> restart

FACULTAD DE INGENIERIA
DIVISION DE CIENCIAS BASICAS
ECUACIONES DIFERENCIALES
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SERIE 3
solución

> restart

1) Transforme la ecuación diferencial en un sistema de 2 ecuaciones de primer orden y resuélvalo

> $Ecua := diff(y(t), t\$2) - 6 \cdot diff(y(t), t) + 9\, y(t) = 0$

$$Ecua := \frac{d^2}{dt^2}\, y(t) - 6\, \frac{d}{dt}\, y(t) + 9\, y(t) = 0 \quad (1)$$

>

RESPUESTA

> $EcuaUno := y(t) = x[1](t)$

$$EcuaUno := y(t) = x_1(t) \quad (2)$$

> $EcuaDos := diff(x[1](t), t) = x[2](t)$

$$EcuaDos := \frac{d}{dt}\, x_1(t) = x_2(t) \quad (3)$$

> $EcuaTres := diff(x[2](t), t) = -9 \cdot x[1](t) + 6 \cdot x[2](t)$

$$EcuaTres := \frac{d}{dt}\, x_2(t) = -9\, x_1(t) + 6\, x_2(t) \quad (4)$$

> $Sistema := [EcuaDos, EcuaTres]:$

> $Sistema[1]$

$$\frac{d}{dt}\, x_1(t) = x_2(t) \quad (5)$$

> $Sistema[2]$

$$\frac{d}{dt}\, x_2(t) = -9\, x_1(t) + 6\, x_2(t) \quad (6)$$

> $AA := array([[0, 1], [-9, 6]])$

$$AA := \begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix} \quad (7)$$

> $with(linalg):$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} e^{3t} - 3\, t\, e^{3t} & t\, e^{3t} \\ -9\, t\, e^{3t} & e^{3t} + 3\, t\, e^{3t} \end{bmatrix} \quad (8)$$

> $Xcero := array([\_C1, \_C2])$

$$Xcero := \begin{bmatrix} \_C1 & \_C2 \end{bmatrix} \quad (9)$$

> $SolGral := evalm(MatExp \&* Xcero)$

(10)

$$SolGral := \left[\; \left(e^{3t} - 3\,t\,e^{3t}\right)\_C1 + t\,e^{3t}\_C2 \quad -9\,t\,e^{3t}\_C1 + \left(e^{3t} + 3\,t\,e^{3t}\right)\_C2 \;\right] \tag{10}$$

> Solucion[1] := x[1](t) = SolGral[1]

$$Solucion_1 := x_1(t) = \left(e^{3t} - 3\,t\,e^{3t}\right)\_C1 + t\,e^{3t}\_C2 \tag{11}$$

> Solucion[2] := x[2](t) = SolGral[2]

$$Solucion_2 := x_2(t) = -9\,t\,e^{3t}\_C1 + \left(e^{3t} + 3\,t\,e^{3t}\right)\_C2 \tag{12}$$

> SolucionGeneral := y(t) = rhs(Solucion[1])

$$SolucionGeneral := y(t) = \left(e^{3t} - 3\,t\,e^{3t}\right)\_C1 + t\,e^{3t}\_C2 \tag{13}$$

> Ecua

$$\frac{d^2}{dt^2}\,y(t) - 6\,\frac{d}{dt}\,y(t) + 9\,y(t) = 0 \tag{14}$$

> Comprobar := simplify(eval(subs(y(t) = rhs(SolucionGeneral), Ecua)))

$$Comprobar := 0 = 0 \tag{15}$$

> ComprobarUno := simplify(eval(subs(x[1](t) = rhs(Solucion[1]), x[2](t) = rhs(Solucion[2]), lhs(Sistema[1]) − rhs(Sistema[1]) = 0)))

$$ComprobarUno := 0 = 0 \tag{16}$$

> ComprobarDos := simplify(eval(subs(x[1](t) = rhs(Solucion[1]), x[2](t) = rhs(Solucion[2]), lhs(Sistema[2]) − rhs(Sistema[2]) = 0)))

$$ComprobarDos := 0 = 0 \tag{17}$$

> restart

2)

> Sistema := diff(x(t), t$2) + diff(y(t), t$2) = exp(2 t), 2·diff(x(t), t) + diff(y(t), t$2) = −exp(2 t) : Sistema[1]; Sistema[2]

$$\frac{d^2}{dt^2}\,x(t) + \frac{d^2}{dt^2}\,y(t) = e^{2t}$$

$$2\,\frac{d}{dt}\,x(t) + \frac{d^2}{dt^2}\,y(t) = -e^{2t} \tag{18}$$

> CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0

$$CondIni := x(0) = 0,\ D(x)(0) = 0,\ y(0) = 0,\ D(y)(0) = 0 \tag{19}$$

>

RESPUESTA

> SistUno := diff(x[1](t), t) = x[2](t)

$$SistUno := \frac{d}{dt}\,x_1(t) = x_2(t) \tag{20}$$

> SistDos := diff(x[2](t), t) = 2·x[2](t) + 2·exp(2·t)

$$SistDos := \frac{d}{dt}\,x_2(t) = 2\,x_2(t) + 2\,e^{2t} \tag{21}$$

> SistTres := diff(y[1](t), t) = y[2](t)

$$SistTres := \frac{d}{dt}\,y_1(t) = y_2(t) \tag{22}$$

> SistCuatro := diff(y[2](t), t) =−2·x[2](t) − exp(2 t)

$$SistCuatro := \frac{d}{dt}\, y_2(t) = -2\, x_2(t) - e^{2\,t} \qquad \textbf{(23)}$$

> AA := array([[0, 1, 0, 0], [0, 2, 0, 0], [0, 0, 0, 1], [0, −2, 0, 0]])

$$AA := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix} \qquad \textbf{(24)}$$

> with(linalg) :

> MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} 1 & \frac{e^{2\,t}}{2} - \frac{1}{2} & 0 & 0 \\ 0 & e^{2\,t} & 0 & 0 \\ 0 & -\frac{e^{2\,t}}{2} + \frac{1}{2} + t & 1 & t \\ 0 & -e^{2\,t} + 1 & 0 & 1 \end{bmatrix} \qquad \textbf{(25)}$$

> Xcero := array([0, 0, 0, 0])

$$Xcero := \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \qquad \textbf{(26)}$$

> SolHom := evalm(MatExp &* Xcero)

$$SolHom := \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \qquad \textbf{(27)}$$

> MatExpTau := map(rcurry(eval, t ='t − tau'), MatExp)

$$MatExpTau := \begin{bmatrix} 1 & \frac{e^{2\,t-2\,\tau}}{2} - \frac{1}{2} & 0 & 0 \\ 0 & e^{2\,t-2\,\tau} & 0 & 0 \\ 0 & -\frac{e^{2\,t-2\,\tau}}{2} + \frac{1}{2} + t - \tau & 1 & t - \tau \\ 0 & -e^{2\,t-2\,\tau} + 1 & 0 & 1 \end{bmatrix} \qquad \textbf{(28)}$$

> BB := array([0, 2·exp(2 t), 0, −exp(2 t)])

$$BB := \begin{bmatrix} 0 & 2\, e^{2\,t} & 0 & -e^{2\,t} \end{bmatrix} \qquad \textbf{(29)}$$

> BBtau := map(rcurry(eval, t ='tau'), BB)

$$BBtau := \begin{bmatrix} 0 & 2\, e^{2\,\tau} & 0 & -e^{2\,\tau} \end{bmatrix} \qquad \textbf{(30)}$$

> ProdTau := simplify(evalm(MatExpTau &* BBtau))

$$ProdTau := \begin{bmatrix} (e^{2\,t-2\,\tau} - 1)\, e^{2\,\tau} & 2\, e^{2\,t} & (t - \tau + 1)\, e^{2\,\tau} - e^{2\,t} & -2\, e^{2\,t} + e^{2\,\tau} \end{bmatrix} \qquad \textbf{(31)}$$

> ProdTau[1]

(32)

$$\left(e^{2t-2\tau}-1\right)e^{2\tau} \quad \textbf{(32)}$$

> *ProdTau*[2]

$$2\,e^{2t} \quad \textbf{(33)}$$

> *ProdTau*[3]

$$(t-\tau+1)\,e^{2\tau}-e^{2t} \quad \textbf{(34)}$$

> *ProdTau*[4]

$$-2\,e^{2t}+e^{2\tau} \quad \textbf{(35)}$$

> *SolNoHom* := *map*(*int*, *ProdTau*, tau = 0 ..*t*) :

> *SolFinal* := *x*[1](*t*) = *SolNoHom*[1], *x*[2](*t*) = *SolNoHom*[2], *y*[1](*t*) = *SolNoHom*[3], *y*[2](*t*) = *SolNoHom*[4] : *SolFinal*[1]; *SolFinal*[2]; *SolFinal*[3]; *SolFinal*[4]

$$x_1(t)=\frac{1}{2}+e^{2t}\,t-\frac{e^{2t}}{2}$$

$$x_2(t)=2\,e^{2t}\,t$$

$$y_1(t)=-\frac{3}{4}-\frac{t}{2}+\frac{3\,e^{2t}}{4}-e^{2t}\,t$$

$$y_2(t)=-\frac{1}{2}-2\,e^{2t}\,t+\frac{e^{2t}}{2} \quad \textbf{(36)}$$

> *ComprobarUno* := *simplify*(*eval*(*subs*(*x*(*t*) = *rhs*(*SolFinal*[1]), *y*(*t*) = *rhs*(*SolFinal*[3]), *Sistema*[1])))

$$ComprobarUno := e^{2t}=e^{2t} \quad \textbf{(37)}$$

> *ComprobarDos* := *simplify*(*eval*(*subs*(*x*(*t*) = *rhs*(*SolFinal*[1]), *y*(*t*) = *rhs*(*SolFinal*[3]), *Sistema*[2])))

$$ComprobarDos := -e^{2t}=-e^{2t} \quad \textbf{(38)}$$

> *Sistema*[1]; *Sistema*[2];

$$\frac{d^2}{dt^2}\,x(t)+\frac{d^2}{dt^2}\,y(t)=e^{2t}$$

$$2\,\frac{d}{dt}\,x(t)+\frac{d^2}{dt^2}\,y(t)=-e^{2t} \quad \textbf{(39)}$$

> *restart*

3)

> *Ecua* := *diff*(*y*(*t*), *t*\$2) − 4·*diff*(*y*(*t*), *t*) + 13·*y*(*t*) = Dirac(*t* − 4)

$$Ecua := \frac{d^2}{dt^2}\,y(t)-4\,\frac{d}{dt}\,y(t)+13\,y(t)=\mathrm{Dirac}(t-4) \quad \textbf{(40)}$$

> *CondIni* := *y*(0) = 1, D(*y*)(0) =−1

$$CondIni := y(0)=1,\ \mathrm{D}(y)(0)=-1 \quad \textbf{(41)}$$

RESPUESTA

> *with*(*inttrans*) :

> *EcuaTL* := *subs*(*CondIni*, *laplace*(*Ecua*, *t*, *s*))

$$EcuaTL := s^2\,\mathcal{L}(y(t),t,s)+5-s-4\,s\,\mathcal{L}(y(t),t,s)+13\,\mathcal{L}(y(t),t,s)=e^{-4s} \quad \textbf{(42)}$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{e^{-4s} + s - 5}{s^2 - 4s + 13} \quad \textbf{(43)}$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \frac{(1 - \text{Heaviside}(4 - t))\sin(3t - 12)\, e^{2t-8}}{3} + e^{2t}(\cos(3t) - \sin(3t)) \quad \textbf{(44)}$$

> $ComprobarUno := simplify(eval(subs(y(t) = rhs(SolPart), Ecua)))$

$$ComprobarUno := \text{Dirac}(t - 4) = \text{Dirac}(t - 4) \quad \textbf{(45)}$$

> $ComprobarDos := simplify(eval(subs(t = 0, SolPart)))$

$$ComprobarDos := y(0) = 1 \quad \textbf{(46)}$$

> $ComprobarTres := D(y)(0) = simplify(eval(subs(t = 0, rhs(diff(SolPart, t)))))$

$$ComprobarTres := D(y)(0) = -1 \quad \textbf{(47)}$$

> restart

4)

> $F := -\frac{\exp(-\text{Pi}\cdot s)}{s^2+1} + \frac{\exp\left(-\frac{\text{Pi}\cdot s}{2}\right)}{s} - \frac{\exp(-\text{Pi}\cdot s)}{s}$

$$F := -\frac{e^{-\pi s}}{s^2 + 1} + \frac{e^{-\frac{\pi s}{2}}}{s} - \frac{e^{-\pi s}}{s} \quad \textbf{(48)}$$

> $with(inttrans):$

> $f := invlaplace(F, s, t)$

$$f := \text{Heaviside}\left(t - \frac{\pi}{2}\right) + \text{Heaviside}(t - \pi)\,(\sin(t) - 1) \quad \textbf{(49)}$$

> restart

5)

> $Sistema := diff(x(t), t) = x(t) - y(t), diff(y(t), t) = -x(t) + 2\cdot y(t) + \exp(-t) : Sistema[1];$
$Sistema[2]$

$$\frac{d}{dt}\, x(t) = x(t) - y(t)$$

$$\frac{d}{dt}\, y(t) = -x(t) + 2\, y(t) + e^{-t} \quad \textbf{(50)}$$

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RESPUESTA 5a) Obtenga una ecuación diferencial equivalente

> $SistemaUno := isolate(Sistema[1], y(t))$

$$SistemaUno := y(t) = -\frac{d}{dt}\, x(t) + x(t) \quad \textbf{(51)}$$

> $SistemaDos := isolate(eval(subs(y(t) = rhs(SistemaUno), Sistema[2])), \exp(-t))$

$$SistemaDos := e^{-t} = -\frac{d^2}{dt^2}\, x(t) + 3\,\frac{d}{dt}\, x(t) - x(t) \quad \textbf{(52)}$$

> $Ecua := -rhs(SistemaDos) = -lhs(SistemaDos)$

$$Ecua := \frac{d^2}{dt^2}\,x(t) - 3\,\frac{d}{dt}\,x(t) + x(t) = -e^{-t} \quad \textbf{(53)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dt^2}\,x(t) - 3\,\frac{d}{dt}\,x(t) + x(t) = 0 \quad \textbf{(54)}$$

> $Q := rhs(Ecua)$

$$Q := -e^{-t} \quad \textbf{(55)}$$

> $EcuaCarac := m^2 - 3\cdot m + 1 = 0$

$$EcuaCarac := m^2 - 3\,m + 1 = 0 \quad \textbf{(56)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \frac{3}{2} + \frac{\sqrt{5}}{2}, \frac{3}{2} - \frac{\sqrt{5}}{2} \quad \textbf{(57)}$$

> $yy[1] := \exp(Raiz[1]\cdot t); evalf(\%, 2); yy[2] := \exp(Raiz[2]\cdot t); evalf(\%, 2)$

$$yy_1 := e^{\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)t}$$

$$e^{2.6\,t}$$

$$yy_2 := e^{\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)t}$$

$$e^{0.4\,t} \quad \textbf{(58)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], t)$

$$WW := \begin{bmatrix} e^{\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)t} & e^{\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)t} \\ \left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right) e^{\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)t} & \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right) e^{\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)t} \end{bmatrix} \quad \textbf{(59)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & -e^{-t} \end{bmatrix} \quad \textbf{(60)}$$

> $Para := simplify(linsolve(WW, BB)) : Aprima := Para[1]; evalf(\%, 2); Bprima := Para[2];$
$evalf(\%, 2)$

$$Aprima := -\frac{\sqrt{5}\,e^{-\frac{t\left(5+\sqrt{5}\right)}{2}}}{5}$$

$$-0.44\,e^{-3.6\,t}$$

$$Bprima := \frac{\sqrt{5}\,e^{\frac{t\left(-5+\sqrt{5}\right)}{2}}}{5}$$

$$0.44\,e^{-1.4\,t} \quad \textbf{(61)}$$

> $SolGralX := x(t) = evalf(simplify((int(Aprima, t) + \_C1) \cdot yy[1] + (int(Bprima, t) + \_C2) \cdot yy[2]), 2)$

$$SolGralX := x(t) = -0.20\,e^{-1.\,t} + e^{2.6\,t}\,\_C1 + e^{0.40\,t}\,\_C2 \quad \textbf{(62)}$$

>

RESPUESTA 5b) resolver el sistema

> $Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\,x(t) = x(t) - y(t)$$

$$\frac{d}{dt}\,y(t) = -x(t) + 2\,y(t) + e^{-t} \quad \textbf{(63)}$$

> $AA := array([[1, -1], [-1, 2]])$

$$AA := \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \quad \textbf{(64)}$$

> $BBB := array([0, \exp(-t)])$

$$BBB := \begin{bmatrix} 0 & e^{-t} \end{bmatrix} \quad \textbf{(65)}$$

> $Xcero := array([\_C1, \_C2])$

$$Xcero := \begin{bmatrix} \_C1 & \_C2 \end{bmatrix} \quad \textbf{(66)}$$

> $with(linalg):$

> $MatExp := exponential(AA, t) : evalf(MatExp[1, 1], 2); evalf(MatExp[1, 2], 2); evalf(MatExp[2, 1], 2); evalf(MatExp[2, 2], 2)$

$$0.72\,e^{0.40\,t} + 0.28\,e^{2.6\,t}$$

$$0.44\,e^{0.40\,t} - 0.44\,e^{2.6\,t}$$

$$0.44\,e^{0.40\,t} - 0.44\,e^{2.6\,t}$$

$$0.28\,e^{0.40\,t} + 0.72\,e^{2.6\,t} \quad \textbf{(67)}$$

> $SolGralHom := evalm(MatExp \&* Xcero) : SolGralHomX := x(t) = evalf(simplify(SolGralHom[1]), 2)$

$$SolGralHomX := x(t) = 0.10\,(2.8\,\_C1 - 4.4\,\_C2)\,e^{2.6\,t} + 0.10\,e^{0.40\,t}\,(7.2\,\_C1 + 4.4\,\_C2) \quad \textbf{(68)}$$

> $SolGralHomY := y(t) = evalf(simplify(SolGralHom[2]), 2)$

$$SolGralHomY := y(t) = 0.10\,(-4.4\,\_C1 + 7.2\,\_C2)\,e^{2.6\,t} + 0.20\,e^{0.40\,t}\,(2.2\,\_C1 + 1.4\,\_C2) \quad \textbf{(69)}$$

> $MatExpTau := map(rcurry(eval, t = 't - \text{tau}'), MatExp) : evalf(\%, 2)$

$$\begin{bmatrix} 0.72\,e^{0.40\,t - 0.40\,\tau} + 0.28\,e^{2.6\,t - 2.6\,\tau} & 0.44\,e^{0.40\,t - 0.40\,\tau} - 0.44\,e^{2.6\,t - 2.6\,\tau} \\ 0.44\,e^{0.40\,t - 0.40\,\tau} - 0.44\,e^{2.6\,t - 2.6\,\tau} & 0.28\,e^{0.40\,t - 0.40\,\tau} + 0.72\,e^{2.6\,t - 2.6\,\tau} \end{bmatrix} \quad \textbf{(70)}$$

> $BBBtau := map(rcurry(eval, t = '\text{tau}'), BBB)$

$$BBBtau := \begin{bmatrix} 0 & e^{-\tau} \end{bmatrix} \quad \textbf{(71)}$$

> $ProdTau := evalm(MatExpTau \&* BBBtau) : evalf(simplify(ProdTau[1]), 2); evalf(simplify(ProdTau[2]), 2);$

$$0.44\,\left(e^{0.40\,t-0.40\,\tau}-1.\,e^{2.6\,t-2.6\,\tau}\right)e^{-1.\,\tau}$$

$$0.10\,e^{-1.\,\tau}\left(7.2\,e^{2.6\,t-2.6\,\tau}+2.8\,e^{0.40\,t-0.40\,\tau}\right) \quad (72)$$

> $SolPartNoHom := simplify(map(int, ProdTau, tau = 0 .. t))$:

> $SolPartNoHomX := x(t) = evalf(SolPartNoHom[1], 2)$

$$SolPartNoHomX := x(t) = -0.12\,e^{2.6\,t}+0.32\,e^{0.40\,t}-0.20\,e^{-1.\,t} \quad (73)$$

> $SolPartNoHomY := y(t) = evalf(SolPartNoHom[2], 2)$

$$SolPartNoHomY := y(t) = 0.20\,e^{0.40\,t}+0.20\,e^{2.6\,t}-0.40\,e^{-1.\,t} \quad (74)$$

> $rhs(SolGralHomX); rhs(SolPartNoHomX)$

$$0.10\,(2.8\,\_C1-4.4\,\_C2)\,e^{2.6\,t}+0.10\,e^{0.40\,t}\,(7.2\,\_C1+4.4\,\_C2)$$

$$-0.12\,e^{2.6\,t}+0.32\,e^{0.40\,t}-0.20\,e^{-1.\,t} \quad (75)$$

> $SolFinal[1] := x(t) = simplify(evalm(rhs(SolGralHomX) + rhs(SolPartNoHomX)))$

$$SolFinal_1 := x(t) = (0.280\,\_C1-0.440\,\_C2-0.12)\,e^{2.6\,t}+(0.720\,\_C1+0.440\,\_C2+0.32)\,e^{0.4\,t}-0.20\,e^{-1.\,t} \quad (76)$$

> $SolFinal[2] := y(t) = simplify(evalm(rhs(SolGralHomY + rhs(SolPartNoHomY))))$

$$SolFinal_2 := y(t) = (-0.440\,\_C1+0.720\,\_C2+0.20)\,e^{2.6\,t}+(0.440\,\_C1+0.280\,\_C2+0.20)\,e^{0.4\,t}-0.40\,e^{-1.\,t} \quad (77)$$

> $simplify(subs(t = 0, SolFinal[1]))$

$$x(0) = 1.\,\_C1 \quad (78)$$

> $simplify(subs(t = 0, SolFinal[2]))$

$$y(0) = 1.\,\_C2 \quad (79)$$

> restart

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