FACULTAD DE INGENIERIA
DIVISION DE CIENCIAS BASICAS
ECUACIONES DIFERENCIALES
GRUPO 13 SEMESTRE 2025-1
SERIE 2
SOLUCIÓN

```
> restart
```

1)

```
> Ecua := expand( ((diff((x·diff(y(x),x) − y(x)),x) + (x·diff(y(x),x) − y(x))) + y(x)) / x )
```

$$= \frac{(x\cdot\exp(-x) - y(x) + y(x))}{x}$$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) = e^{-x} \quad \textbf{(1)}$$

RESPUESTA

```
> EcuaHom := lhs(Ecua) = 0
```

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) = 0 \quad \textbf{(2)}$$

```
> Q := rhs(Ecua)
```

$$Q := e^{-x} \quad \textbf{(3)}$$

```
> EcuaCarac := m^2 + m = 0
```

$$EcuaCarac := m^2 + m = 0 \quad \textbf{(4)}$$

```
> Raiz := solve(EcuaCarac)
```

$$Raiz := -1, 0 \quad \textbf{(5)}$$

```
> yy[1] := exp(Raiz[1]·x)
```

$$yy_1 := e^{-x} \quad \textbf{(6)}$$

```
> yy[2] := exp(Raiz[2]·x)
```

$$yy_2 := 1 \quad \textbf{(7)}$$

```
> with(linalg) :
> WW := wronskian([yy[1], yy[2]], x)
```

$$WW := \begin{bmatrix} e^{-x} & 1 \\ -e^{-x} & 0 \end{bmatrix} \quad \textbf{(8)}$$

```
> BB := array([0, Q])
```

$$BB := \begin{bmatrix} 0 & e^{-x} \end{bmatrix} \quad \textbf{(9)}$$

```
> Para := linsolve(WW, BB)
```

$$Para := \begin{bmatrix} -1 & e^{-x} \end{bmatrix} \quad \textbf{(10)}$$

```
> Aprima := Para[1]
```

$$Aprima := -1 \quad \textbf{(11)}$$

```
> Bprima := Para[2]
```

$$Bprima := e^{-x} \quad (12)$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = (-x + \_C1 - 1)\, e^{-x} + \_C2 \quad (13)$$

> $SolFinal := subs(\_C1 = \_C10 + 1, \_C2 = \_C20, SolGral)$

$$SolFinal := y(x) = (-x + \_C10)\, e^{-x} + \_C20 \quad (14)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecua)))$

$$Comprobar := e^{-x} = e^{-x} \quad (15)$$

> $restart$

2)

> $Ecua := y'' + y = \sec(x) \cdot \csc(x)$

$$Ecua := \frac{d^2}{dx^2} y(x) + y(x) = \sec(x)\, \csc(x) \quad (16)$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + y(x) = 0 \quad (17)$$

> $Q := rhs(Ecua)$

$$Q := \sec(x)\, \csc(x) \quad (18)$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \quad (19)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := I, -I \quad (20)$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(x) \quad (21)$$

> $yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_2 := \sin(x) \quad (22)$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad (23)$$

> $BB := array([0, Q,])$

$$BB := \begin{bmatrix} 0 & \sec(x)\, \csc(x) \end{bmatrix} \quad (24)$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} -\sec(x) & \csc(x) \end{bmatrix} \quad (25)$$

> $Aprima := Para[1]$

$$Aprima := -\sec(x) \quad (26)$$

> $Bprima := Para[2]$

$$Bprima := \csc(x) \quad (27)$$

> $SolGral := y(x) = expand((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGral := y(x) = -\sin(x)\ln(\csc(x) + \cot(x)) - \cos(x)\ln(\sec(x) + \tan(x)) + \cos(x)\ \_C1 + \sin(x)\ \_C2 \quad (28)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))$

$$Comprobar := \sec(x)\csc(x) = \sec(x)\csc(x) \quad (29)$$

> $restart$

3)

> $Ecua := diff(y(t), t\$2) - diff(y(t), t) - 2\cdot y(t) = -3\cdot\exp(-t) + 2$

$$Ecua := \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2\,y(t) = -3\,e^{-t} + 2 \quad (30)$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2\,y(t) = 0 \quad (31)$$

> $Q := rhs(Ecua)$

$$Q := -3\,e^{-t} + 2 \quad (32)$$

> $EcuaCarac := m^2 - m - 2 = 0$

$$EcuaCarac := m^2 - m - 2 = 0 \quad (33)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2, -1 \quad (34)$$

> $yy[1] := \exp(Raiz[1]\cdot t)$

$$yy_1 := e^{2t} \quad (35)$$

> $yy[2] := \exp(Raiz[2]\cdot t)$

$$yy_2 := e^{-t} \quad (36)$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], t)$

$$WW := \begin{bmatrix} e^{2t} & e^{-t} \\ 2\,e^{2t} & -e^{-t} \end{bmatrix} \quad (37)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & -3\,e^{-t} + 2 \end{bmatrix} \quad (38)$$

> $Para := expand(linsolve(WW, BB))$

$$Para := \begin{bmatrix} -\frac{3\,e^{-t} - 2}{3\,e^{2t}} & \frac{3\,e^{-t} - 2}{3\,e^{-t}} \end{bmatrix} \quad (39)$$

> $Aprima := expand(simplify(Para[1]))$

$$Aprima := -\frac{1}{(e^t)^3} + \frac{2}{3\,(e^t)^2} \quad (40)$$

> $Bprima := expand(simplify(Para[2]))$

$$Bprima := 1 - \frac{2\,e^{t}}{3} \tag{41}$$

> $SolGral := y(t) = expand(simplify((int(Aprima, t) + \_C1)\cdot yy[1] + (int(Bprima, t) + \_C2)\cdot yy[2]))$

$$SolGral := y(t) = \frac{1}{3\,e^{t}} - 1 + \left(e^{t}\right)^{2}\,\_C1 + \frac{t}{e^{t}} + \frac{\_C2}{e^{t}} \tag{42}$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(SolGral), Ecua)))$

$$Comprobar := -3\,e^{-t} + 2 = -3\,e^{-t} + 2 \tag{43}$$

> $restart$

4)

> $Ecua := y'' - 4\cdot y = \exp(2\cdot x) - 4\cdot\cos(2\,x)$

$$Ecua := \frac{d^{2}}{dx^{2}}\,y(x) - 4\,y(x) = e^{2\,x} - 4\cos(2\,x) \tag{44}$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^{2}}{dx^{2}}\,y(x) - 4\,y(x) = 0 \tag{45}$$

> $Q := rhs(Ecua)$

$$Q := e^{2\,x} - 4\cos(2\,x) \tag{46}$$

> $EcuaCarac := m^{2} - 4 = 0$

$$EcuaCarac := m^{2} - 4 = 0 \tag{47}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2, -2 \tag{48}$$

> $yy[1] := \exp(Raiz[1]\cdot x)$

$$yy_{1} := e^{2\,x} \tag{49}$$

> $yy[2] := \exp(Raiz[2]\cdot x)$

$$yy_{2} := e^{-2\,x} \tag{50}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{2\,x} & e^{-2\,x} \\ 2\,e^{2\,x} & -2\,e^{-2\,x} \end{bmatrix} \tag{51}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^{2\,x} - 4\cos(2\,x) \end{bmatrix} \tag{52}$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} \frac{e^{2\,x} - 4\cos(2\,x)}{4\,e^{2\,x}} & -\frac{e^{2\,x} - 4\cos(2\,x)}{4\,e^{-2\,x}} \end{bmatrix} \tag{53}$$

> $Aprima := Para[1]$

$$Aprima := \frac{e^{2x} - 4\cos(2x)}{4e^{2x}} \quad (54)$$

> $Bprima := Para[2]$

$$Bprima := -\frac{e^{2x} - 4\cos(2x)}{4e^{-2x}} \quad (55)$$

> $SolGral := y(x) = expand(simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2]))$

$$SolGral := y(x) = \cos(x)^2 - \frac{1}{2} + (e^x)^2 \_C1 + \frac{(e^x)^2 x}{4} - \frac{(e^x)^2}{16} + \frac{\_C2}{(e^x)^2} \quad (56)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (57)$$

> $restart$

5)

> $Ecua := diff(y(x), x\$4) - y(x) = 0$

$$Ecua := \frac{d^4}{dx^4} y(x) - y(x) = 0 \quad (58)$$

> $EcuaCarac := m^4 - 1 = 0$

$$EcuaCarac := m^4 - 1 = 0 \quad (59)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -1, I, -I \quad (60)$$

> $yy[1] := \exp(Raiz[1]\cdot x)$

$$yy_1 := e^x \quad (61)$$

> $yy[2] := \exp(Raiz[2]\cdot x)$

$$yy_2 := e^{-x} \quad (62)$$

> $yy[3] := \cos(\mathrm{Im}(Raiz[3])\cdot x)$

$$yy_3 := \cos(x) \quad (63)$$

> $yy[4] := \sin(\mathrm{Im}(Raiz[3])\cdot x)$

$$yy_4 := \sin(x) \quad (64)$$

> $SolGral := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2] + \_C3\cdot yy[3] + \_C4\cdot yy[4]$

$$SolGral := y(x) = \_C1\, e^x + \_C2\, e^{-x} + \_C3\cos(x) + \_C4\sin(x) \quad (65)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))$

$$Comprobar := 0 = 0 \quad (66)$$

> $restart$

6)

> $Ecua := y'' + 2\cdot y' - 3\cdot y = 15\cdot\cos(3\cdot x)$

$$Ecua := \frac{d^2}{dx^2} y(x) + 2\frac{d}{dx} y(x) - 3y(x) = 15\cos(3x) \quad (67)$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) - 3 y(x) = 0 \quad (68)$$

> $Q := rhs(Ecua)$

$$Q := 15 \cos(3 x) \quad (69)$$

> $EcuaCarac := m^2 + 2 \cdot m - 3 = 0$

$$EcuaCarac := m^2 + 2 m - 3 = 0 \quad (70)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -3 \quad (71)$$

> $yy[1] := \exp(Raiz[1] \cdot x)$

$$yy_1 := e^{x} \quad (72)$$

> $yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_2 := e^{-3 x} \quad (73)$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{x} & e^{-3 x} \\ e^{x} & -3 e^{-3 x} \end{bmatrix} \quad (74)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 15 \cos(3 x) \end{bmatrix} \quad (75)$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} \frac{15 \cos(3 x)}{4 e^{x}} & -\frac{15 \cos(3 x)}{4 e^{-3 x}} \end{bmatrix} \quad (76)$$

> $Aprima := Para[1]$

$$Aprima := \frac{15 \cos(3 x)}{4 e^{x}} \quad (77)$$

> $Bprima := Para[2]$

$$Bprima := -\frac{15 \cos(3 x)}{4 e^{-3 x}} \quad (78)$$

> $SolGral := y(x) = expand(simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2]))$

$$SolGral := y(x) = 2 \sin(x) \cos(x)^2 - \frac{\sin(x)}{2} - 4 \cos(x)^3 + 3 \cos(x) + e^{x} \_C1 + \frac{\_C2}{(e^{x})^3} \quad (79)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (80)$$

> $restart$

7)

> $Ecua := (x^2 + 1) \cdot y'' - 2 \cdot x \cdot y' + 2 \cdot y = 0$

$$Ecua := (x^2+1)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2x\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0 \quad (81)$$

RESULTADO

> $EcuaNorm := expand\left(simplify\left(\frac{lhs(Ecua)}{(x^2+1)}\right)\right) = 0$

$$EcuaNorm := \frac{\left(\frac{d^2}{dx^2}\,y(x)\right)x^2}{x^2+1} - \frac{2x\left(\frac{d}{dx}\,y(x)\right)}{x^2+1} + \frac{\frac{d^2}{dx^2}\,y(x)}{x^2+1} + \frac{2\,y(x)}{x^2+1} = 0 \quad (82)$$

> $EcuaNormDos := diff(y(x), x\$2) - \frac{2\cdot x\cdot diff(y(x), x)}{(x^2+1)} + \frac{2\cdot y(x)}{(x^2+1)} = 0$

$$EcuaNormDos := \frac{d^2}{dx^2}\,y(x) - \frac{2x\left(\frac{d}{dx}\,y(x)\right)}{x^2+1} + \frac{2\,y(x)}{x^2+1} = 0 \quad (83)$$

> $yy[1] := x$

$$yy_1 := x \quad (84)$$

> $yy[2] := x^2 - 1$

$$yy_2 := x^2 - 1 \quad (85)$$

>

> $with(linalg):$

> $AA := wronskian([yy[1], yy[2]], x)$

$$AA := \begin{bmatrix} x & x^2-1 \\ 1 & 2x \end{bmatrix} \quad (86)$$

> $EcuaNoHom := lhs(EcuaNormDos) = \frac{(6\cdot(x^2+1)^2)}{(x^2+1)}$

$$EcuaNoHom := \frac{d^2}{dx^2}\,y(x) - \frac{2x\left(\frac{d}{dx}\,y(x)\right)}{x^2+1} + \frac{2\,y(x)}{x^2+1} = 6x^2 + 6 \quad (87)$$

> $Q := rhs(EcuaNoHom)$

$$Q := 6x^2 + 6 \quad (88)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} x & x^2-1 \\ 1 & 2x \end{bmatrix} \quad (89)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 6x^2+6 \end{bmatrix} \quad (90)$$

> $Para := linsolve(WW, BB)$

(91)

$$Para := \begin{bmatrix} -6x^2+6 & 6x \end{bmatrix} \quad (91)$$

> $Aprima := Para[1]$

$$Aprima := -6x^2+6 \quad (92)$$

> $Bprima := Para[2]$

$$Bprima := 6x \quad (93)$$

> $SolGralNoHom := y(x) = simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGralNoHom := y(x) = x^4 + (\_C2+3)x^2 + \_C1\,x - \_C2 \quad (94)$$

> $ComprobarFinal := simplify(eval(subs(y(x) = rhs(SolGralNoHom), EcuaNoHom)))$

$$ComprobarFinal := 6x^2+6 = 6x^2+6 \quad (95)$$

> $restart$

8)

> $Ecua := y'' + y = -\cos(x)$

$$Ecua := \frac{d^2}{dx^2}y(x) + y(x) = -\cos(x) \quad (96)$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}y(x) + y(x) = 0 \quad (97)$$

> $Q := rhs(Ecua)$

$$Q := -\cos(x) \quad (98)$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \quad (99)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := I, -I \quad (100)$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot x)$

$$yy_1 := \cos(x) \quad (101)$$

> $yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot x)$

$$yy_2 := \sin(x) \quad (102)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad (103)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & -\cos(x) \end{bmatrix} \quad (104)$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} \cos(x)\sin(x) & -\cos(x)^2 \end{bmatrix} \quad (105)$$

> $Aprima := Para[1]$

$$Aprima := \cos(x)\,\sin(x) \tag{106}$$

> $Bprima := Para[2]$

$$Bprima := -\cos(x)^2 \tag{107}$$

> $SolGral := y(x) = expand(simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2]))$

$$SolGral := y(x) = \sin(x)\ \_C2 - \frac{\sin(x)\,x}{2} + \cos(x)\ \_C1 \tag{108}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))$

$$Comprobar := -\cos(x) = -\cos(x) \tag{109}$$

> $restart$

9)

> $Ecua := diff(y(\text{theta}), \text{theta\$2}) + y(\text{theta}) = \sin(\text{theta})^2 - \cos(\text{theta})^2$

$$Ecua := \frac{d^2}{d\theta^2}\,y(\theta) + y(\theta) = \sin(\theta)^2 - \cos(\theta)^2 \tag{110}$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{d\theta^2}\,y(\theta) + y(\theta) = 0 \tag{111}$$

> $Q := rhs(Ecua)$

$$Q := \sin(\theta)^2 - \cos(\theta)^2 \tag{112}$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \tag{113}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := I, -I \tag{114}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot\text{theta})$

$$yy_1 := \cos(\theta) \tag{115}$$

> $yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot\text{theta})$

$$yy_2 := \sin(\theta) \tag{116}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], \text{theta})$

$$WW := \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \tag{117}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \sin(\theta)^2 - \cos(\theta)^2 \end{bmatrix} \tag{118}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} 2\sin(\theta)\cos(\theta)^2 - \sin(\theta) & -2\cos(\theta)^3 + \cos(\theta) \end{bmatrix} \tag{119}$$

> $Aprima := Para[1]$

$$Aprima := 2\sin(\theta)\cos(\theta)^2 - \sin(\theta) \quad \textbf{(120)}$$

> $Bprima := Para[2]$

$$Bprima := -2\cos(\theta)^3 + \cos(\theta) \quad \textbf{(121)}$$

> $SolGral := y(\text{theta}) = expand(simplify((int(Aprima, \text{theta}) + \_C1)\cdot yy[1] + (int(Bprima, \text{theta}) + \_C2)\cdot yy[2]))$

$$SolGral := y(\theta) = \cos(\theta)\,\_C1 + \sin(\theta)\,\_C2 + \frac{2\cos(\theta)^2}{3} - \frac{1}{3} \quad \textbf{(122)}$$

> $Comprobar := simplify(eval(subs(y(\text{theta}) = rhs(SolGral), Ecua)))$

$$Comprobar := -2\cos(\theta)^2 + 1 = -2\cos(\theta)^2 + 1 \quad \textbf{(123)}$$

> $restart$

FIN DE SERIE 2