> *restart*

FACULTAD DE INGENIERIA
DIVISION DE CIENCIAS BASICAS
ECUACIONES DIFERENCIALES
GRUPO 13 SEMESTRE 2025-1
SERIE 4
solución

> *restart*

1) Separación de variables con una constante positiva

> $Ecua := y \cdot diff(u(x,y), x, y) + u(x,y) = 0$

$$Ecua := y\left(\frac{\partial^2}{\partial x \partial y} u(x,y)\right) + u(x,y) = 0 \tag{1}$$

>

SOLUCIÓN

> $EcuaSeparable := eval(subs(u(x,y) = F(x) \cdot G(y), Ecua))$

$$EcuaSeparable := y\left(\frac{d}{dx} F(x)\right)\left(\frac{d}{dy} G(y)\right) + F(x)\,G(y) = 0 \tag{2}$$

> $EcuaSeparada := \frac{(lhs(EcuaSeparable) - F(x) \cdot G(y))}{y \cdot diff(G(y), y) \cdot F(x)} = \frac{(rhs(EcuaSeparable) - F(x) \cdot G(y))}{y \cdot diff(G(y), y) \cdot F(x)}$

$$EcuaSeparada := \frac{\frac{d}{dx} F(x)}{F(x)} = -\frac{G(y)}{y\left(\frac{d}{dy} G(y)\right)} \tag{3}$$

> $EcuaX := lhs(EcuaSeparada) = \beta^2$

$$EcuaX := \frac{\frac{d}{dx} F(x)}{F(x)} = \beta^2 \tag{4}$$

> $EcuaY := rhs(EcuaSeparada) = \beta^2$

$$EcuaY := -\frac{G(y)}{y\left(\frac{d}{dy} G(y)\right)} = \beta^2 \tag{5}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1\, e^{\beta^2 x} \tag{6}$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = c_1\, y^{-\frac{1}{\beta^2}} \tag{7}$$

> $SolGral := u(x,y) = rhs(SolX) \cdot subs(c_1 = 1, rhs(SolY))$

(8)

$$SolGral := u(x, y) = c_1 \, e^{\beta^2 x} \, y^{-\frac{1}{\beta^2}} \quad (8)$$

> $comprobar := simplify(eval(subs(u(x, y) = rhs(SolGral), Ecua)))$

$$comprobar := 0 = 0 \quad (9)$$

> $restart$

2) Obtener la serie coseno de Fourier

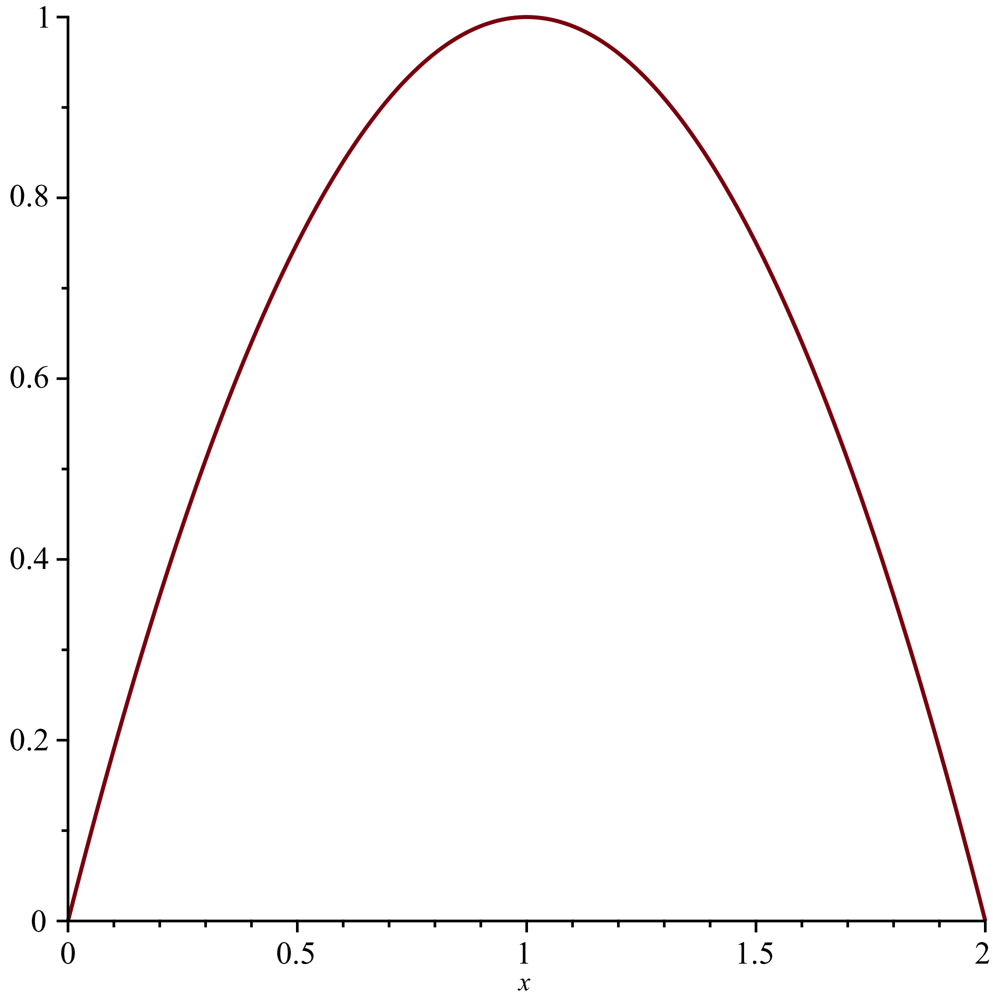
> $f := x \cdot (2 - x)$

$$f := x\,(2 - x) \quad (10)$$

> $Intervalo := 0 < x < 2$

$$Intervalo := 0 < x < 2 \quad (11)$$

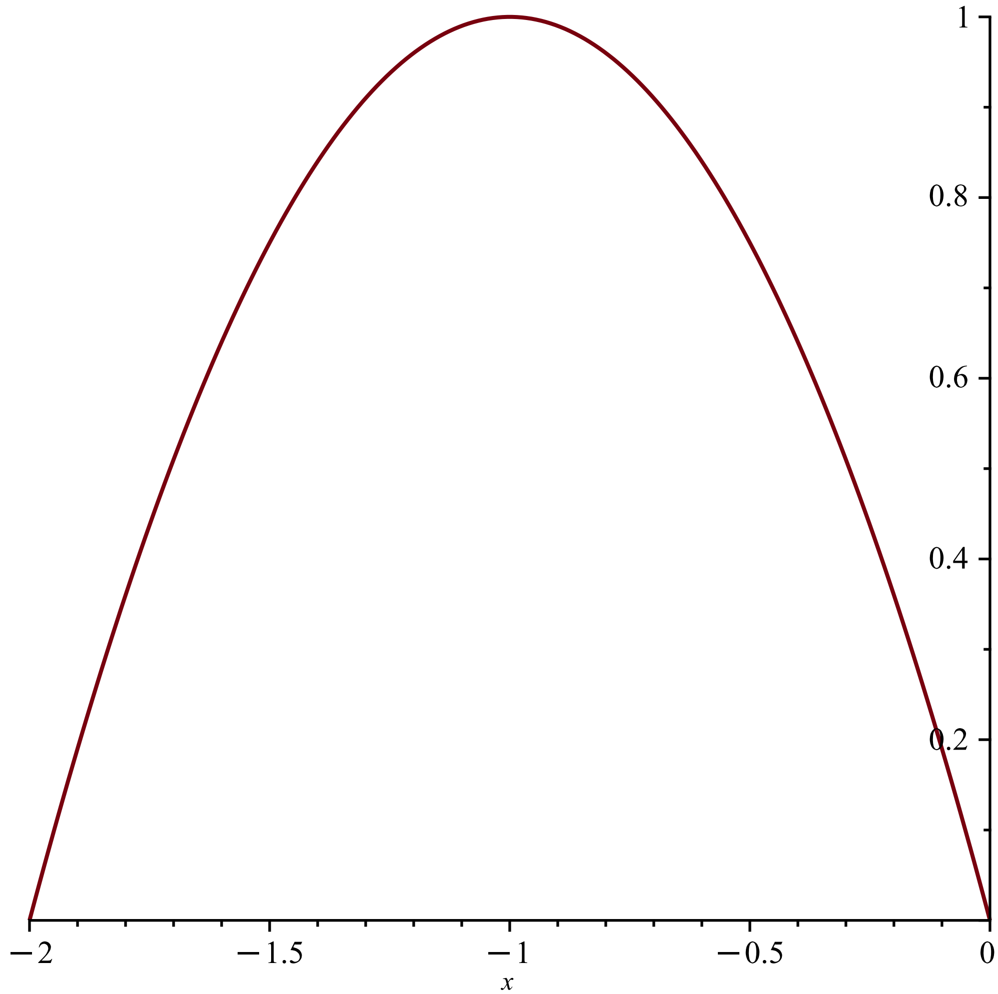
> $plot(f, x = 0..2)$



> $g := -x \cdot (2 + x)$

$$g := -x\,(2 + x) \quad (12)$$

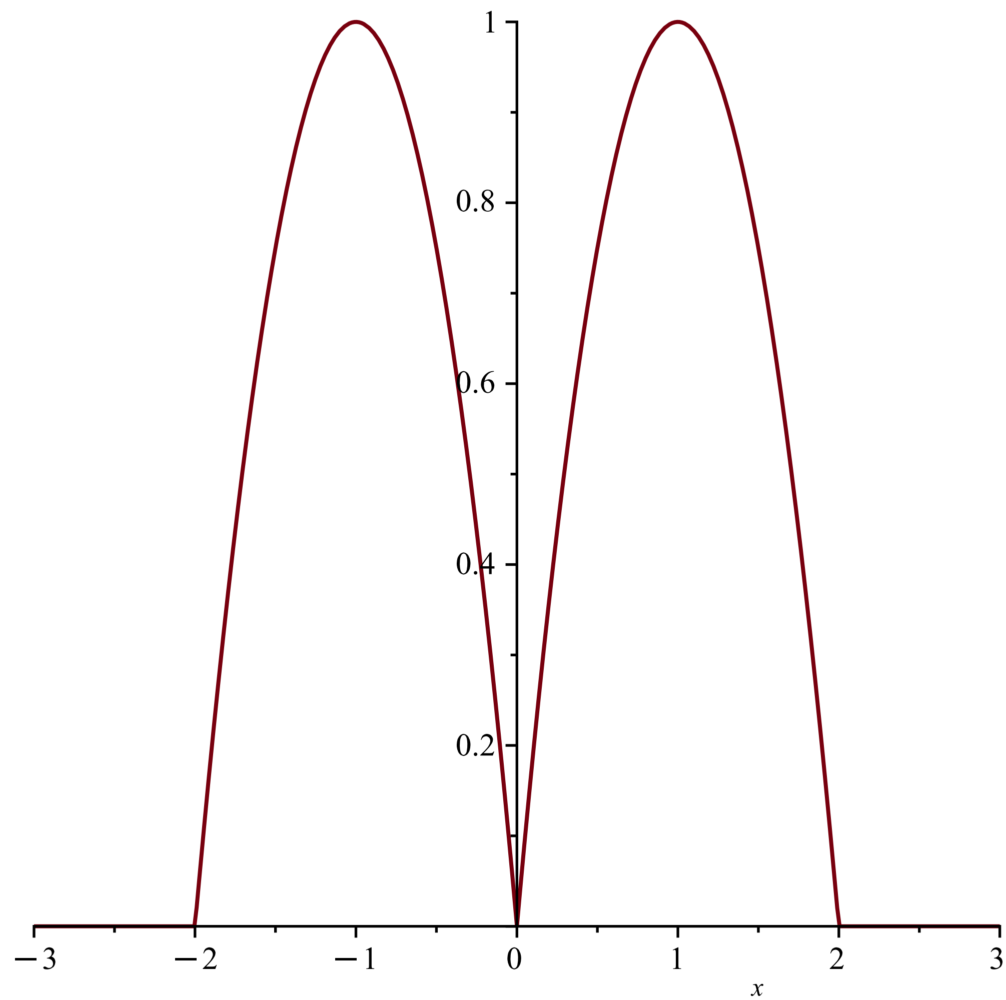
> $plot(g, x = -2..0)$



> $h := \text{Heaviside}(x+2)\cdot g - \text{Heaviside}(x)\cdot g + \text{Heaviside}(x)\cdot f - \text{Heaviside}(x-2)\cdot f$

$$h := -\text{Heaviside}(2+x)\, x\,(2+x) + \text{Heaviside}(x)\, x\,(2+x) + \text{Heaviside}(x)\, x\,(2-x) - \text{Heaviside}(x-2)\, x\,(2-x) \quad \textbf{(13)}$$

> $plot(h, x=-3..3)$



> $L := 2$

$$L := 2 \tag{14}$$

> $a[0] := \frac{1}{L} \cdot int(h, x = -L..L)$

$$a_0 := \frac{4}{3} \tag{15}$$

> $a[n] := simplify\left(subs\left(\cos(n \cdot \mathrm{Pi}) = (-1)^n, \sin(n \cdot \mathrm{Pi}) = 0, \frac{1}{L} \cdot int\left(h \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)\right)$

$$a_n := \frac{-8 - 8\,(-1)^n}{n^2\,\pi^2} \tag{16}$$

> $b[n] := simplify\left(subs\left(\cos(n \cdot \mathrm{Pi}) = (-1)^n, \sin(n \cdot \mathrm{Pi}) = 0, \frac{1}{L} \cdot int\left(h \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)\right)$

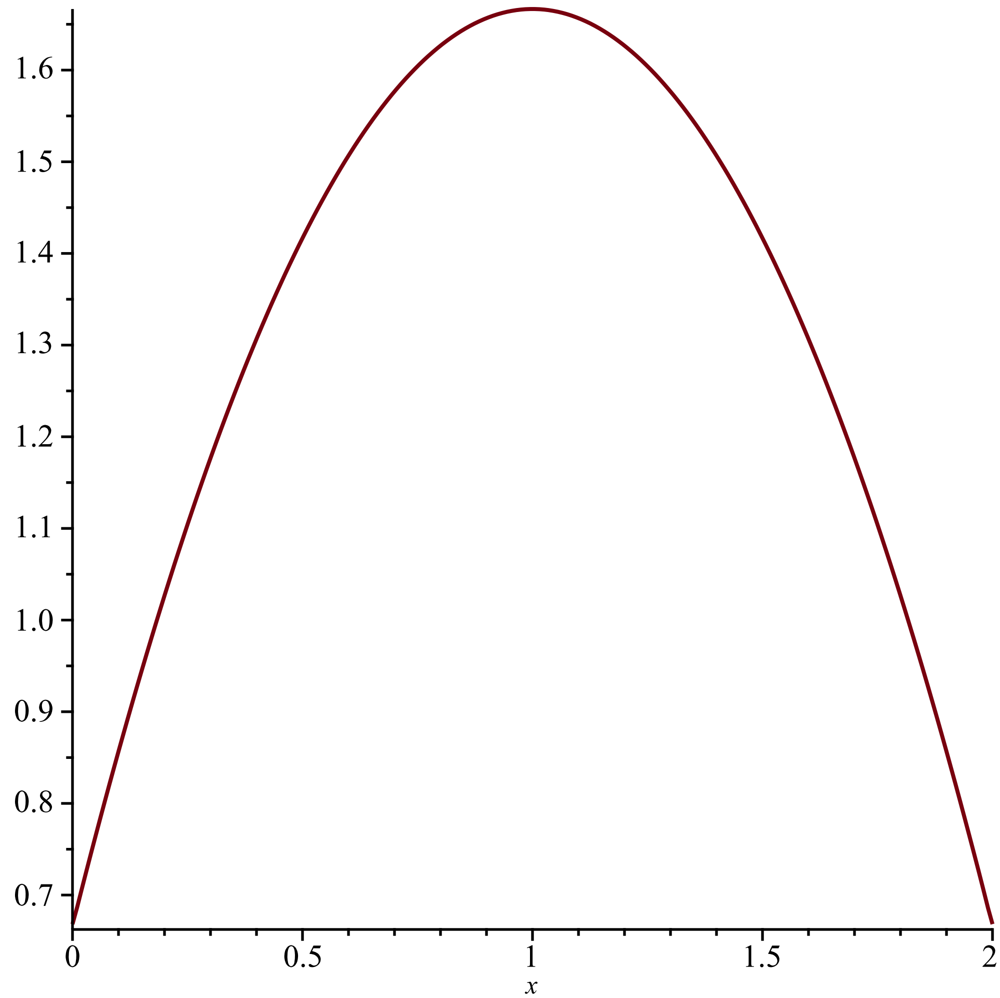
$$b_n := 0 \tag{17}$$

> $STF := a[0] + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..\mathrm{infinity}\right)$

$$STF := \frac{4}{3} + \left(\sum_{n=1}^{\infty} \frac{(-8 - 8(-1)^n)\cos\left(\frac{n\pi x}{2}\right)}{n^2\pi^2}\right) \quad \mathbf{(18)}$$

> $STF500 := a[0] + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..500\right)$:

> $plot(STF500, x = 0..2)$



> *restart*

3) serie coseno

> $f := \mathrm{Heaviside}(x + a)\cdot\exp(-a\cdot x) - \mathrm{Heaviside}(x)\exp(-a\cdot x) + \mathrm{Heaviside}(x)\cdot\exp(a\cdot x) - \mathrm{Heaviside}(x - a)\cdot\exp(a\cdot x)$

$$f := \mathrm{Heaviside}(x + a)\,e^{-a x} - \mathrm{Heaviside}(x)\,e^{-a x} + \mathrm{Heaviside}(x)\,e^{a x} - \mathrm{Heaviside}(x - a)\,e^{a x} \quad \mathbf{(19)}$$

> $L := a$

$$L := a \quad \mathbf{(20)}$$

> $a[0] := expand\left(\frac{1}{a}\cdot int(f, x = -L..L)\right)$

(21)

$$a_0 := \frac{2\,e^{a^2}\,\mathrm{Heaviside}(a)}{a^2} - \frac{2\,\mathrm{Heaviside}(a)}{a^2} - \frac{2\,e^{a^2}\,\mathrm{Heaviside}(-a)}{a^2} + \frac{2\,\mathrm{Heaviside}(-a)}{a^2} \quad \mathbf{(21)}$$

> $a[n] := expand\left(subs\left(\cos(n\cdot\mathrm{Pi}) = (-1)^n, \sin(n\cdot\mathrm{Pi}) = 0, \frac{1}{L}\cdot int\left(f\cdot\cos\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)\right)\right)$

$$a_n := \frac{2\,a^2\,\mathrm{Heaviside}(a)\,e^{a^2}\,(-1)^n}{\pi^2\,n^2 + a^4} - \frac{2\,a^2\,\mathrm{Heaviside}(a)}{\pi^2\,n^2 + a^4} - \frac{2\,a^2\,\mathrm{Heaviside}(-a)\,e^{a^2}\,(-1)^n}{\pi^2\,n^2 + a^4} + \frac{2\,a^2\,\mathrm{Heaviside}(-a)}{\pi^2\,n^2 + a^4} \quad \mathbf{(22)}$$

> $b[n] := expand\left(\frac{1}{L}\cdot int\left(f\cdot\sin\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)\right)$

$$b_n := 0 \quad \mathbf{(23)}$$

> $STF := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), n = 1..\mathrm{infinity}\right)$

$$STF := \frac{e^{a^2}\,\mathrm{Heaviside}(a)}{a^2} - \frac{\mathrm{Heaviside}(a)}{a^2} - \frac{e^{a^2}\,\mathrm{Heaviside}(-a)}{a^2} + \frac{\mathrm{Heaviside}(-a)}{a^2} + \left(\sum_{n=1}^{\infty}\left(\frac{2\,a^2\,\mathrm{Heaviside}(a)\,e^{a^2}\,(-1)^n}{\pi^2\,n^2 + a^4} - \frac{2\,a^2\,\mathrm{Heaviside}(a)}{\pi^2\,n^2 + a^4} - \frac{2\,a^2\,\mathrm{Heaviside}(-a)\,e^{a^2}\,(-1)^n}{\pi^2\,n^2 + a^4} + \frac{2\,a^2\,\mathrm{Heaviside}(-a)}{\pi^2\,n^2 + a^4}\right)\cos\left(\frac{n\,\pi\,x}{a}\right)\right) \quad \mathbf{(24)}$$

> *restart*

4) obtener la solución para constante positiva

> $Ecua := diff(u(x,t), t\$2) = 9\cdot diff(u(x,t), x\$2)$

$$Ecua := \frac{\partial^2}{\partial t^2}\,u(x,t) = 9\,\frac{\partial^2}{\partial x^2}\,u(x,t) \quad \mathbf{(25)}$$

> $EcuaSeparable := eval(subs(u(x,t) = F(x)\cdot G(t), Ecua))$

$$EcuaSeparable := F(x)\left(\frac{d^2}{dt^2}\,G(t)\right) = 9\left(\frac{d^2}{dx^2}\,F(x)\right)G(t) \quad \mathbf{(26)}$$

> $EcuaSeparada := \frac{lhs(EcuaSeparable)}{9\cdot F(x)\cdot G(t)} = \frac{rhs(EcuaSeparable)}{9\cdot F(x)\cdot G(t)}$

$$EcuaSeparada := \frac{\frac{d^2}{dt^2}\,G(t)}{9\,G(t)} = \frac{\frac{d^2}{dx^2}\,F(x)}{F(x)} \quad \mathbf{(27)}$$

> $EcuaT := lhs(EcuaSeparada) = \mathrm{beta}^2$

$$EcuaT := \frac{\frac{d^2}{dt^2}\,G(t)}{9\,G(t)} = \beta^2 \quad \mathbf{(28)}$$

> $EcuaX := rhs(EcuaSeparada) = \text{beta}^2$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \quad \textbf{(29)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 e^{\beta x} + c_2 e^{-\beta x} \quad \textbf{(30)}$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = c_1 e^{-3 \beta t} + c_2 e^{3 \beta t} \quad \textbf{(31)}$$

> $SolGral := u(x, t) = simplify(subs(c_1 = 1, c_2 = 1, rhs(SolX)) \cdot rhs(SolT))$

$$SolGral := u(x, t) = e^{\beta(-3t - x)} (e^{2 \beta x} + 1) (c_2 e^{6 \beta t} + c_1) \quad \textbf{(32)}$$

> restart

5)

> $f := x$

$$f := x \quad \textbf{(33)}$$

> $L := \text{Pi}$

$$L := \pi \quad \textbf{(34)}$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := 0 \quad \textbf{(35)}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)$

$$a_n := 0 \quad \textbf{(36)}$$

> $b[n] := subs\left(\cos(n \cdot \text{Pi}) = (-1)^n, \sin(n \cdot \text{Pi}) = 0, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

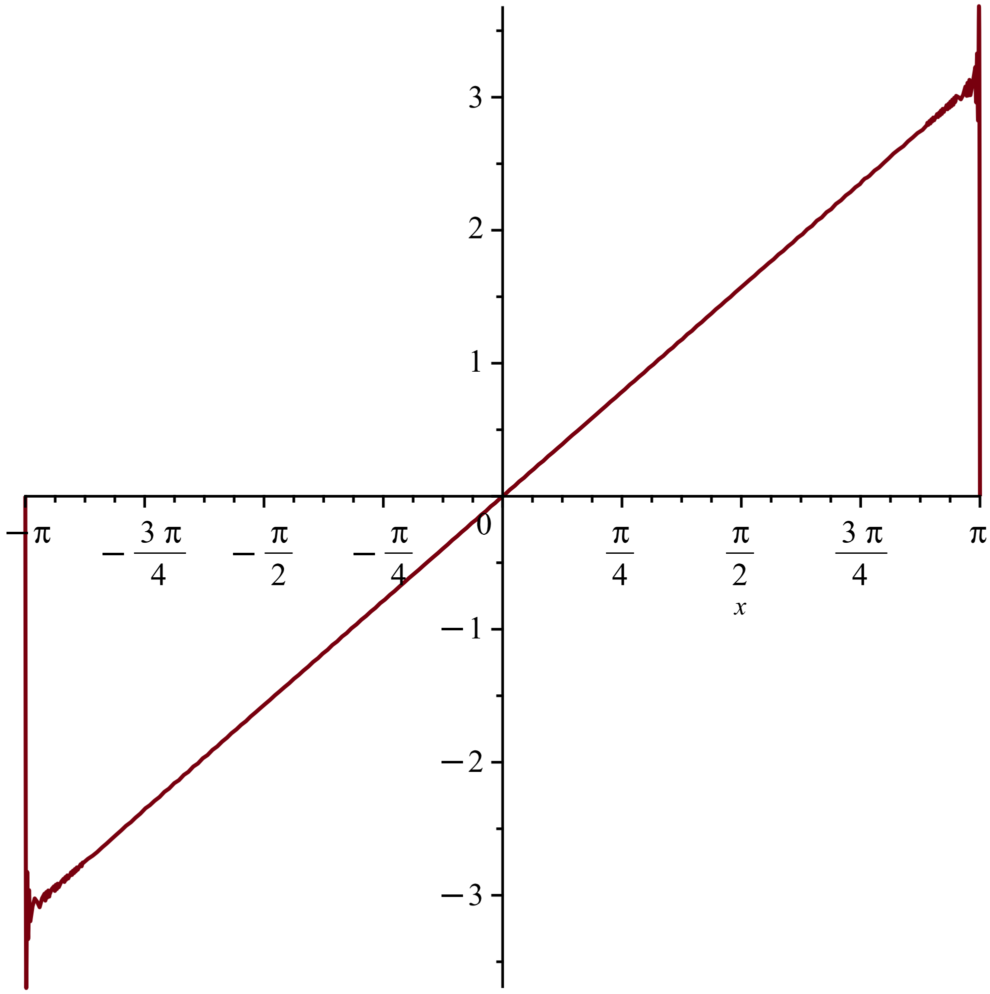
$$b_n := -\frac{2 (-1)^n}{n} \quad \textbf{(37)}$$

> $STF4 := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..4\right)$

$$STF4 := 2 \sin(x) - \sin(2 x) + \frac{2 \sin(3 x)}{3} - \frac{\sin(4 x)}{2} \quad \textbf{(38)}$$

> $STF500 := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..500\right) :$

> $plot(STF500, x = -L..L)$



> *restart*

6) obtener la Ecuación cuya solución

> $f(x)\cdot\exp(x\cdot y)+g(x)\cdot\exp(-x\cdot y)+\dfrac{\exp(y)}{1-x^2}$

$$f(x)\,e^{xy}+g(x)\,e^{-xy}+\frac{e^{y}}{-x^2+1} \quad \textbf{(39)}$$

> $u := f(x)\cdot\exp(x\cdot y)+g(x)\cdot\exp(-x\cdot y)$

$$u := f(x)\,e^{xy}+g(x)\,e^{-xy} \quad \textbf{(40)}$$

> $Q := \dfrac{\exp(y)}{1-x^2}$

$$Q := \frac{e^{y}}{-x^2+1} \quad \textbf{(41)}$$

> $DerYY := diff(u, y\$2)$

$$DerYY := f(x)\,x^2\,e^{xy}+g(x)\,x^2\,e^{-xy} \quad \textbf{(42)}$$

> $expand(DerYY - x^2\cdot u)$

$$0 \quad \textbf{(43)}$$

> $EcuaHom := diff(v(x,y), y\$2) - x^2\cdot v(x,y) = 0$

$$EcuaHom := \frac{\partial^2}{\partial y^2} v(x,y) - x^2 v(x,y) = 0 \quad \textbf{(44)}$$

> $pdsolve(EcuaHom)$

$$v(x,y) = f_1(x)\, e^{-x y} + f_2(x)\, e^{x y} \quad \textbf{(45)}$$

> $QQ := simplify(eval(subs(v(x,y) = Q, lhs(EcuaHom))))$

$$QQ := e^{y} \quad \textbf{(46)}$$

> $EcuaNoHom := lhs(EcuaHom) = QQ$

$$EcuaNoHom := \frac{\partial^2}{\partial y^2} v(x,y) - x^2 v(x,y) = e^{y} \quad \textbf{(47)}$$

> $pdsolve(EcuaNoHom)$

$$v(x,y) = e^{-x y} f_2(x) + e^{x y} f_1(x) - \frac{e^{y}}{x^2 - 1} \quad \textbf{(48)}$$

> $restart$

7) resuelva para constante separacion positiva

> $Ecua := k \cdot (diff(u(r,t), r\$2) + diff(u(r,t), r)) = diff(u(r,t), t)$

$$Ecua := k\left(\frac{\partial^2}{\partial r^2} u(r,t) + \frac{\partial}{\partial r} u(r,t)\right) = \frac{\partial}{\partial t} u(r,t) \quad \textbf{(49)}$$

> $EcuaSep := eval(subs(u(r,t) = F(r) \cdot G(t), Ecua))$

$$EcuaSep := k\left(\left(\frac{d^2}{dr^2} F(r)\right) G(t) + \left(\frac{d}{dr} F(r)\right) G(t)\right) = F(r)\left(\frac{d}{dt} G(t)\right) \quad \textbf{(50)}$$

> $EcuaSeparada := factor\left(\frac{lhs(EcuaSep)}{k \cdot F(r) \cdot G(t)}\right) = \frac{rhs(EcuaSep)}{k \cdot F(r) \cdot G(t)}$

$$EcuaSeparada := \frac{\frac{d^2}{dr^2} F(r) + \frac{d}{dr} F(r)}{F(r)} = \frac{\frac{d}{dt} G(t)}{k\, G(t)} \quad \textbf{(51)}$$

> $EcuaR := lhs(EcuaSeparada) = \beta^2$

$$EcuaR := \frac{\frac{d^2}{dr^2} F(r) + \frac{d}{dr} F(r)}{F(r)} = \beta^2 \quad \textbf{(52)}$$

> $EcuaT := rhs(EcuaSeparada) = \beta^2$

$$EcuaT := \frac{\frac{d}{dt} G(t)}{k\, G(t)} = \beta^2 \quad \textbf{(53)}$$

> $SolR := dsolve(EcuaR)$

$$SolR := F(r) = c_1\, e^{\left(-\frac{1}{2} + \frac{\sqrt{4\beta^2+1}}{2}\right) r} + c_2\, e^{\left(-\frac{1}{2} - \frac{\sqrt{4\beta^2+1}}{2}\right) r} \quad \textbf{(54)}$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = c_1\, e^{\beta^2 k t} \quad \textbf{(55)}$$

> $SolGralPos := u(r,t) = rhs(SolR) \cdot subs(c_1 = 1, rhs(SolT))$

$$SolGralPos := u(r,t) = \left(c_1 e^{\left(-\frac{1}{2}+\frac{\sqrt{4\beta^2+1}}{2}\right)r} + c_2 e^{\left(-\frac{1}{2}-\frac{\sqrt{4\beta^2+1}}{2}\right)r}\right) e^{\beta^2 k t} \quad \textbf{(56)}$$

> $Comprobar := simplify(eval(subs(u(r,t) = rhs(SolGralPos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad \textbf{(57)}$$

> restart

8) Serie de Fourier

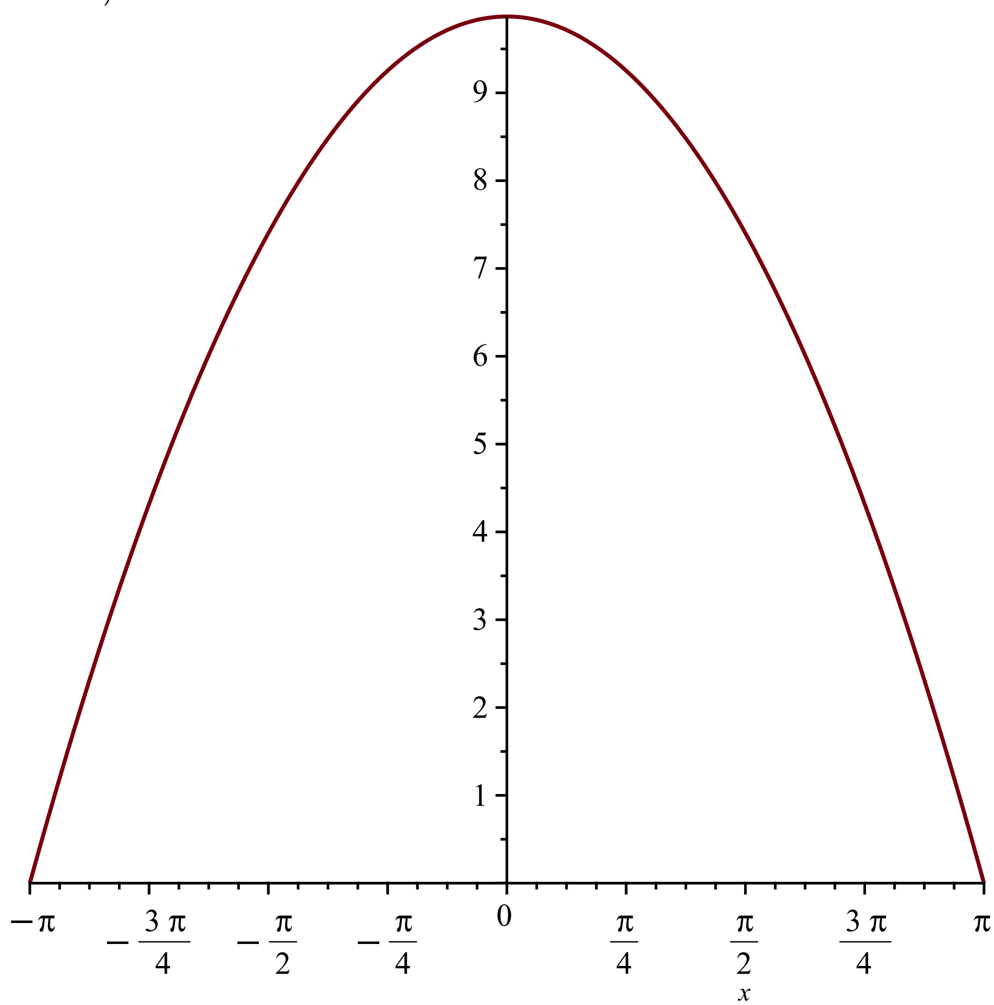
> $f := \pi^2 - x^2$

$$f := \pi^2 - x^2 \quad \textbf{(58)}$$

> $L := \text{Pi}$

$$L := \pi \quad \textbf{(59)}$$

> $plot(f, x = -L..L)$



> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := \frac{4\pi^2}{3} \quad \textbf{(60)}$$

> $a[n] := subs\left(\cos(n\cdot \mathrm{Pi}) = (-1)^n, \sin(n\cdot \mathrm{Pi}) = 0, \frac{1}{L}\cdot int\left(f\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)\right)$

$$a_n := -\frac{4\,(-1)^n}{n^2} \quad (61)$$

> $b[n] := subs\left(\cos(n\cdot \mathrm{Pi}) = (-1)^n, \sin(n\cdot \mathrm{Pi}) = 0, \frac{1}{L}\cdot int\left(f\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)\right)$

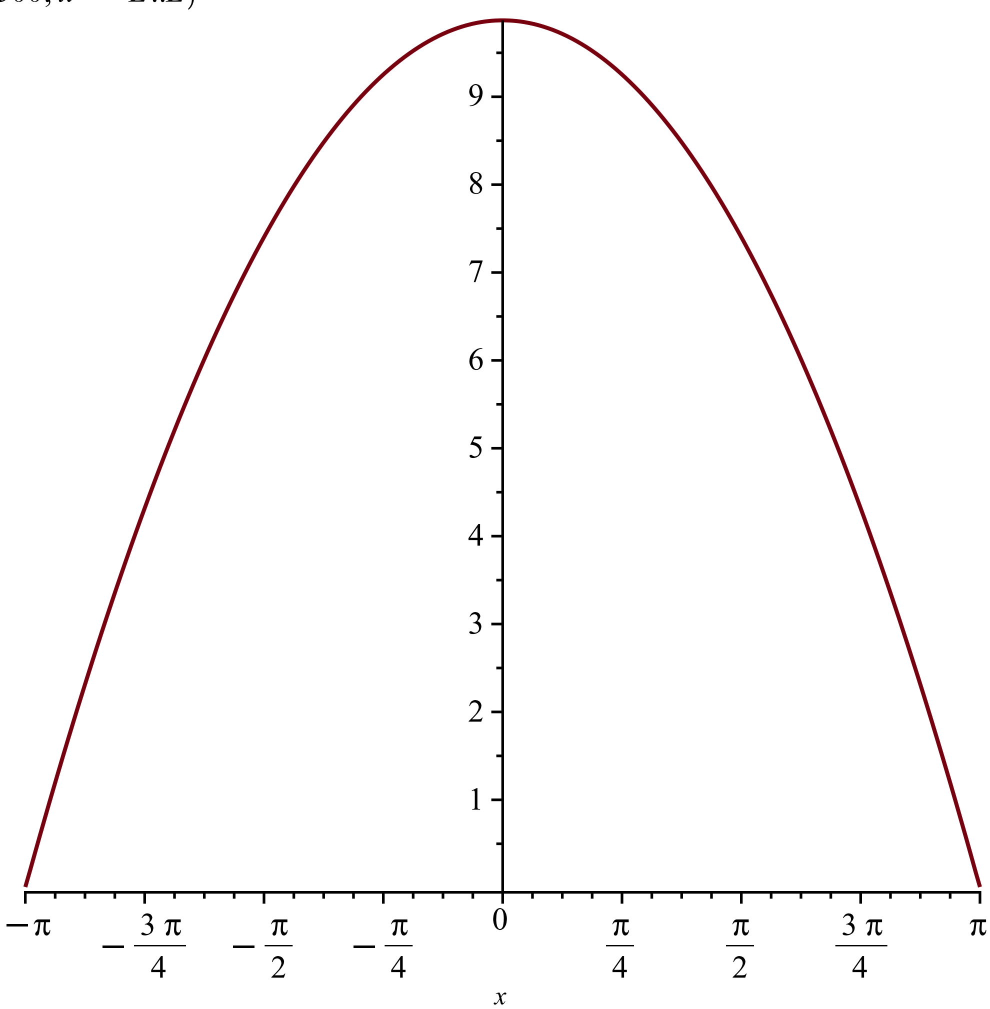
$$b_n := 0 \quad (62)$$

> $STF := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..\mathrm{infinity}\right)$

$$STF := \frac{2\,\pi^2}{3} + \sum_{n=1}^{\infty}\left(-\frac{4\,(-1)^n\cos(n\,x)}{n^2}\right) \quad (63)$$

> $STF500 := \frac{a[0]}{2} + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..500\right)$:

> $plot(STF500, x = -L..L)$



> $restart$

9) resolver EDenDP para alpha = 1

> $Ecua := diff(u(x,t), x) - 3\cdot diff(u(x,t), t, x) + diff(u(x,t), t\$2) = 0$

(64)

$$Ecua := \frac{\partial}{\partial x} u(x,t) - 3 \frac{\partial^2}{\partial t \partial x} u(x,t) + \frac{\partial^2}{\partial t^2} u(x,t) = 0 \quad (64)$$

> EcuaSep := eval(subs(u(x, t) = F(x)·G(t), Ecua))

$$EcuaSep := \left(\frac{d}{dx} F(x)\right) G(t) - 3 \left(\frac{d}{dx} F(x)\right)\left(\frac{d}{dt} G(t)\right) + F(x)\left(\frac{d^2}{dt^2} G(t)\right) = 0 \quad (65)$$

> $EcuaSeparada := simplify\left(\frac{\left(lhs(EcuaSep) - \left(F(x)\left(\frac{d^2}{dt^2} G(t)\right)\right)\right)}{-(F(x)\cdot(G(t) - 3\cdot diff(G(t), t)))}\right) = simplify\left(\frac{\left(rhs(EcuaSep) - \left(F(x)\left(\frac{d^2}{dt^2} G(t)\right)\right)\right)}{-(F(x)\cdot(G(t) - 3\cdot diff(G(t), t)))}\right)$

$$EcuaSeparada := -\frac{\frac{d}{dx} F(x)}{F(x)} = \frac{\frac{d^2}{dt^2} G(t)}{G(t) - 3 \frac{d}{dt} G(t)} \quad (66)$$

> EcuaX := lhs(EcuaSeparada) = 1

$$EcuaX := -\frac{\frac{d}{dx} F(x)}{F(x)} = 1 \quad (67)$$

> EcuaT := rhs(EcuaSeparada) = 1

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{G(t) - 3 \frac{d}{dt} G(t)} = 1 \quad (68)$$

> SolX := dsolve(EcuaX)

$$SolX := F(x) = c_1 e^{-x} \quad (69)$$

> SolT := dsolve(EcuaT)

$$SolT := G(t) = c_1 e^{\frac{(-3+\sqrt{13})t}{2}} + c_2 e^{-\frac{(3+\sqrt{13})t}{2}} \quad (70)$$

> $SolGral := u(x,t) = subs(c_1 = 1, rhs(SolX))\cdot rhs(SolT)$

$$SolGral := u(x,t) = e^{-x}\left(c_1 e^{\frac{(-3+\sqrt{13})t}{2}} + c_2 e^{-\frac{(3+\sqrt{13})t}{2}}\right) \quad (71)$$

> Comprobar := simplify(eval(subs(u(x, t) = rhs(SolGral), Ecua)))

$$Comprobar := 0 = 0 \quad (72)$$

> restart

>