> *restart*

FACULTAD DE INGENIERIA
DIVISION DE CIENCIAS BASICAS
ECUACIONES DIFERENCIALES
GRUPO 13 SEMESTRE 2025-1
SERIE 1
SOLUCIÓN

> *restart*

1)

> $Ecua := (\exp(x) \cdot \log(y)) + (2^{-1} \cdot \exp(2 \cdot x) \cdot y^{-1}) \cdot y' = 0$

$$Ecua := e^x \ln(y(x)) + \frac{e^{2x}\left(\dfrac{d}{dx}\, y(x)\right)}{2\, y(x)} = 0 \tag{1}$$

> $CondIni := y(0) = \exp(1)$

$$CondIni := y(0) = e \tag{2}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_separable] \tag{3}$$

> $M := e^x \ln(y)$

$$M := e^x \ln(y) \tag{4}$$

> $N := \dfrac{e^{2x}}{2\, y}$

$$N := \frac{e^{2x}}{2\, y} \tag{5}$$

> $P := e^x; Q := \ln(y); R := e^{2x}; S := \dfrac{1}{2\, y}$

$$P := e^x$$

$$Q := \ln(y)$$

$$R := e^{2x}$$

$$S := \frac{1}{2\, y} \tag{6}$$

> $SolGral := simplify\left(int\left(\dfrac{P}{R}, x\right) + int\left(\dfrac{S}{Q}, y\right)\right) = \_C1$

$$SolGral := -e^{-x} + \frac{\ln(\ln(y))}{2} = \_C1 \tag{7}$$

> $Para := simplify(subs(x = 0, y = \exp(1), SolGral))$

$$Para := -1 = \_C1 \tag{8}$$

> $SolPart := simplify(subs(\_C1 = lhs(Para), SolGral))$

$$\tag{9}$$

$$SolPart := -\mathrm{e}^{-x} + \frac{\ln(\ln(y))}{2} = -1 \tag{9}$$

> $SolPartFinal := -\mathrm{e}^{-x} + \dfrac{\ln(\ln(y(x)))}{2} = -1$

$$SolPartFinal := -\mathrm{e}^{-x} + \frac{\ln(\ln(y(x)))}{2} = -1 \tag{10}$$

> $DerSolPart := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPart := \frac{\mathrm{d}}{\mathrm{d}x} y(x) = -2 y(x) \ln(y(x)) \mathrm{e}^{-x} \tag{11}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{\mathrm{d}}{\mathrm{d}x} y(x) = -2 y(x) \ln(y(x)) \mathrm{e}^{-x} \tag{12}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolPart)) = 0$

$$Comprobar := 0 = 0 \tag{13}$$

> $restart$

2)

> $SolGral := 4 \cdot x^2 - y(x)^2 = \_C1$

$$SolGral := 4 x^2 - y(x)^2 = \_C1 \tag{14}$$

> $Ecua := y \cdot y' - 4 \cdot x = 0$

$$Ecua := y(x) \left(\frac{\mathrm{d}}{\mathrm{d}x} y(x) \right) - 4 x = 0 \tag{15}$$

> $DerSolGral := isolate(diff(SolGral, x), diff(y(x), x))$

$$DerSolGral := \frac{\mathrm{d}}{\mathrm{d}x} y(x) = \frac{4 x}{y(x)} \tag{16}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{\mathrm{d}}{\mathrm{d}x} y(x) = \frac{4 x}{y(x)} \tag{17}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprobar := 0 = 0 \tag{18}$$

> $SolPartUno := subs(\_C1 = 0, SolGral)$

$$SolPartUno := 4 x^2 - y(x)^2 = 0 \tag{19}$$

> $SolPartOnce := isolate(SolPartUno, y(x)^2)$

$$SolPartOnce := y(x)^2 = 4 x^2 \tag{20}$$

> $SolPartOnceDos := y(x) = 2 \cdot x$

$$SolPartOnceDos := y(x) = 2 x \tag{21}$$

> $plot(rhs(SolPartOnceDos), x = 0 .. 5)$
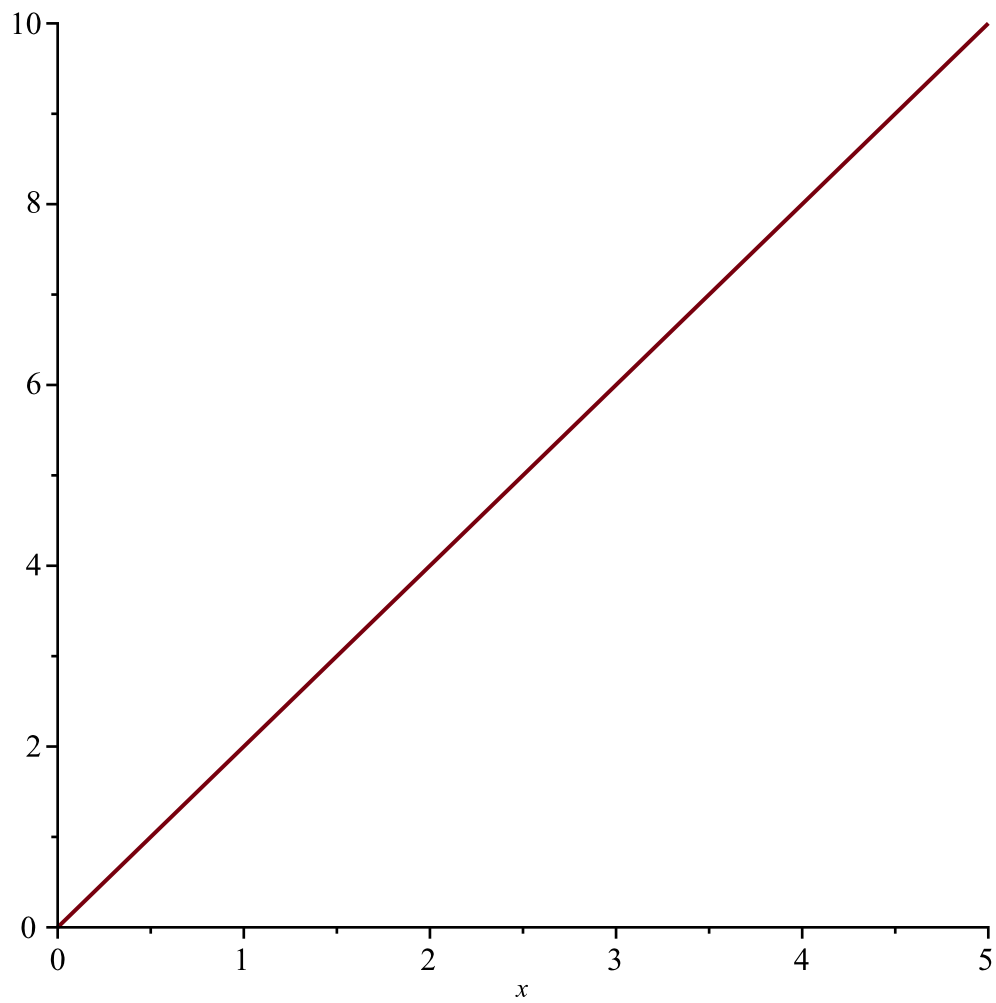
> $SolPartDos := subs(\_C1 = 1, SolGral)$

$$SolPartDos := 4\,x^2 - y(x)^2 = 1 \tag{22}$$

> $SolPartDosDoce := y(x) = \text{sqrt}\left(-1 + 4\,x^2\right)$

$$SolPartDosDoce := y(x) = \sqrt{4\,x^2 - 1} \tag{23}$$

> $plot\left(rhs(SolPartDosDoce), x = 0\,..5, y = 0\,..10\right)$
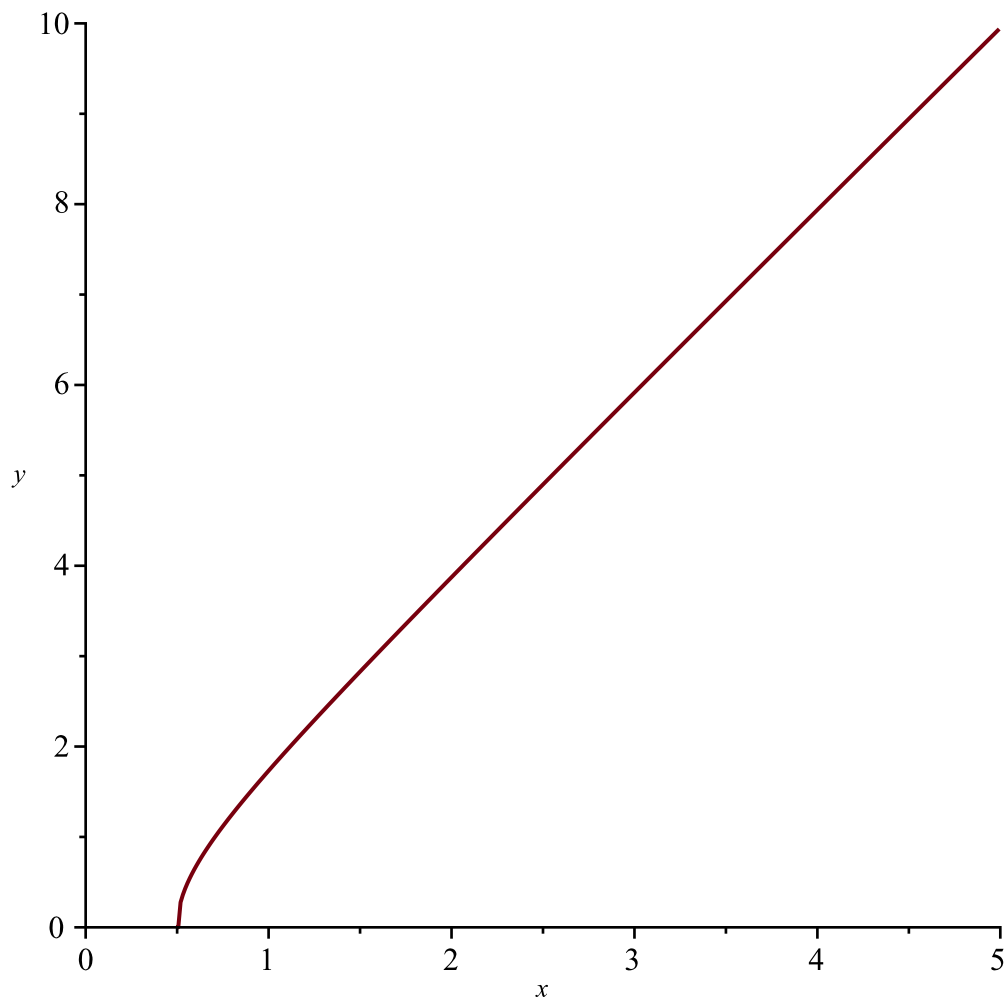
> $SolPartTres := subs(\_C1 = -1, SolGral)$

$$SolPartTres := 4\,x^2 - y(x)^2 = -1 \tag{24}$$

> $SolPartTresTrece := y(x) = \text{sqrt}\left(4\,x^2 + 1\right)$

$$SolPartTresTrece := y(x) = \sqrt{4\,x^2 + 1} \tag{25}$$

> $plot(rhs(SolPartTresTrece), x = 0..5, y = 0..10)$
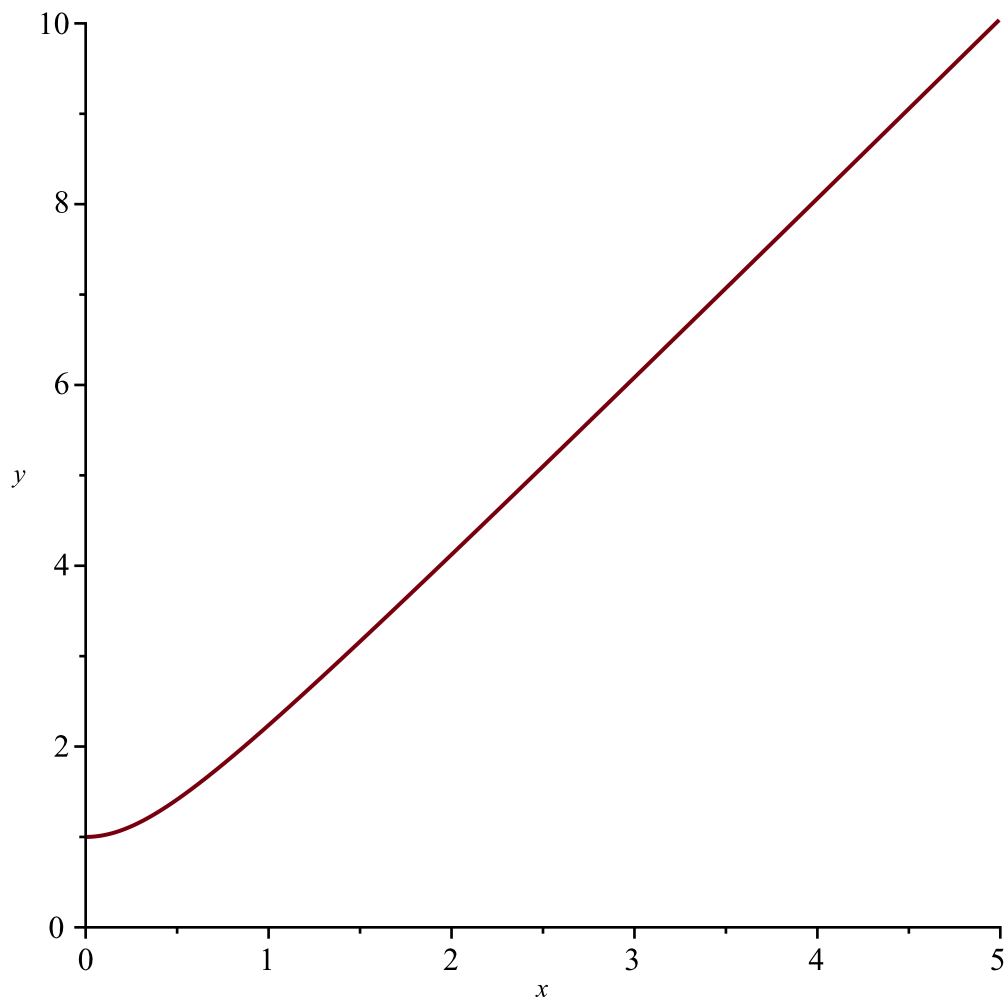
> *restart*

3)

> $Ecua := (x^2 + 4) \cdot y' = 2 \cdot x - 8 \cdot x \cdot y$

$$Ecua := (x^2 + 4)\left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) = 2\,x - 8\,x\,y(x) \tag{26}$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \tag{27}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_separable\,] \tag{28}$$

> $M := -(2\,x - 8\,x\,y)$

$$M := 8\,x\,y - 2\,x \tag{29}$$

> $N := (x^2 + 4)$

$$N := x^2 + 4 \tag{30}$$

> $P := 2 \cdot x;\ Q := 4\,y - 1;\ R := N;\ S := 1$

$$P := 2\,x$$

$$Q := 4y - 1$$
$$R := x^2 + 4$$
$$S := 1 \tag{31}$$

> $SolGral := int\left(\dfrac{P}{R}, x\right) + int\left(\dfrac{S}{Q}, y\right) = \_C1$

$$SolGral := \ln(x^2 + 4) + \frac{\ln(4y - 1)}{4} = \_C1 \tag{32}$$

> $Para := subs(x = 0, y = 0, SolGral)$

$$Para := \ln(4) + \frac{\ln(-1)}{4} = \_C1 \tag{33}$$

> $SolPart := isolate(subs(\_C1 = lhs(Para), SolGral), y)$

$$SolPart := y = -\frac{64}{(x^2 + 4)^4} + \frac{1}{4} \tag{34}$$

> $SolPartFinal := y(x) = -\dfrac{64}{(x^2 + 4)^4} + \dfrac{1}{4}$

$$SolPartFinal := y(x) = -\frac{64}{(x^2 + 4)^4} + \frac{1}{4} \tag{35}$$

> $Comprobar := subs(x = 0, SolPartFinal)$

$$Comprobar := y(0) = 0 \tag{36}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolPartFinal), lhs(Ecua) - rhs(Ecua)$
   $= 0)))$

$$ComprobarDos := 0 = 0 \tag{37}$$

> $restart$

4)

> $SolGral := y(x) = \_C1 \cdot x$

$$SolGral := y(x) = \_C1\, x \tag{38}$$

> $DerSolGral := diff(SolGral, x)$

$$DerSolGral := \frac{d}{dx} y(x) = \_C1 \tag{39}$$

> $Para := isolate(SolGral, \_C1)$

$$Para := \_C1 = \frac{y(x)}{x} \tag{40}$$

> $Ecua := lhs(DerSolGral) = rhs(Para)$

$$Ecua := \frac{d}{dx} y(x) = \frac{y(x)}{x} \tag{41}$$

> $ParaAA := isolate(subs(x = 2, y(2) = 1, SolGral), \_C1)$

$$ParaAA := \_C1 = \frac{1}{2} \tag{42}$$

> $SolPartAA := subs(\_C1 = rhs(ParaAA), SolGral)$

$$\tag{43}$$

$$SolPartAA := y(x) = \frac{x}{2} \tag{43}$$
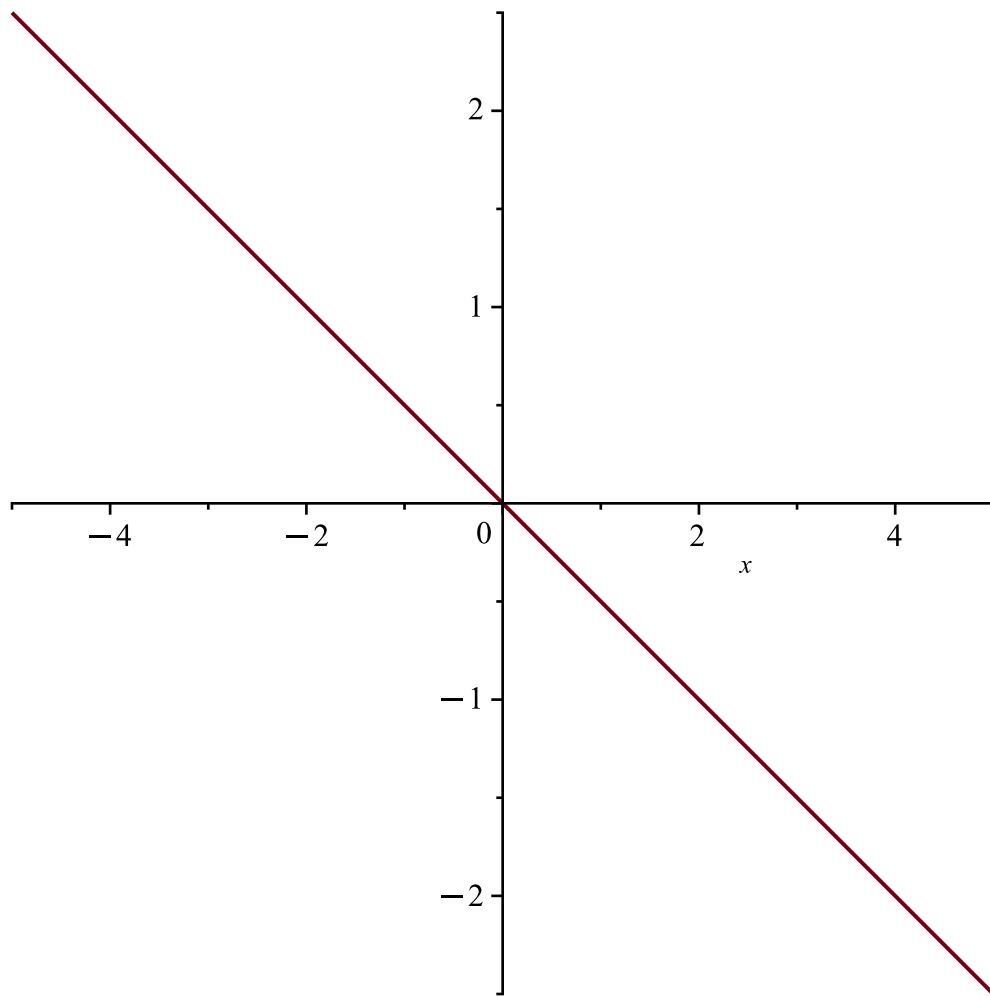
> $plot(rhs(SolPartAA), x = -5..5)$
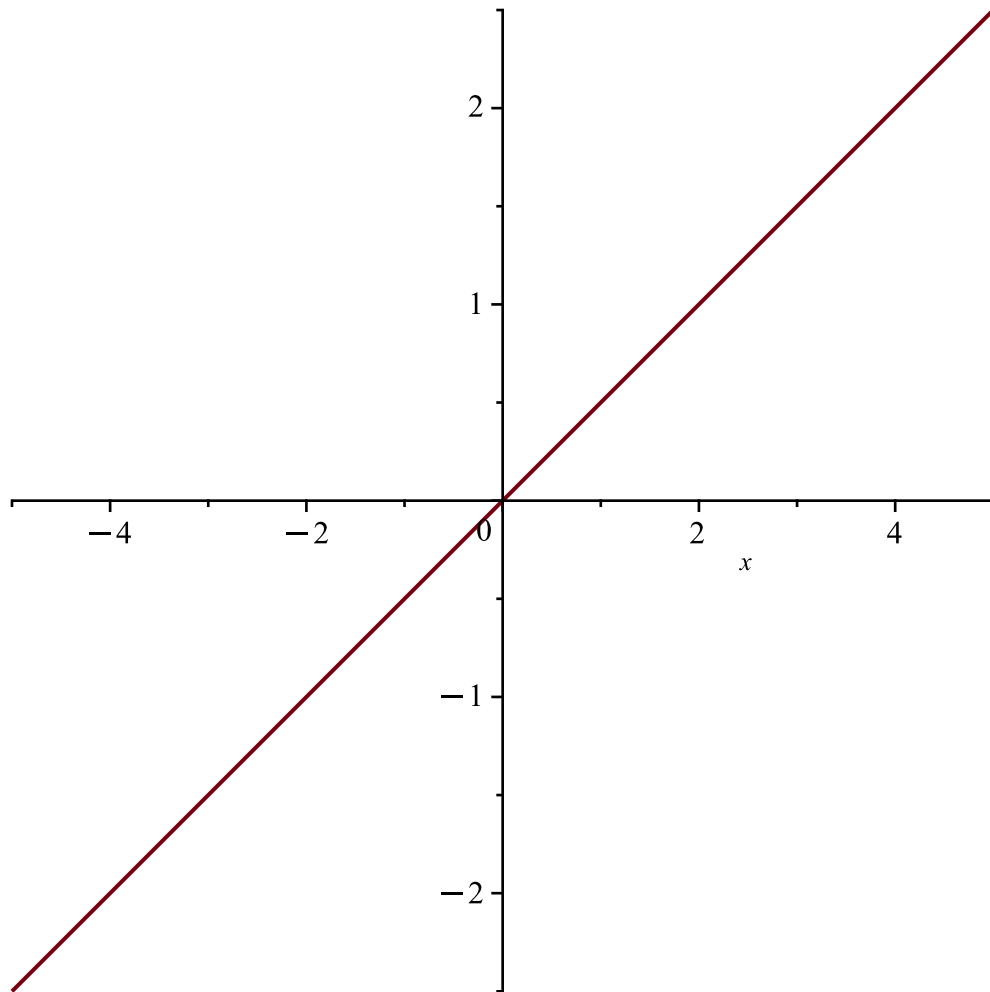
> $ParaBB := isolate(subs(x = -2, y(-2) = 1, SolGral), \_C1)$

$$ParaBB := \_C1 = -\frac{1}{2} \tag{44}$$

> $SolPartBB := subs(\_C1 = rhs(ParaBB), SolGral)$

$$SolPartBB := y(x) = -\frac{x}{2} \tag{45}$$

> $plot(rhs(SolPartBB), x = -5..5)$

> *restart*

5)

> $Ecua := (\exp(y) + \exp(-x)) + (\exp(y) + 2 \cdot y \cdot \exp(-x)) \cdot y' = 0$

$$Ecua := e^{y(x)} + e^{-x} + \left(e^{y(x)} + 2\, y(x)\, e^{-x} \right) \left(\frac{d}{dx}\, y(x) \right) = 0 \qquad \textbf{(46)}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[y = \_G(x, y')] \qquad \textbf{(47)}$$

> $FI := intfactor(Ecua)$

$$FI := e^x \qquad \textbf{(48)}$$

> $M := e^y + e^{-x}$

$$M := e^y + e^{-x} \qquad \textbf{(49)}$$

> $N := \left(e^y + 2\, y\, e^{-x} \right)$

$$N := e^y + 2\, y\, e^{-x} \qquad \textbf{(50)}$$

> $diff\,(M, y) \neq diff\,(N, x)$

$$e^y \neq -2\, y\, e^{-x} \qquad \textbf{(51)}$$

No es exacta

> $MM := expand(M \cdot FI)$

$$MM := e^y\,e^x + 1 \tag{52}$$

> $NN := expand(N \cdot FI)$

$$NN := e^y\,e^x + 2\,y \tag{53}$$

> $diff\,(MM, y) = diff\,(NN, x)$

$$e^y\,e^x = e^y\,e^x \tag{54}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := x + e^y\,e^x \tag{55}$$

> $SolGral := IntMMx + int((NN - diff\,(IntMMx, y)), y) = \_C1$

$$SolGral := x + e^y\,e^x + y^2 = c_1 \tag{56}$$

> $SolGralFinal := x + e^{y(x)}\,e^x + y(x)^2 = c_1$

$$SolGralFinal := x + e^{y(x)}\,e^x + y(x)^2 = c_1 \tag{57}$$

> $DerSolGral := simplify(isolate(diff\,(SolGralFinal, x), diff\,(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\,y(x) = \frac{-1 - e^{y(x)+x}}{e^{y(x)+x} + 2\,y(x)} \tag{58}$$

> $DerEcua := isolate(Ecua, diff\,(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{-e^{y(x)} - e^{-x}}{e^{y(x)} + 2\,y(x)\,e^{-x}} \tag{59}$$

> $Comprueba := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprueba := 0 = 0 \tag{60}$$

> $restart$
>
>
> $restart$

6)

> $Ecua := -y + (x - 2 \cdot x^2 \cdot y^2) \cdot y' = 0$

$$Ecua := -y(x) + (x - 2\,x^2\,y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{61}$$

> $CondIni := y(-1) = 2$

$$CondIni := y(-1) = 2 \tag{62}$$

>

RESPUESTA

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[[\_homogeneous,\ class\ G],\ \_rational] \tag{63}$$

> $M := -y;\ N := x - 2\,x^2\,y^2$

$$M := -y$$

$$N := -2\,x^2\,y^2 + x \tag{64}$$

> $diff(M, y)$

$$-1 \tag{65}$$

> $diff(N, x)$

$$-4\,x\,y^2 + 1 \tag{66}$$

No es exacta

> $FI := intfactor(Ecua)$

$$FI := \frac{1}{x^2} \tag{67}$$

> $MM := M \cdot FI$

$$MM := -\frac{y}{x^2} \tag{68}$$

> $NN := simplify(N \cdot FI)$

$$NN := \frac{-2\,x\,y^2 + 1}{x} \tag{69}$$

> $diff(MM, y)$

$$-\frac{1}{x^2} \tag{70}$$

> $simplify(diff(NN, x))$

$$-\frac{1}{x^2} \tag{71}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y}{x} \tag{72}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y}{x} - \frac{2\,y^3}{3} = \_C1 \tag{73}$$

> $Para := subs(x = -1, y = 2, SolGral)$

$$Para := -\frac{22}{3} = \_C1 \tag{74}$$

> $SolPart := subs(c_1 = lhs(Para), SolGral)$

$$SolPart := \frac{y}{x} - \frac{2\,y^3}{3} = \_C1 \tag{75}$$

> $SolPartDos := lhs(SolPart) \cdot 3 = rhs(SolPart) \cdot 3$

$$SolPartDos := \frac{3\,y}{x} - 2\,y^3 = 3\,\_C1 \tag{76}$$

> $SolPartFinal := \frac{3\,y(x)}{x} - 2\,y(x)^3 = -22$

$$\tag{77}$$

$$SolPartFinal := \frac{3\,y(x)}{x} - 2\,y(x)^3 = -22 \tag{77}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{y(x)}{x - 2\,x^2\,y(x)^2} \tag{78}$$

> $DerSolPart := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\,y(x) = -\frac{y(x)}{2\,x^2\,y(x)^2 - x} \tag{79}$$

> $ComprobarUno := simplify(rhs(DerSolPart) - rhs(DerEcua)) = 0$

$$ComprobarUno := 0 = 0 \tag{80}$$

> $CondIni$

$$y(-1) = 2 \tag{81}$$

> $ComprobarDos := isolate(subs(y = 2, SolPartFinal), x)$

$$ComprobarDos := x = -1 \tag{82}$$

> $restart$

7)

> $Ecua := (y^2 + x \cdot y^3) + (5 \cdot y^2 - x \cdot y + y^3 \cdot \sin(y)) \cdot y' = 0$

$$Ecua := y(x)^2 + x\,y(x)^3 + \left(5\,y(x)^2 - x\,y(x) + y(x)^3\,\sin(y(x))\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{83}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[x = \_G(y, y')] \tag{84}$$

> $M := (y^2 + x \cdot y^3)$

$$M := x\,y^3 + y^2 \tag{85}$$

> $N := (5 \cdot y^2 - x \cdot y + y^3 \cdot \sin(y))$

$$N := 5\,y^2 - x\,y + y^3\,\sin(y) \tag{86}$$

> $diff(M, y)$

$$3\,x\,y^2 + 2\,y \tag{87}$$

> $diff(N, x)$

$$-y \tag{88}$$

No es exacta

> $intfactor(Ecua)$

$$\frac{1}{y(x)^3} \tag{89}$$

> $FI := \frac{1}{y^3}$

$$\tag{90}$$

$$FI := \frac{1}{y^3} \tag{90}$$

> $MM := expand(simplify(M \cdot FI))$

$$MM := x + \frac{1}{y} \tag{91}$$

> $NN := expand(simplify(N \cdot FI))$

$$NN := \sin(y) - \frac{x}{y^2} + \frac{5}{y} \tag{92}$$

> $diff(MM, y)$

$$-\frac{1}{y^2} \tag{93}$$

> $diff(NN, x)$

$$-\frac{1}{y^2} \tag{94}$$

Es exacta

> $IntMMx := expand(int(MM, x))$

$$IntMMx := \frac{x^2}{2} + \frac{x}{y} \tag{95}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{x^2}{2} + \frac{x}{y} + 5\ln(y) - \cos(y) = c_1 \tag{96}$$

> $SolGralFinal := \frac{x^2}{2} + \frac{x}{y(x)} + 5\ln(y(x)) - \cos(y(x)) = c_1$

$$SolGralFinal := \frac{x^2}{2} + \frac{x}{y(x)} + 5\ln(y(x)) - \cos(y(x)) = c_1 \tag{97}$$

> $DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx} y(x) = \frac{y(x)(x\, y(x) + 1)}{-\sin(y(x))\, y(x)^2 + x - 5\, y(x)} \tag{98}$$

> $DerEcua := factor(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx} y(x) = -\frac{y(x)(x\, y(x) + 1)}{\sin(y(x))\, y(x)^2 - x + 5\, y(x)} \tag{99}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprobar := 0 = 0 \tag{100}$$

> $restart$

8)

> $Ecua := y^2 \cdot \sin(x) + (1 + 5 \cdot y \cdot \cos(x)) \cdot y' = 0$

$$Ecua := y(x)^2 \sin(x) + (1 + 5\, y(x)\cos(x)) \left(\frac{d}{dx} y(x)\right) = 0 \tag{101}$$

>

RESPUESTA

> *with(DEtools)* :

> *odeadvisor(Ecua)*

$$[[\_1st\_order, \_with\_symmetry\_[F(x)*G(y),0]], [\_Abel, 2nd\ type, class\ B]] \quad \textbf{(102)}$$

> $M := y^2 \cdot \sin(x); N := (1 + 5\,y\cdot\cos(x))$

$$M := y^2 \sin(x)$$
$$N := 1 + 5\,y\cos(x) \quad \textbf{(103)}$$

> $diff\,(M, y); diff\,(N, x)$

$$2\sin(x)\,y$$
$$-5\sin(x)\,y \quad \textbf{(104)}$$

No es exacta

> *intfactor(Ecua)*

$$\frac{1}{y(x)^7} \quad \textbf{(105)}$$

> $FI := \dfrac{1}{y^7}$

$$FI := \frac{1}{y^7} \quad \textbf{(106)}$$

> $MM := M\cdot FI$

$$MM := \frac{\sin(x)}{y^5} \quad \textbf{(107)}$$

> $NN := expand(N\cdot FI)$

$$NN := \frac{1}{y^7} + \frac{5\cos(x)}{y^6} \quad \textbf{(108)}$$

> $diff\,(MM, y); diff\,(NN, x)$

$$-\frac{5\sin(x)}{y^6}$$
$$-\frac{5\sin(x)}{y^6} \quad \textbf{(109)}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{\cos(x)}{y^5} \quad \textbf{(110)}$$

> $SolGral := IntMMx + int((NN - diff\,(IntMMx, y)), y) = \_C1$

$$SolGral := -\frac{\cos(x)}{y^5} - \frac{1}{6\,y^6} = c_1 \quad \textbf{(111)}$$

> $SolGralFinal := -\dfrac{\cos(x)}{y(x)^5} - \dfrac{1}{6\,y(x)^6} = c_1$

$$\quad \textbf{(112)}$$

$$SolGralFinal := -\frac{\cos(x)}{y(x)^5} - \frac{1}{6\,y(x)^6} = c_1 \tag{112}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = -\frac{y(x)^2\,\sin(x)}{1 + 5\,y(x)\,\cos(x)} \tag{113}$$

> $DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\,y(x) = -\frac{y(x)^2\,\sin(x)}{1 + 5\,y(x)\,\cos(x)} \tag{114}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprobar := 0 = 0 \tag{115}$$

> $restart$

9)

> $Ecua := \frac{1}{x}\cdot y' - \frac{2\cdot y}{x^2} = x\cdot\cos(x)$

$$Ecua := \frac{\frac{d}{dx}\,y(x)}{x} - \frac{2\,y(x)}{x^2} = x\,\cos(x) \tag{116}$$

> $CondIni := y\left(\frac{Pi}{2}\right) = 3$

$$CondIni := y\left(\frac{\pi}{2}\right) = 3 \tag{117}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_linear] \tag{118}$$

> $FI := intfactor(Ecua)$

$$FI := \frac{1}{x} \tag{119}$$

> $M := -\frac{2\,y}{x^2} - x\,\cos(x)$

$$M := -\frac{2\,y}{x^2} - x\,\cos(x) \tag{120}$$

> $N := \frac{1}{x}$

$$N := \frac{1}{x} \tag{121}$$

> $diff(M, y) \neq diff(N, x)$

$$-\frac{2}{x^2} = -\frac{1}{x^2} \tag{122}$$

No es exacta

> $MM := expand(simplify(M \cdot FI))$

$$MM := -\cos(x) - \frac{2\,y}{x^3} \tag{123}$$

> $NN := N \cdot FI$

$$NN := \frac{1}{x^2} \tag{124}$$

> $diff(MM, y) = diff(NN, x)$

$$-\frac{2}{x^3} = -\frac{2}{x^3} \tag{125}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y}{x^2} - \sin(x) \tag{126}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y}{x^2} - \sin(x) = \_C1 \tag{127}$$

> $Para := subs\left(x = \frac{Pi}{2}, y = 3, SolGral\right)$

$$Para := \frac{12}{\pi^2} - \sin\left(\frac{\pi}{2}\right) = \_C1 \tag{128}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$SolPart := \frac{y}{x^2} - \sin(x) = \frac{12}{\pi^2} - 1 \tag{129}$$

> $SolPartFinal := \frac{y(x)}{x^2} - \sin(x) = \frac{12}{\pi^2} - 1$

$$SolPartFinal := \frac{y(x)}{x^2} - \sin(x) = \frac{12}{\pi^2} - 1 \tag{130}$$

> $DerSolPart := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\,y(x) = \frac{x^3 \cos(x) + 2\,y(x)}{x} \tag{131}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{x^3 \cos(x) + 2\,y(x)}{x} \tag{132}$$

> $Comprobar := simplify(rhs(DerSolPart) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \tag{133}$$

> $restart$

10)

> $Ecua := y' = (x - 4 \cdot y - 1)^2$

$$Ecua := \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = (x - 4\, y(x) - 1)^2 \qquad (134)$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \qquad (135)$$

>

RESPUESTA

> $EcuaDos := isolate(eval(subs(isolate(z(x) = x - 4 \cdot y(x) - 1, y(x)), Ecua)), diff(z(x), x))$

$$EcuaDos := \frac{\mathrm{d}}{\mathrm{d}x}\, z(x) = -4\, z(x)^2 + 1 \qquad (136)$$

> $with(DEtools):$
> $odeadvisor(EcuaDos)$

$$[\,\_quadrature\,] \qquad (137)$$

> $intfactor(EcuaDos)$

$$\frac{1}{(2\, z(x) - 1)\, (2\, z(x) + 1)} \qquad (138)$$

> $FI := simplify\left(\dfrac{1}{(2\, z - 1)\, (2\, z + 1)}\right)$

$$FI := \frac{1}{4\, z^2 - 1} \qquad (139)$$

> $M := 4\, z^2 - 1$

$$M := 4\, z^2 - 1 \qquad (140)$$

> $N := 1$

$$N := 1 \qquad (141)$$

> $MM := M \cdot FI$

$$MM := 1 \qquad (142)$$

> $NN := N \cdot FI$

$$NN := \frac{1}{4\, z^2 - 1} \qquad (143)$$

> $P := 1;\ Q := 1;\ R := 1;\ S := \dfrac{1}{4\, z^2 - 1}$

$$P := 1$$
$$Q := 1$$
$$R := 1$$
$$S := \frac{1}{4\, z^2 - 1} \qquad (144)$$

> $SolGral := simplify\left(int\left(\dfrac{P}{R}, x\right) + int\left(\dfrac{S}{Q}, z\right)\right) = \_C1$

$$SolGral := x - \frac{\ln(2\, z + 1)}{4} + \frac{\ln(2\, z - 1)}{4} = \_C1 \qquad (145)$$

> $subs(x = 0, y(0) = 0, z(x) = x - 4 \cdot y(x) - 1)$

$$z(0) = -1 \tag{146}$$

> $Para := simplify(subs(x = 0, z = -1, SolGral))$

$$Para := \frac{\ln(3)}{4} = \_C1 \tag{147}$$

> $SolPartDos := subs(\_C1 = lhs(Para), SolGral)$

$$SolPartDos := x - \frac{\ln(2z+1)}{4} + \frac{\ln(2z-1)}{4} = \frac{\ln(3)}{4} \tag{148}$$

> $SolPartFinal := simplify(isolate(simplify(subs(z = x - 4 \cdot y(x) - 1, SolPartDos)), y(x)))$

$$SolPartFinal := y(x) = \frac{(2x-3)\,e^{4x} - 6x + 3}{8\,e^{4x} - 24} \tag{149}$$

> $Ecua$

$$\frac{d}{dx}\,y(x) = (x - 4\,y(x) - 1)^2 \tag{150}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolPartFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \tag{151}$$

> $restart$
>
>
> $restart$

6)

> $Ecua := -y + (x - 2 \cdot x^2 \cdot y^2) \cdot y' = 0$

$$Ecua := -y(x) + (x - 2\,x^2\,y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{152}$$

> $CondIni := y(-1) = 2$

$$CondIni := y(-1) = 2 \tag{153}$$

>

RESPUESTA

> $with(DEtools):$
> $odeadvisor(Ecua)$

$$[\,[\_homogeneous, class\ G], \_rational\,] \tag{154}$$

> $M := -y; N := x - 2\,x^2\,y^2$

$$M := -y$$
$$N := -2\,x^2\,y^2 + x \tag{155}$$

> $diff(M, y)$

$$-1 \tag{156}$$

> $diff(N, x)$

$$-4\,x\,y^2 + 1 \tag{157}$$

No es exacta

> $FI := intfactor(Ecua)$

$$FI := \frac{1}{x^2} \tag{158}$$

> $MM := M \cdot FI$

$$MM := -\frac{y}{x^2} \tag{159}$$

> $NN := simplify(N \cdot FI)$

$$NN := \frac{-2\,x\,y^2 + 1}{x} \tag{160}$$

> $diff(MM, y)$

$$-\frac{1}{x^2} \tag{161}$$

> $simplify(diff(NN, x))$

$$-\frac{1}{x^2} \tag{162}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y}{x} \tag{163}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y}{x} - \frac{2\,y^3}{3} = \_C1 \tag{164}$$

> $Para := subs(x = -1, y = 2, SolGral)$

$$Para := -\frac{22}{3} = \_C1 \tag{165}$$

> $SolPart := subs\left(c_1 = lhs(Para), SolGral\right)$

$$SolPart := \frac{y}{x} - \frac{2\,y^3}{3} = \_C1 \tag{166}$$

> $SolPartDos := lhs(SolPart) \cdot 3 = rhs(SolPart) \cdot 3$

$$SolPartDos := \frac{3\,y}{x} - 2\,y^3 = 3\,\_C1 \tag{167}$$

> $SolPartFinal := \frac{3\,y(x)}{x} - 2\,y(x)^3 = -22$

$$SolPartFinal := \frac{3\,y(x)}{x} - 2\,y(x)^3 = -22 \tag{168}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{y(x)}{x - 2\,x^2\,y(x)^2} \tag{169}$$

> $DerSolPart := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\,y(x) = -\frac{y(x)}{2\,x^2\,y(x)^2 - x} \tag{170}$$

> $ComprobarUno := simplify(rhs(DerSolPart) - rhs(DerEcua)) = 0$

$$ComprobarUno := 0 = 0 \tag{171}$$

> $CondIni$

$$y(-1) = 2 \tag{172}$$

> $ComprobarDos := isolate(subs(y = 2, SolPartFinal), x)$

$$ComprobarDos := x = -1 \tag{173}$$

> $restart$

7)

> $Ecua := (y^2 + x \cdot y^3) + (5 \cdot y^2 - x \cdot y + y^3 \cdot \sin(y)) \cdot y' = 0$

$$Ecua := y(x)^2 + x\, y(x)^3 + \left(5\, y(x)^2 - x\, y(x) + y(x)^3 \sin(y(x))\right) \left(\frac{d}{dx}\, y(x)\right) = 0 \tag{174}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[x = \_G(y, y')] \tag{175}$$

> $M := (y^2 + x \cdot y^3)$

$$M := x\, y^3 + y^2 \tag{176}$$

> $N := (5 \cdot y^2 - x \cdot y + y^3 \cdot \sin(y))$

$$N := 5\, y^2 - x\, y + y^3 \sin(y) \tag{177}$$

> $diff(M, y)$

$$3\, x\, y^2 + 2\, y \tag{178}$$

> $diff(N, x)$

$$-y \tag{179}$$

No es exacta

> $intfactor(Ecua)$

$$\frac{1}{y(x)^3} \tag{180}$$

> $FI := \dfrac{1}{y^3}$

$$FI := \frac{1}{y^3} \tag{181}$$

> $MM := expand(simplify(M \cdot FI))$

$$MM := x + \frac{1}{y} \tag{182}$$

> $NN := expand(simplify(N \cdot FI))$

$$NN := \sin(y) - \frac{x}{y^2} + \frac{5}{y} \tag{183}$$

> $diff(MM, y)$

$$\tag{184}$$

$$-\frac{1}{y^2} \tag{184}$$

> $diff(NN, x)$

$$-\frac{1}{y^2} \tag{185}$$

Es exacta

> $IntMMx := expand(int(MM, x))$

$$IntMMx := \frac{x^2}{2} + \frac{x}{y} \tag{186}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{x^2}{2} + \frac{x}{y} + 5\ln(y) - \cos(y) = c_1 \tag{187}$$

> $SolGralFinal := \frac{x^2}{2} + \frac{x}{y(x)} + 5\ln(y(x)) - \cos(y(x)) = c_1$

$$SolGralFinal := \frac{x^2}{2} + \frac{x}{y(x)} + 5\ln(y(x)) - \cos(y(x)) = c_1 \tag{188}$$

> $DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}y(x) = \frac{y(x)(xy(x)+1)}{-\sin(y(x))y(x)^2 + x - 5y(x)} \tag{189}$$

> $DerEcua := factor(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}y(x) = -\frac{y(x)(xy(x)+1)}{\sin(y(x))y(x)^2 - x + 5y(x)} \tag{190}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprobar := 0 = 0 \tag{191}$$

> $restart$

8)

> $Ecua := y^2 \cdot \sin(x) + (1 + 5 \cdot y \cdot \cos(x)) \cdot y' = 0$

$$Ecua := y(x)^2 \sin(x) + (1 + 5y(x)\cos(x))\left(\frac{d}{dx}y(x)\right) = 0 \tag{192}$$

>

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[[\_1st\_order, \_with\_symmetry\_[F(x)*G(y),0]], [\_Abel, 2nd\ type,\ class\ B]] \tag{193}$$

> $M := y^2 \cdot \sin(x); N := (1 + 5y \cdot \cos(x))$

$$M := y^2 \sin(x)$$

$$N := 1 + 5y\cos(x) \tag{194}$$

> $diff(M, y); diff(N, x)$

$$2\sin(x)y$$

$$\tag{195}$$

$$-5 \sin(x)\, y \tag{195}$$

No es exacta

> $intfactor(Ecua)$

$$\frac{1}{y(x)^7} \tag{196}$$

> $FI := \dfrac{1}{y^7}$

$$FI := \frac{1}{y^7} \tag{197}$$

> $MM := M \cdot FI$

$$MM := \frac{\sin(x)}{y^5} \tag{198}$$

> $NN := expand(N \cdot FI)$

$$NN := \frac{1}{y^7} + \frac{5 \cos(x)}{y^6} \tag{199}$$

> $diff(MM, y);\ diff(NN, x)$

$$-\frac{5 \sin(x)}{y^6}$$

$$-\frac{5 \sin(x)}{y^6} \tag{200}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{\cos(x)}{y^5} \tag{201}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := -\frac{\cos(x)}{y^5} - \frac{1}{6\,y^6} = c_1 \tag{202}$$

> $SolGralFinal := -\dfrac{\cos(x)}{y(x)^5} - \dfrac{1}{6\,y(x)^6} = c_1$

$$SolGralFinal := -\frac{\cos(x)}{y(x)^5} - \frac{1}{6\,y(x)^6} = c_1 \tag{203}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = -\frac{y(x)^2 \sin(x)}{1 + 5\,y(x)\cos(x)} \tag{204}$$

> $DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = -\frac{y(x)^2 \sin(x)}{1 + 5\,y(x)\cos(x)} \tag{205}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprobar := 0 = 0 \tag{206}$$

> restart

9)

> $Ecua := \dfrac{1}{x} \cdot y' - \dfrac{2 \cdot y}{x^2} = x \cdot \cos(x)$

$$Ecua := \dfrac{\dfrac{d}{dx} y(x)}{x} - \dfrac{2\, y(x)}{x^2} = x \cos(x) \qquad (207)$$

> $CondIni := y\left(\dfrac{Pi}{2}\right) = 3$

$$CondIni := y\left(\dfrac{\pi}{2}\right) = 3 \qquad (208)$$

>

RESPUESTA

> $with(DEtools) :$
> $odeadvisor(Ecua)$

$$[\_linear] \qquad (209)$$

> $FI := intfactor(Ecua)$

$$FI := \dfrac{1}{x} \qquad (210)$$

> $M := -\dfrac{2\,y}{x^2} - x\cos(x)$

$$M := -\dfrac{2\,y}{x^2} - x\cos(x) \qquad (211)$$

> $N := \dfrac{1}{x}$

$$N := \dfrac{1}{x} \qquad (212)$$

> $diff(M, y) \neq diff(N, x)$

$$-\dfrac{2}{x^2} = -\dfrac{1}{x^2} \qquad (213)$$

No es exacta

> $MM := expand(simplify(M \cdot FI))$

$$MM := -\cos(x) - \dfrac{2\,y}{x^3} \qquad (214)$$

> $NN := N \cdot FI$

$$NN := \dfrac{1}{x^2} \qquad (215)$$

> $diff(MM, y) = diff(NN, x)$

$$-\dfrac{2}{x^3} = -\dfrac{2}{x^3} \qquad (216)$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y}{x^2} - \sin(x) \tag{217}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y}{x^2} - \sin(x) = \_C1 \tag{218}$$

> $Para := subs\left(x = \frac{Pi}{2}, y = 3, SolGral\right)$

$$Para := \frac{12}{\pi^2} - \sin\left(\frac{\pi}{2}\right) = \_C1 \tag{219}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$SolPart := \frac{y}{x^2} - \sin(x) = \frac{12}{\pi^2} - 1 \tag{220}$$

> $SolPartFinal := \frac{y(x)}{x^2} - \sin(x) = \frac{12}{\pi^2} - 1$

$$SolPartFinal := \frac{y(x)}{x^2} - \sin(x) = \frac{12}{\pi^2} - 1 \tag{221}$$

> $DerSolPart := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx} y(x) = \frac{x^3 \cos(x) + 2\, y(x)}{x} \tag{222}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx} y(x) = \frac{x^3 \cos(x) + 2\, y(x)}{x} \tag{223}$$

> $Comprobar := simplify(rhs(DerSolPart) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \tag{224}$$

> $restart$

10)

> $Ecua := y' = (x - 4 \cdot y - 1)^2$

$$Ecua := \frac{d}{dx} y(x) = (x - 4\, y(x) - 1)^2 \tag{225}$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \tag{226}$$

>

RESPUESTA

> $EcuaDos := isolate(eval(subs(isolate(z(x) = x - 4 \cdot y(x) - 1, y(x)), Ecua)), diff(z(x), x))$

$$EcuaDos := \frac{d}{dx} z(x) = -4\, z(x)^2 + 1 \tag{227}$$

> $with(DEtools) :$

> $odeadvisor(EcuaDos)$

$$[\_quadrature]\tag{228}$$

> $intfactor(EcuaDos)$

$$\frac{1}{(2\,z(x)-1)\,(2\,z(x)+1)}\tag{229}$$

> $FI := simplify\left(\dfrac{1}{(2\,z-1)\,(2\,z+1)}\right)$

$$FI := \frac{1}{4\,z^2-1}\tag{230}$$

> $M := 4\,z^2-1$

$$M := 4\,z^2-1\tag{231}$$

> $N := 1$

$$N := 1\tag{232}$$

> $MM := M \cdot FI$

$$MM := 1\tag{233}$$

> $NN := N \cdot FI$

$$NN := \frac{1}{4\,z^2-1}\tag{234}$$

> $P := 1; Q := 1; R := 1; S := \dfrac{1}{4\,z^2-1}$

$$P := 1$$
$$Q := 1$$
$$R := 1$$
$$S := \frac{1}{4\,z^2-1}\tag{235}$$

> $SolGral := simplify\left(int\left(\dfrac{P}{R},x\right) + int\left(\dfrac{S}{Q},z\right)\right) = \_C1$

$$SolGral := x - \frac{\ln(2\,z+1)}{4} + \frac{\ln(2\,z-1)}{4} = \_C1\tag{236}$$

> $subs(x=0, y(0)=0, z(x)=x-4\cdot y(x)-1)$

$$z(0) = -1\tag{237}$$

> $Para := simplify(subs(x=0, z=-1, SolGral))$

$$Para := \frac{\ln(3)}{4} = \_C1\tag{238}$$

> $SolPartDos := subs(\_C1 = lhs(Para), SolGral)$

$$SolPartDos := x - \frac{\ln(2\,z+1)}{4} + \frac{\ln(2\,z-1)}{4} = \frac{\ln(3)}{4}\tag{239}$$

> $SolPartFinal := simplify(isolate(simplify(subs(z=x-4\cdot y(x)-1, SolPartDos)), y(x)))$

$$SolPartFinal := y(x) = \frac{(2\,x-3)\,e^{4\,x} - 6\,x + 3}{8\,e^{4\,x} - 24}\tag{240}$$

> *Ecua*

$$\frac{d}{dx} y(x) = (x - 4\, y(x) - 1)^2 \tag{241}$$

> *ComprobarDos* := *simplify*(*eval*(*subs*(*y*(*x*) = *rhs*(*SolPartFinal*), *lhs*(*Ecua*) − *rhs*(*Ecua*) = 0)))

$$ComprobarDos := 0 = 0 \tag{242}$$

> *restart*

>