac{d}{dx}y(x) = \frac{y(x)}{x} - 3y(x)^2 \tag{111}$$

> $Comprobar := simplify(rhs(Ecua) - rhs(DerSolFinal)) = 0$

$$Comprobar := 0 = 0 \tag{112}$$

> $restart$

9)

> $Ecua := (y^2 + 4) = (2y + 8y \cdot x) \cdot y'$

$$Ecua := y(x)^2 + 4 = (2y(x) + 8y(x)x)\left(\frac{d}{dx}y(x)\right) \tag{113}$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \tag{114}$$

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[\_separable] \tag{115}$$

> $M := -(y^2 + 4)$

$$M := -y^2 - 4 \tag{116}$$

> $N := factor(2y + 8yx)$

$$N := 2y(4x + 1) \tag{117}$$

> $P := -1; Q := (y^2 + 4); R := (4x + 1); S := 2y$

$$P := -1$$
$$Q := y^2 + 4$$
$$R := 4x + 1$$
$$S := 2y \tag{118}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := -\frac{\ln(4x + 1)}{4} + \ln(y^2 + 4) = \_C1 \tag{119}$$

> $SolFinal := simplify(\exp(lhs(SolGral))) = \_C1$

$$SolFinal := \frac{y^2 + 4}{(4\,x + 1)^{1/4}} = \_C1 \tag{120}$$

> $SolFinalDos := \dfrac{y(x)^2 + 4}{(4\,x + 1)^{1/4}} = \_C1$

$$SolFinalDos := \frac{y(x)^2 + 4}{(4\,x + 1)^{1/4}} = \_C1 \tag{121}$$

> $DerSolFinal := isolate(diff(SolFinalDos, x), diff(y(x), x))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{y(x)^2 + 4}{2\,(4\,x + 1)\,y(x)} \tag{122}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{y(x)^2 + 4}{(8\,x + 2)\,y(x)} \tag{123}$$

> $Comprobar := simplify(lhs(DerEcua) - lhs(DerSolFinal)) = 0$

$$Comprobar := 0 = 0 \tag{124}$$

> restart

>