>

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SERIE TEMA 2
GRUPO 11
SOLUCIÓN

>

> *restart*

1) Resuelva

> $Ecua := y'' + y = \sec(x)^2$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + y(x) = \sec(x)^2 \qquad (1)$$

Respuesta

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) + y(x) = 0 \qquad (2)$$

> $Q := rhs(Ecua)$

$$Q := \sec(x)^2 \qquad (3)$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \qquad (4)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \mathrm{I}, -\mathrm{I} \qquad (5)$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot x); yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot x)$

$$yy_1 := \cos(x)$$
$$yy_2 := \sin(x) \qquad (6)$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(x) = \_C1\,\cos(x) + \_C2\,\sin(x) \qquad (7)$$

> $SolNoHom := y(x) = A\cdot yy[1] + B\cdot yy[2]$

$$SolNoHom := y(x) = A\,\cos(x) + B\,\sin(x) \qquad (8)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \qquad (9)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \sec(x)^2 \end{bmatrix} \qquad (10)$$

> $Parametro := simplify(linsolve(WW, BB))$

$$Parametro := \begin{bmatrix} -\tan(x)\,\sec(x) & \sec(x) \end{bmatrix} \qquad (11)$$

> $Aprima := Parametro[1]; Bprima := Parametro[2]$

$$Aprima := -\tan(x)\sec(x)$$
$$Bprima := \sec(x) \quad \textbf{(12)}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\sec(x) + \_C1$$
$$B := \ln(\sec(x) + \tan(x)) + \_C2 \quad \textbf{(13)}$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = -\cos(x)\sec(x) + \_C1\cos(x) + \sin(x)\ln(\sec(x) + \tan(x)) + \_C2\sin(x) \quad \textbf{(14)}$$

Fin respuesta 1)

> $restart$

2) Obtenga la solución general de la ecuacion diferencial

> $Ecua := x^2 \cdot y'' + x \cdot (5 \cdot x - 1) \cdot y' - 5 \cdot x \cdot y = x^3 \cdot \exp(-5 \cdot x)$

$$Ecua := x^2 \left(\frac{d^2}{dx^2} y(x)\right) + x(5x - 1)\left(\frac{d}{dx} y(x)\right) - 5xy(x) = x^3 e^{-5x} \quad \textbf{(15)}$$

> $yy[1] := 5 \cdot x - 1; yy[2] := \exp(-5 \cdot x)$

$$yy_1 := 5x - 1$$
$$yy_2 := e^{-5x} \quad \textbf{(16)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := x^2 \left(\frac{d^2}{dx^2} y(x)\right) + x(5x - 1)\left(\frac{d}{dx} y(x)\right) - 5xy(x) = 0 \quad \textbf{(17)}$$

Respuesta

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1\,(5x - 1) + \_C2\,e^{-5x} \quad \textbf{(18)}$$

> $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHom)))$

$$ComprobarUno := 0 = 0 \quad \textbf{(19)}$$

> $EcuaHomNormal := expand\left(\frac{lhs(EcuaHom)}{x^2}\right) = 0$

$$EcuaHomNormal := \frac{d^2}{dx^2} y(x) + 5\frac{d}{dx} y(x) - \frac{\frac{d}{dx} y(x)}{x} - \frac{5y(x)}{x} = 0 \quad \textbf{(20)}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHomNormal)))$

$$ComprobarDos := 0 = 0 \quad \textbf{(21)}$$

> $EcuaNoHomNormal := expand\left(\frac{lhs(Ecua)}{x^2}\right) = expand\left(\frac{rhs(Ecua)}{x^2}\right)$

$$EcuaNoHomNormal := \frac{d^2}{dx^2} y(x) + 5\frac{d}{dx} y(x) - \frac{\frac{d}{dx} y(x)}{x} - \frac{5y(x)}{x} = \frac{x}{(e^x)^5} \quad \textbf{(22)}$$

> $Q := rhs(EcuaNoHomNormal)$

$$Q := \frac{x}{(e^x)^5} \quad \textbf{(23)}$$

> $SolNoHom := y(x) = A\cdot (5x-1) + B\cdot e^{-5x}$

$$SolNoHom := y(x) = A\,(5x-1) + B\,e^{-5x} \tag{24}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} 5x-1 & e^{-5x} \\ 5 & -5\,e^{-5x} \end{bmatrix} \tag{25}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{x}{(e^{x})^{5}} \end{bmatrix} \tag{26}$$

> $Parametro := simplify(linsolve(WW, BB))$

$$Parametro := \begin{bmatrix} \frac{e^{-5x}}{25} & -\frac{x}{5} + \frac{1}{25} \end{bmatrix} \tag{27}$$

> $Aprima := Parametro[1]; Bprima := Parametro[2]$

$$Aprima := \frac{e^{-5x}}{25}$$

$$Bprima := -\frac{x}{5} + \frac{1}{25} \tag{28}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\frac{e^{-5x}}{125} + \_C1$$

$$B := -\frac{1}{10}x^2 + \frac{1}{25}x + \_C2 \tag{29}$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = \frac{1}{125\,(e^{x})^{5}} + 5\,\_C1\,x - \_C1 - \frac{x^2}{10\,(e^{x})^{5}} + \frac{\_C2}{(e^{x})^{5}} \tag{30}$$

> $SolHomFinal := y(x) = \_C1\cdot(5\cdot x - 1) + \_C2\cdot \exp(-5\cdot x)$

$$SolHomFinal := y(x) = \_C1\,(5x-1) + \_C2\,e^{-5x} \tag{31}$$

> $SolPartFinal := y(x) = -\frac{x^2\cdot \exp(-5\cdot x)}{10}$

$$SolPartFinal := y(x) = -\frac{x^2\,e^{-5x}}{10} \tag{32}$$

> $SolFinalDos := y(x) = rhs(SolHomFinal) + rhs(SolPartFinal)$

$$SolFinalDos := y(x) = \_C1\,(5x-1) + \_C2\,e^{-5x} - \frac{x^2\,e^{-5x}}{10} \tag{33}$$

> $Ecua$

$$x^2\left(\frac{d^2}{dx^2}\,y(x)\right) + x\,(5x-1)\left(\frac{d}{dx}\,y(x)\right) - 5\,x\,y(x) = x^3\,e^{-5x} \tag{34}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinalDos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \tag{35}$$

Fin respuesta 2)

> $restart$

3) Resuelva la ecuacion diferencial

> $Ecua := x^3 \cdot y'' - 3 \cdot x^2 \cdot y' + 3 \cdot x \cdot y = x^5 + 2 \cdot x^3$

$$Ecua := x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3 x^2 \left(\frac{d}{dx} y(x)\right) + 3 x y(x) = x^5 + 2 x^3 \tag{36}$$

> $yy[1] := x; yy[2] := x^3; yy[3] := 2 \cdot x^3 - x$

$$yy_1 := x$$

$$yy_2 := x^3$$

$$yy_3 := 2 x^3 - x \tag{37}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3 x^2 \left(\frac{d}{dx} y(x)\right) + 3 x y(x) = 0 \tag{38}$$

Respuesta

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C2 x^3 + \_C1 x \tag{39}$$

> $SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$

$$SolNoHom := y(x) = B x^3 + A x \tag{40}$$

> $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHom)))$

$$ComprobarUno := 0 = 0 \tag{41}$$

> $EcuaHomNormal := expand\left(\frac{lhs(EcuaHom)}{x^3}\right) = 0$

$$EcuaHomNormal := \frac{d^2}{dx^2} y(x) - \frac{3 \left(\frac{d}{dx} y(x)\right)}{x} + \frac{3 y(x)}{x^2} = 0 \tag{42}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHomNormal)))$

$$ComprobarDos := 0 = 0 \tag{43}$$

> $EcuaNoHomNormal := expand\left(\frac{lhs(Ecua)}{x^3}\right) = expand\left(\frac{rhs(Ecua)}{x^3}\right)$

$$EcuaNoHomNormal := \frac{d^2}{dx^2} y(x) - \frac{3 \left(\frac{d}{dx} y(x)\right)}{x} + \frac{3 y(x)}{x^2} = x^2 + 2 \tag{44}$$

> $Q := rhs(EcuaNoHomNormal)$

$$Q := x^2 + 2 \tag{45}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} x & x^3 \\ 1 & 3x^2 \end{bmatrix} \quad \textbf{(46)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x^2 + 2 \end{bmatrix} \quad \textbf{(47)}$$

> $Parametro := linsolve(WW, BB)$

$$Parametro := \begin{bmatrix} -\frac{x^2}{2} - 1 & \frac{x^2+2}{2x^2} \end{bmatrix} \quad \textbf{(48)}$$

> $Aprima := Parametro[1]; Bprima := expand(Parametro[2])$

$$Aprima := -\frac{x^2}{2} - 1$$

$$Bprima := \frac{1}{2} + \frac{1}{x^2} \quad \textbf{(49)}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\frac{1}{6}x^3 - x + \_C1$$

$$B := \frac{x}{2} - \frac{1}{x} + \_C2 \quad \textbf{(50)}$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = \frac{1}{3}x^4 - 2x^2 + x^3\,\_C2 + x\,\_C1 \quad \textbf{(51)}$$

> $Ecua$

$$x^3\left(\frac{d^2}{dx^2}y(x)\right) - 3x^2\left(\frac{d}{dx}y(x)\right) + 3x\,y(x) = x^5 + 2x^3 \quad \textbf{(52)}$$

> $ComprobarTres := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarTres := 0 = 0 \quad \textbf{(53)}$$

Fin respuesta 3)

> $restart$

4) Determine la solución general

> $Ecua := 4\cdot t\cdot diff(y(t), t\$2) + t\cdot y(t) = t + 2\cdot t\cdot \sin(3\cdot t)$

$$Ecua := 4\,t\left(\frac{d^2}{dt^2}y(t)\right) + t\,y(t) = t + 2\,t\sin(3\,t) \quad \textbf{(54)}$$

Respuesta

> $EcuaNorm := expand\left(\frac{lhs(Ecua)}{4\cdot t}\right) = simplify\left(\frac{rhs(Ecua)}{4\cdot t}\right)$

$$EcuaNorm := \frac{d^2}{dt^2}y(t) + \frac{y(t)}{4} = \frac{1}{4} + \frac{\sin(3\,t)}{2} \quad \textbf{(55)}$$

> $EcuaHom := lhs(EcuaNorm) = 0$

(56)

$$EcuaHom := \frac{d^2}{dt^2}\, y(t) + \frac{y(t)}{4} = 0 \quad \textbf{(56)}$$

> $Q := rhs(EcuaNorm)$

$$Q := \frac{1}{4} + \frac{\sin(3\,t)}{2} \quad \textbf{(57)}$$

> $EcuaCarac := m^2 + \frac{1}{4} = 0$

$$EcuaCarac := m^2 + \frac{1}{4} = 0 \quad \textbf{(58)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \frac{I}{2}, -\frac{I}{2} \quad \textbf{(59)}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot t); yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot t)$

$$yy_1 := \cos\left(\frac{t}{2}\right)$$

$$yy_2 := \sin\left(\frac{t}{2}\right) \quad \textbf{(60)}$$

> $SolHom := y(t) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(t) = \_C1\, \cos\left(\frac{t}{2}\right) + \_C2\, \sin\left(\frac{t}{2}\right) \quad \textbf{(61)}$$

> $SolNoHom := y(t) = A\cdot yy[1] + B\cdot yy[2]$

$$SolNoHom := y(t) = A\, \cos\left(\frac{t}{2}\right) + B\, \sin\left(\frac{t}{2}\right) \quad \textbf{(62)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], t)$

$$WW := \begin{bmatrix} \cos\left(\frac{t}{2}\right) & \sin\left(\frac{t}{2}\right) \\ -\frac{\sin\left(\frac{t}{2}\right)}{2} & \frac{\cos\left(\frac{t}{2}\right)}{2} \end{bmatrix} \quad \textbf{(63)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{1}{4} + \frac{\sin(3\,t)}{2} \end{bmatrix} \quad \textbf{(64)}$$

> $Parametro := simplify(linsolve(WW, BB))$

$$Parametro := \begin{bmatrix} -\frac{\sin\left(\frac{t}{2}\right)\,(1 + 2\sin(3\,t))}{2} & \frac{\cos\left(\frac{t}{2}\right)\,(1 + 2\sin(3\,t))}{2} \end{bmatrix} \quad \textbf{(65)}$$

> $Aprima := Parametro[1]; Bprima := Parametro[2]$

$$Aprima := -\frac{\sin\left(\frac{t}{2}\right)\left(1+2\sin(3\,t)\right)}{2}$$

$$Bprima := \frac{\cos\left(\frac{t}{2}\right)\left(1+2\sin(3\,t)\right)}{2} \quad \textbf{(66)}$$

> $A := int(Aprima, t) + \_C1; B := int(Bprima, t) + \_C2$

$$A := \frac{\sin\left(\frac{7\,t}{2}\right)}{7} - \frac{\sin\left(\frac{5\,t}{2}\right)}{5} + \cos\left(\frac{t}{2}\right) + \_C1$$

$$B := \sin\left(\frac{t}{2}\right) - \frac{\cos\left(\frac{5\,t}{2}\right)}{5} - \frac{\cos\left(\frac{7\,t}{2}\right)}{7} + \_C2 \quad \textbf{(67)}$$

> $SolFinal := simplify(SolNoHom)$

$$SolFinal := y(t) = 1 + \_C1\cos\left(\frac{t}{2}\right) + \_C2\sin\left(\frac{t}{2}\right) - \frac{4\cos\left(\frac{t}{2}\right)\left(16\cos\left(\frac{t}{2}\right)^4 - 16\cos\left(\frac{t}{2}\right)^2 + 3\right)\sin\left(\frac{t}{2}\right)}{35} \quad \textbf{(68)}$$

> $Ecua$

$$4\,t\left(\frac{d^2}{dt^2}\,y(t)\right) + t\,y(t) = t + 2\,t\sin(3\,t) \quad \textbf{(69)}$$

> $ComprobarUno := simplify(eval(subs(y(t) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarUno := 0 = 0 \quad \textbf{(70)}$$

Fin respuesta 4)

> $restart$

5) Obtener la solución general

> $Ecua := x \cdot y'' + (1 - 2\,x) \cdot y' + (x - 1) \cdot y = x \cdot \exp(x)$

$$Ecua := x\left(\frac{d^2}{dx^2}\,y(x)\right) + (1 - 2\,x)\left(\frac{d}{dx}\,y(x)\right) + (x - 1)\,y(x) = x\,e^x \quad \textbf{(71)}$$

> $yy[1] := \exp(x); yy[2] := \exp(x) \cdot \log(x)$

$$yy_1 := e^x$$

$$yy_2 := e^x \ln(x) \quad \textbf{(72)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := x\left(\frac{d^2}{dx^2}\,y(x)\right) + (1 - 2\,x)\left(\frac{d}{dx}\,y(x)\right) + (x - 1)\,y(x) = 0 \quad \textbf{(73)}$$

Respuesta

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1\,e^x + \_C2\,e^x \ln(x) \quad \textbf{(74)}$$

> $ComprobarCero := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHom)))$

$$ComprobarCero := 0 = 0 \quad (75)$$

> $EcuaHomNormal := expand\left(\frac{lhs(EcuaHom)}{x}\right) = 0$

$$EcuaHomNormal := \frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} - 2\,\frac{d}{dx}\, y(x) + y(x) - \frac{y(x)}{x} = 0 \quad (76)$$

> $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHomNormal)))$

$$ComprobarUno := 0 = 0 \quad (77)$$

> $EcuaNorm := expand\left(\frac{lhs(Ecua)}{x}\right) = expand\left(\frac{rhs(Ecua)}{x}\right)$

$$EcuaNorm := \frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} - 2\,\frac{d}{dx}\, y(x) + y(x) - \frac{y(x)}{x} = e^x \quad (78)$$

> $Q := rhs(EcuaNorm)$

$$Q := e^x \quad (79)$$

> $SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$

$$SolNoHom := y(x) = A\, e^x + B\, e^x \ln(x) \quad (80)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & e^x \ln(x) \\ e^x & e^x \ln(x) + \frac{e^x}{x} \end{bmatrix} \quad (81)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^x \end{bmatrix} \quad (82)$$

> $Parametro := linsolve(WW, BB)$

$$Parametro := \begin{bmatrix} -\ln(x)\, x & x \end{bmatrix} \quad (83)$$

> $Aprima := Parametro[1];\ Bprima := Parametro[2]$

$$Aprima := -\ln(x)\, x$$

$$Bprima := x \quad (84)$$

> $A := int(Aprima, x) + \_C1;\ B := int(Bprima, x) + \_C2$

$$A := -\frac{\ln(x)\, x^2}{2} + \frac{x^2}{4} + \_C1$$

$$B := \frac{x^2}{2} + \_C2 \quad (85)$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = \frac{e^x x^2}{4} + \_C1\, e^x + \_C2\, e^x \ln(x) \quad (86)$$

> *Ecua*

$$x\left(\frac{d^2}{dx^2}\,y(x)\right)+(1-2\,x)\left(\frac{d}{dx}\,y(x)\right)+(x-1)\,y(x)=x\,e^x \quad \textbf{(87)}$$

> *ComprobarDos* := *simplify*(*eval*(*subs*($y(x)$ = *rhs*(*SolFinal*), *lhs*(*Ecua*) − *rhs*(*Ecua*) = 0)))

$$ComprobarDos := 0=0 \quad \textbf{(88)}$$

Fin respuesta 5)

> *restart*

6) Obtenga la solución general

> *Ecua* := $y''-y=\exp(x)+\cos(2\,x)$

$$Ecua := \frac{d^2}{dx^2}\,y(x)-y(x)=e^x+\cos(2\,x) \quad \textbf{(89)}$$

Respuesta

> *EcuaHom* := *lhs*(*Ecua*) = 0

$$EcuaHom := \frac{d^2}{dx^2}\,y(x)-y(x)=0 \quad \textbf{(90)}$$

> Q := *rhs*(*Ecua*)

$$Q := e^x+\cos(2\,x) \quad \textbf{(91)}$$

> *EcuaCarac* := $m^2-1=0$

$$EcuaCarac := m^2-1=0 \quad \textbf{(92)}$$

> *Raiz* := *solve*(*EcuaCarac*)

$$Raiz := 1,\,-1 \quad \textbf{(93)}$$

> *yy*[1] := exp(*Raiz*[1]·*x*); *yy*[2] := exp(*Raiz*[2]·*x*)

$$yy_1 := e^x$$

$$yy_2 := e^{-x} \quad \textbf{(94)}$$

> *SolHom* := $y(x)$ = *_C1*·*yy*[1] + *_C2*·*yy*[2]

$$SolHom := y(x)=\_C1\,e^x+\_C2\,e^{-x} \quad \textbf{(95)}$$

> *SolNoHom* := $y(x)$ = *A*·*yy*[1] + *B*·*yy*[2]

$$SolNoHom := y(x)=A\,e^x+B\,e^{-x} \quad \textbf{(96)}$$

> *with*(*linalg*) :

> *WW* := *wronskian*([*yy*[1], *yy*[2]], *x*)

$$WW := \begin{bmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{bmatrix} \quad \textbf{(97)}$$

> *BB* := *array*([0, *Q*])

$$BB := \begin{bmatrix} 0 & e^x+\cos(2\,x) \end{bmatrix} \quad \textbf{(98)}$$

> *ParaVar* := *simplify*(*linsolve*(*WW*, *BB*))

$$ParaVar := \begin{bmatrix} \frac{1}{2}+\frac{\cos(2\,x)\,e^{-x}}{2} & -\frac{\left(e^x+\cos(2\,x)\right)e^x}{2} \end{bmatrix} \quad \textbf{(99)}$$

> $Aprima := ParaVar[1];\ Bprima := ParaVar[2]$

$$Aprima := \frac{1}{2} + \frac{\cos(2\,x)\,e^{-x}}{2}$$

$$Bprima := -\frac{(e^{x} + \cos(2\,x))\,e^{x}}{2} \qquad \textbf{(100)}$$

> $A := int(Aprima, x) + \_C1;\ B := int(Bprima, x) + \_C2$

$$A := \frac{x}{2} - \frac{\cos(2\,x)\,e^{-x}}{10} + \frac{\sin(2\,x)\,e^{-x}}{5} + \_C1$$

$$B := -\frac{(e^{x})^{2}}{4} - \frac{(\cos(x) + 2\sin(x))\,e^{x}\cos(x)}{5} + \frac{e^{x}}{10} + \_C2 \qquad \textbf{(101)}$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = \frac{e^{x}\,x}{2} - \frac{2\cos(x)^{2}}{5} + \frac{1}{5} + \_C1\,e^{x} - \frac{e^{x}}{4} + \frac{\_C2}{e^{x}} \qquad \textbf{(102)}$$

> $SolUltima := dsolve(Ecua)$

$$SolUltima := y(x) = c_2\,e^{-x} + c_1\,e^{x} - \frac{\cos(2\,x)}{5} + \frac{(-1+2\,x)\,e^{x}}{4} \qquad \textbf{(103)}$$

> $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarUno := 0 = 0 \qquad \textbf{(104)}$$

Fin respuesta 6)

> $restart$

7) Sea la función

> $SolPart := y(x) = 4\cdot\cos(\log(x)) + 10\cdot\sin(\log(x))$

$$SolPart := y(x) = 4\cos(\ln(x)) + 10\sin(\ln(x)) \qquad \textbf{(105)}$$

una solución de la ecuación diferencial

> $EcuaHom := x^{2}\cdot y'' + x\cdot y' + y = 0$

$$EcuaHom := x^{2}\left(\frac{d^{2}}{dx^{2}}\,y(x)\right) + x\left(\frac{d}{dx}\,y(x)\right) + y(x) = 0 \qquad \textbf{(106)}$$

que satisface las condiciones

> $CondIniHom := y(1) = 4, D(y)(1) = 10$

$$CondIniHom := y(1) = 4, D(y)(1) = 10 \qquad \textbf{(107)}$$

Resuelva el problema

> $Ecua := lhs(EcuaHom) = \log(x)$

$$Ecua := x^{2}\left(\frac{d^{2}}{dx^{2}}\,y(x)\right) + x\left(\frac{d}{dx}\,y(x)\right) + y(x) = \ln(x) \qquad \textbf{(108)}$$

> $CondIni := CondIniHom$

$$CondIni := y(1) = 4, D(y)(1) = 10 \qquad \textbf{(109)}$$

Respuesta

> $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolPart), EcuaHom)))$

$$ComprobarUno := 0 = 0 \qquad \textbf{(110)}$$

> $EcuaHomNormal := expand\left(\frac{lhs(EcuaHom)}{x^2}\right) = 0$

$$EcuaHomNormal := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} + \frac{y(x)}{x^2} = 0 \quad \mathbf{(111)}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolPart), EcuaHomNormal)))$

$$ComprobarDos := 0 = 0 \quad \mathbf{(112)}$$

> $yy[1] := \cos(\ln(x)); yy[2] := \sin(\log(x))$

$$yy_1 := \cos(\ln(x))$$

$$yy_2 := \sin(\ln(x)) \quad \mathbf{(113)}$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1 \cos(\ln(x)) + \_C2 \sin(\ln(x)) \quad \mathbf{(114)}$$

> $ParaUno := simplify(subs(x = 1, rhs(SolHom))) = 4$

$$ParaUno := \_C1 = 4 \quad \mathbf{(115)}$$

> $ParaDos := simplify(subs(x = 1, rhs(diff(SolHom, x)))) = 10$

$$ParaDos := \_C2 = 10 \quad \mathbf{(116)}$$

> $SolPartDos := subs(ParaUno, ParaDos, SolHom)$

$$SolPartDos := y(x) = 4 \cos(\ln(x)) + 10 \sin(\ln(x)) \quad \mathbf{(117)}$$

> $EcuaNormal := expand\left(\frac{lhs(Ecua)}{x^2}\right) = \frac{rhs(Ecua)}{x^2}$

$$EcuaNormal := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} + \frac{y(x)}{x^2} = \frac{\ln(x)}{x^2} \quad \mathbf{(118)}$$

> $Q := rhs(EcuaNormal)$

$$Q := \frac{\ln(x)}{x^2} \quad \mathbf{(119)}$$

> $SolNoHom := y(x) = A \cdot \cos(\ln(x)) + B \cdot \sin(\ln(x))$

$$SolNoHom := y(x) = A \cos(\ln(x)) + B \sin(\ln(x)) \quad \mathbf{(120)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(\ln(x)) & \sin(\ln(x)) \\ -\frac{\sin(\ln(x))}{x} & \frac{\cos(\ln(x))}{x} \end{bmatrix} \quad \mathbf{(121)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{\ln(x)}{x^2} \end{bmatrix} \quad \mathbf{(122)}$$

> $ParaVar := simplify(linsolve(WW, BB))$

(123)

$$ParaVar := \left[-\frac{\ln(x)\sin(\ln(x))}{x} \quad \frac{\ln(x)\cos(\ln(x))}{x} \right] \quad \textbf{(123)}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{\ln(x)\sin(\ln(x))}{x}$$

$$Bprima := \frac{\ln(x)\cos(\ln(x))}{x} \quad \textbf{(124)}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\sin(\ln(x)) + \cos(\ln(x))\ln(x) + \_C1$$

$$B := \cos(\ln(x)) + \sin(\ln(x))\ln(x) + \_C2 \quad \textbf{(125)}$$

> $SolGralFinal := simplify(expand(SolNoHom))$

$$SolGralFinal := y(x) = \_C1\cos(\ln(x)) + \_C2\sin(\ln(x)) + \ln(x) \quad \textbf{(126)}$$

> $ConstanteUno := simplify(subs(x = 1, rhs(SolGralFinal) = 4))$

$$ConstanteUno := \_C1 = 4 \quad \textbf{(127)}$$

> $ConstanteDos := isolate(simplify(subs(x = 1, rhs(diff(SolGralFinal, x)) = 10)), \_C2)$

$$ConstanteDos := \_C2 = 9 \quad \textbf{(128)}$$

> $SolPartFinal := subs(\_C1 = rhs(ConstanteUno), \_C2 = rhs(ConstanteDos), SolGralFinal)$

$$SolPartFinal := y(x) = 4\cos(\ln(x)) + 9\sin(\ln(x)) + \ln(x) \quad \textbf{(129)}$$

Fin respuesta 7)

> $restart$

8) Obtenga la solución general

> $Ecua := diff(y(\mathrm{theta}), \mathrm{theta\$3}) + diff(y(\mathrm{theta}), \mathrm{theta}) = \csc(\mathrm{theta}) \cdot \cot(\mathrm{theta})$

$$Ecua := \frac{d^3}{d\theta^3} y(\theta) + \frac{d}{d\theta} y(\theta) = \csc(\theta)\cot(\theta) \quad \textbf{(130)}$$

Respuesta

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^3}{d\theta^3} y(\theta) + \frac{d}{d\theta} y(\theta) = 0 \quad \textbf{(131)}$$

> $Q := rhs(Ecua)$

$$Q := \csc(\theta)\cot(\theta) \quad \textbf{(132)}$$

> $EcuaCarac := m^3 + m = 0$

$$EcuaCarac := m^3 + m = 0 \quad \textbf{(133)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 0, \mathrm{I}, -\mathrm{I} \quad \textbf{(134)}$$

> $yy[1] := \exp(Raiz[1] \cdot \mathrm{theta}); yy[2] := \cos(\mathrm{Im}(Raiz[2]) \cdot \mathrm{theta}); yy[3] := \sin(\mathrm{Im}(Raiz[2]) \cdot \mathrm{theta})$

$$yy_1 := 1$$

$$yy_2 := \cos(\theta)$$

(135)

$$yy_3 := \sin(\theta) \quad \textbf{(135)}$$

> $SolHom := y(\text{theta}) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2] + \_C3 \cdot yy[3]$

$$SolHom := y(\theta) = \_C1 + \_C2\cos(\theta) + \_C3\sin(\theta) \quad \textbf{(136)}$$

> $SolNoHom := y(\text{theta}) = AA \cdot yy[1] + BB \cdot yy[2] + DD \cdot yy[3]$

$$SolNoHom := y(\theta) = AA + BB\cos(\theta) + DD\sin(\theta) \quad \textbf{(137)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2], yy[3]], \text{theta})$

$$WW := \begin{bmatrix} 1 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \\ 0 & -\cos(\theta) & -\sin(\theta) \end{bmatrix} \quad \textbf{(138)}$$

> $BB := array([0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & \csc(\theta)\cot(\theta) \end{bmatrix} \quad \textbf{(139)}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} \csc(\theta)\cot(\theta) & -\cot(\theta)^2 & -\cot(\theta) \end{bmatrix} \quad \textbf{(140)}$$

> $AAprima := ParaVar[1]; BBprima := ParaVar[2]; DDprima := ParaVar[3]$

$$AAprima := \csc(\theta)\cot(\theta)$$

$$BBprima := -\cot(\theta)^2$$

$$DDprima := -\cot(\theta) \quad \textbf{(141)}$$

> $AA := int(AAprima, \text{theta}) + \_C1; BB := int(BBprima, \text{theta}) + \_C2; DD := int(DDprima, \text{theta}) + \_C3$

$$AA := -\csc(\theta) + \_C1$$

$$BB := \cot(\theta) - \frac{\pi}{2} + \theta + \_C2$$

$$DD := -\ln(\sin(\theta)) + \_C3 \quad \textbf{(142)}$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(\theta) = -\csc(\theta) + \_C1 + \cos(\theta)\cot(\theta) - \frac{\cos(\theta)\pi}{2} + \cos(\theta)\theta + \_C2\cos(\theta) - \sin(\theta)\ln(\sin(\theta)) + \_C3\sin(\theta) \quad \textbf{(143)}$$

> $SolHomFinal := y(\theta) = \_C1 + \_C2\cos(\theta) + \_C3\sin(\theta)$

$$SolHomFinal := y(\theta) = \_C1 + \_C2\cos(\theta) + \_C3\sin(\theta) \quad \textbf{(144)}$$

> $SolPartNoHom := y(\theta) = -\csc(\theta) + \cos(\theta)\cot(\theta) - \frac{\cos(\theta)\pi}{2} + \cos(\theta)\theta - \sin(\theta)\ln(\sin(\theta))$

$$SolPartNoHom := y(\theta) = -\csc(\theta) + \cos(\theta)\cot(\theta) - \frac{\cos(\theta)\pi}{2} + \cos(\theta)\theta - \sin(\theta)\ln(\sin(\theta)) \quad \textbf{(145)}$$

> $ComprobarUno := expand(eval(subs(y(\theta) = rhs(SolHomFinal), EcuaHom)))$

$$ComprobarUno := 0 = 0 \quad (146)$$

> $ComprobarDos := simplify(eval(subs(y(\theta) = rhs(SolPartNoHom), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \quad (147)$$

Fin respuesta 8)

> restart

9) Obtenga la solución

> $Ecua := x \cdot (y'' + 6\,y' + 9\,y) = -x^2 \exp(4\,x)$

$$Ecua := x \left(\frac{d^2}{dx^2}\, y(x) + 6\, \frac{d}{dx}\, y(x) + 9\, y(x) \right) = -x^2\, e^{4\,x} \quad (148)$$

sujeta a las condiciones

> $CondIni := y(0) = \mathrm{Pi},\ D(y)(0) = 1$

$$CondIni := y(0) = \pi,\ D(y)(0) = 1 \quad (149)$$

Respuesta

> $EcuaNorm := \frac{lhs(Ecua)}{x} = \frac{rhs(Ecua)}{x}$

$$EcuaNorm := \frac{d^2}{dx^2}\, y(x) + 6\, \frac{d}{dx}\, y(x) + 9\, y(x) = -x\, e^{4\,x} \quad (150)$$

> $EcuaNormHom := lhs(EcuaNorm) = 0$

$$EcuaNormHom := \frac{d^2}{dx^2}\, y(x) + 6\, \frac{d}{dx}\, y(x) + 9\, y(x) = 0 \quad (151)$$

> $Q := rhs(EcuaNorm)$

$$Q := -x\, e^{4\,x} \quad (152)$$

> $EcuaCarac := m^2 + 6\,m + 9 = 0$

$$EcuaCarac := m^2 + 6\,m + 9 = 0 \quad (153)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -3,\ -3 \quad (154)$$

como son raíces reales e iguales corresponde al caso 2

> $yy[1] := \exp(Raiz[1] \cdot x);\ yy[2] := x \cdot \exp(Raiz[1] \cdot x)$

$$yy_1 := e^{-3\,x}$$

$$yy_2 := x\, e^{-3\,x} \quad (155)$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1\, e^{-3\,x} + \_C2\, x\, e^{-3\,x} \quad (156)$$

> $SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$

$$SolNoHom := y(x) = A\, e^{-3\,x} + B\, x\, e^{-3\,x} \quad (157)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-3x} & x\,e^{-3x} \\ -3\,e^{-3x} & e^{-3x} - 3\,x\,e^{-3x} \end{bmatrix} \quad \textbf{(158)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & -x\,e^{4x} \end{bmatrix} \quad \textbf{(159)}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} x^2\,e^{7x} & -x\,e^{7x} \end{bmatrix} \quad \textbf{(160)}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := x^2\,e^{7x}$$

$$Bprima := -x\,e^{7x} \quad \textbf{(161)}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := \frac{(49\,x^2 - 14\,x + 2)\,e^{7x}}{343} + \_C1$$

$$B := -\frac{(7\,x - 1)\,e^{7x}}{49} + \_C2 \quad \textbf{(162)}$$

> $SolGralFinal := expand(SolNoHom)$

$$SolGralFinal := y(x) = -\frac{(e^x)^4\,x}{49} + \frac{2\,(e^x)^4}{343} + \frac{\_C1}{(e^x)^3} + \frac{x\,\_C2}{(e^x)^3} \quad \textbf{(163)}$$

> $CondIni$

$$y(0) = \pi, D(y)(0) = 1 \quad \textbf{(164)}$$

> $EcuaUno := expand(subs(x = 0, rhs(SolGralFinal) = \mathrm{Pi}))$

$$EcuaUno := \frac{2}{343} + \_C1 = \pi \quad \textbf{(165)}$$

> $EcuaDos := expand(subs(x = 0, rhs(diff(SolGralFinal, x)) = 1))$

$$EcuaDos := \frac{1}{343} - 3\,\_C1 + \_C2 = 1 \quad \textbf{(166)}$$

> $Para := solve([EcuaUno, EcuaDos])$

$$Para := \left\{\_C1 = -\frac{2}{343} + \pi, \_C2 = \frac{48}{49} + 3\,\pi\right\} \quad \textbf{(167)}$$

> $SolPartFinal := expand(subs(Para[1], Para[2], SolGralFinal))$

$$SolPartFinal := y(x) = -\frac{(e^x)^4\,x}{49} + \frac{2\,(e^x)^4}{343} - \frac{2}{343\,(e^x)^3} + \frac{\pi}{(e^x)^3} + \frac{48\,x}{49\,(e^x)^3} + \frac{3\,x\,\pi}{(e^x)^3} \quad \textbf{(168)}$$

> $SolParticularFinal := y(x) = \left(\frac{2}{343}\right)\cdot\exp(4\,x) - \left(\frac{1}{49}\right)\cdot x\cdot\exp(4\,x) + \left(-\frac{2}{343} + \mathrm{Pi}\right)\cdot\exp(-3\,x) + \left(\frac{48}{49} + 3\cdot\mathrm{Pi}\right)\cdot x\cdot\exp(-3\,x)$

$$SolParticularFinal := y(x) = \frac{2\,e^{4x}}{343} - \frac{x\,e^{4x}}{49} + \left(-\frac{2}{343} + \pi\right)e^{-3x} + \left(\frac{48}{49} + 3\,\pi\right)x\,e^{-3x} \quad \textbf{(169)}$$

> $ComprobarUno := simplify(eval(subs(x=0, SolParticularFinal)))$

$$ComprobarUno := y(0) = \pi \quad (170)$$

> $ComporbarDos := D(y)(0) = simplify(eval(subs(x=0, rhs(diff(SolParticularFinal, x)))))$

$$ComporbarDos := D(y)(0) = 1 \quad (171)$$

> $ComprobarTres := simplify(eval(subs(y(x) = rhs(SolParticularFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarTres := 0 = 0 \quad (172)$$

Fin respuesta 9)

> $restart$

>

>