>

SERIE GRUPAL ECUACIONES DIFERENCIALES
UNIDAD 1
GRUPO 13 SEMESTRE 2026-1
SOLUCIÓN

> *restart*

1) Determinar solución particular

> $Ecua := x \cdot y' + x = y$

$$Ecua := x\left(\frac{d}{dx}\,y(x)\right) + x = y(x) \tag{1}$$

> $Cond := y(1) = 1$

$$Cond := y(1) = 1 \tag{2}$$

Respuesta

> $EcuaDos := simplify\left(\frac{lhs(Ecua)}{x}\right) = \frac{rhs(Ecua)}{x}$

$$EcuaDos := \frac{d}{dx}\,y(x) + 1 = \frac{y(x)}{x} \tag{3}$$

> $EcuaTres := lhs(EcuaDos) - 1 - rhs(EcuaDos) = rhs(EcuaDos) - 1 - rhs(EcuaDos)$

$$EcuaTres := \frac{d}{dx}\,y(x) - \frac{y(x)}{x} = -1 \tag{4}$$

> $P := -\frac{1}{x}; Q := rhs(EcuaTres)$

$$P := -\frac{1}{x}$$

$$Q := -1 \tag{5}$$

> $SolGral := y(x) = \_C1 \cdot \exp(-int(P, x)) + \exp(-int(P, x)) \cdot int(\exp(int(P, x)) \cdot Q, x)$

$$SolGral := y(x) = \_C1\,x - x\ln(x) \tag{6}$$

> $Para := simplify(isolate(subs(x = 1, rhs(SolGral) = 1), \_C1))$

$$Para := \_C1 = 1 \tag{7}$$

> $SolPart := subs(\_C1 = rhs(Para), SolGral)$

$$SolPart := y(x) = x - x\ln(x) \tag{8}$$

Fin respuesta 1

> *restart*

2) Resuelva el problema de valor inicial

> $Ecua := (\exp(x) \cdot \log(y)) + \left(2^{(-1)} \cdot \exp(2x) \cdot y^{(-1)}\right) \cdot y' = 0$

$$Ecua := e^{x}\ln(y(x)) + \frac{e^{2x}\left(\frac{d}{dx}\,y(x)\right)}{2\,y(x)} = 0 \tag{9}$$

> $Cond := y(0) = \exp(1)$

$$Cond := y(0) = e \tag{10}$$

Respuesta

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_separable] \quad (11)$$

> $M := e^{x}\ln(y)$

$$M := e^{x}\ln(y) \quad (12)$$

> $N := \frac{e^{2x}}{2y}$

$$N := \frac{e^{2x}}{2y} \quad (13)$$

> $P := \exp(x); Q := \log(y); R := \exp(2x); S := \frac{1}{2\cdot y}$

$$P := e^{x}$$
$$Q := \ln(y)$$
$$R := e^{2x}$$
$$S := \frac{1}{2y} \quad (14)$$

> $SolGral := simplify\left(int\left(\frac{P}{R}, x\right)\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := -e^{-x} + \frac{\ln(\ln(y))}{2} = \_C1 \quad (15)$$

> $SolGralFinal := isolate\left(-e^{-x} + \frac{\ln(\ln(y(x)))}{2} = \_C1, y(x)\right)$

$$SolGralFinal := y(x) = e^{e^{\frac{2(\_C1\, e^{x}+1)}{e^{x}}}} \quad (16)$$

> $Para := simplify(isolate(subs(x=0, rhs(SolGralFinal) = \exp(1)), \_C1))$

$$Para := \_C1 = -1 \quad (17)$$

> $SolPart := simplify(subs(\_C1 = rhs(Para), SolGralFinal))$

$$SolPart := y(x) = e^{e^{-2+2e^{-x}}} \quad (18)$$

> $Comprobar := simplify(subs(x=0, rhs(SolPart)))$

$$Comprobar := e \quad (19)$$

> $Cond$

$$y(0) = e \quad (20)$$

Fin respuesta 2

> $restart$

3) Resuelva

> $Ecua := \left(y^{2}\cdot\cos(x) - 3\cdot x^{2}\cdot y - 2x\right) + \left(2\cdot y\cdot\sin(x) - x^{3} + \log(y)\right)\cdot y' = 0$

$$Ecua := y(x)^{2}\cos(x) - 3x^{2}y(x) - 2x + \left(2y(x)\sin(x) - x^{3} + \ln(y(x))\right)\left(\frac{d}{dx}y(x)\right) = 0 \quad (21)$$

> $Cond := y(0) = \exp(1)$

(22)

$$Cond := y(0) = e \tag{22}$$

Respuesta

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_exact] \tag{23}$$

> $M := \left(y^2 \cdot \cos(x) - 3 \cdot x^2 \cdot y - 2\,x\right)$

$$M := y^2 \cos(x) - 3\,x^2\,y - 2\,x \tag{24}$$

> $N := \left(2 \cdot y \cdot \sin(x) - x^3 + \log(y)\right)$

$$N := 2\,y \sin(x) - x^3 + \ln(y) \tag{25}$$

> $IntMx := int(M, x)$

$$IntMx := y^2 \sin(x) - x^2 - x^3\,y \tag{26}$$

> $SolGral := IntMx + int(\left(N - diff\left(IntMx, y\right)\right), y) = \_C1$

$$SolGral := y^2 \sin(x) - x^2 - x^3\,y + \ln(y)\,y - y = c_1 \tag{27}$$

> $SolGralFinal := y(x)^2 \sin(x) - x^2 - x^3\,y(x) + \ln(y(x))\,y(x) - y(x) = c_1$

$$SolGralFinal := y(x)^2 \sin(x) - x^2 - x^3\,y(x) + \ln(y(x))\,y(x) - y(x) = c_1 \tag{28}$$

> $Para := simplify(subs(x = 0, y(0) = \exp(1), SolGralFinal))$

$$Para := 0 = c_1 \tag{29}$$

> $SolPart := subs\left(c_1 = lhs(Para), SolGralFinal\right)$

$$SolPart := y(x)^2 \sin(x) - x^2 - x^3\,y(x) + \ln(y(x))\,y(x) - y(x) = 0 \tag{30}$$

> $Comprobar := simplify(subs(x = 0, y(0) = \exp(1), SolPart))$

$$Comprobar := 0 = 0 \tag{31}$$

> $DerSolPart := simplify(isolate(diff\left(SolPart, x\right), diff\left(y(x), x\right)))$

$$DerSolPart := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = \frac{y(x)^2 \cos(x) - 3\,x^2\,y(x) - 2\,x}{x^3 - 2\,y(x) \sin(x) - \ln(y(x))} \tag{32}$$

> $Ecua$

$$y(x)^2 \cos(x) - 3\,x^2\,y(x) - 2\,x + \left(2\,y(x) \sin(x) - x^3 + \ln(y(x))\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) = 0 \tag{33}$$

> $DerEcua := simplify(isolate(Ecua, diff\left(y(x), x\right)))$

$$DerEcua := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = \frac{y(x)^2 \cos(x) - 3\,x^2\,y(x) - 2\,x}{x^3 - 2\,y(x) \sin(x) - \ln(y(x))} \tag{34}$$

> $ComprobarDos := simplify(rhs(DerSolPart) - rhs(DerEcua) = 0)$

$$ComprobarDos := 0 = 0 \tag{35}$$

Fin respuesta 3

> $restart$

4) Resuelva

> $Ecua := \frac{y}{x} + \left(y^3 - \log(x)\right) \cdot y' = 0$

(36)

$$Ecua := \frac{y(x)}{x} + \left(y(x)^3 - \ln(x)\right)\left(\frac{d}{dx}\, y(x)\right) = 0 \tag{36}$$

> $Cond := y(1) = 2$

$$Cond := y(1) = 2 \tag{37}$$

Respuesta

> $with(DEtools)$:

> $odeadvisor(Ecua)$

$$[[\_1st\_order, \_with\_symmetry\_[F(x),G(y)]]] \tag{38}$$

> $intfactor(Ecua)$

$$\frac{1}{y(x)^2} \tag{39}$$

> $FactInt := \frac{1}{y^2}$

$$FactInt := \frac{1}{y^2} \tag{40}$$

> $M := \frac{y}{x}$

$$M := \frac{y}{x} \tag{41}$$

> $N := y^3 - \ln(x)$

$$N := y^3 - \ln(x) \tag{42}$$

> $MM := FactInt \cdot M$

$$MM := \frac{1}{y\,x} \tag{43}$$

> $NN := expand(FactInt \cdot N)$

$$NN := y - \frac{\ln(x)}{y^2} \tag{44}$$

> $diff(MM, y) = diff(NN, x)$

$$-\frac{1}{y^2\,x} = -\frac{1}{y^2\,x} \tag{45}$$

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{\ln(x)}{y} \tag{46}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{\ln(x)}{y} + \frac{y^2}{2} = c_1 \tag{47}$$

> $Para := simplify(subs(x = 1, y = 2, SolGral))$

$$Para := 2 = c_1 \tag{48}$$

> $SolPart := subs(c_1 = lhs(Para), SolGral)$

$$SolPart := \frac{\ln(x)}{y} + \frac{y^2}{2} = 2 \tag{49}$$

> $SolPartFinal := \frac{\ln(x)}{y(x)} + \frac{y(x)^2}{2} = 2$

$$SolPartFinal := \frac{\ln(x)}{y(x)} + \frac{y(x)^2}{2} = 2 \tag{50}$$

> $DerSolPartFinal := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPartFinal := \frac{d}{dx} y(x) = \frac{y(x)}{x\left(-y(x)^3 + \ln(x)\right)} \tag{51}$$

> $Ecua$

$$\frac{y(x)}{x} + \left(y(x)^3 - \ln(x)\right)\left(\frac{d}{dx} y(x)\right) = 0 \tag{52}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx} y(x) = \frac{y(x)}{x\left(-y(x)^3 + \ln(x)\right)} \tag{53}$$

> $Comprobar := simplify(rhs(DerSolPartFinal) - rhs(DerEcua) = 0)$

$$Comprobar := 0 = 0 \tag{54}$$

Fin respuesta 4

> $restart$

5) Resuelva

> $Ecua := x^2 \cdot y' = 1 - x^2 + y^2 - x^2 \cdot y^2$

$$Ecua := x^2\left(\frac{d}{dx} y(x)\right) = 1 - x^2 + y(x)^2 - x^2 y(x)^2 \tag{55}$$

Respuesta

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_separable] \tag{56}$$

> $M := -factor(1 - x^2 + y^2 - x^2 y^2)$

$$M := \left(y^2 + 1\right)(x - 1)(x + 1) \tag{57}$$

> $N := x^2$

$$N := x^2 \tag{58}$$

> $P := (x - 1)(x + 1); Q := \left(y^2 + 1\right); R := N; S := 1$

$$P := (x - 1)(x + 1)$$
$$Q := y^2 + 1$$
$$R := x^2$$
$$S := 1 \tag{59}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

(60)

$$SolGral := x + \frac{1}{x} + \arctan(y) = \_C1 \quad (60)$$

> $SolFinal := x + \frac{1}{x} + \arctan(y(x)) = \_C1$

$$SolFinal := x + \frac{1}{x} + \arctan(y(x)) = \_C1 \quad (61)$$

> $DerSolFinal := expand(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx} y(x) = -y(x)^2 - 1 + \frac{y(x)^2}{x^2} + \frac{1}{x^2} \quad (62)$$

> $Ecua$

$$x^2 \left(\frac{d}{dx} y(x) \right) = 1 - x^2 + y(x)^2 - x^2 y(x)^2 \quad (63)$$

> $DerEcua := expand(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx} y(x) = -y(x)^2 - 1 + \frac{y(x)^2}{x^2} + \frac{1}{x^2} \quad (64)$$

> $Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua) = 0)$

$$Comprobar := 0 = 0 \quad (65)$$

Fin respuesta 5

> $restart$

6) Determine la Ecuación Difrencial

> $SolGral := y(x) = \frac{2 \cdot \_C1 \cdot \exp(2x)}{(1 + 2 \cdot \_C1 \cdot \exp(2x))}$

$$SolGral := y(x) = \frac{2 \_C1 \, e^{2x}}{1 + 2 \_C1 \, e^{2x}} \quad (66)$$

> $DerSolGral := diff(SolGral, x)$

$$DerSolGral := \frac{d}{dx} y(x) = \frac{4 \_C1 \, e^{2x}}{1 + 2 \_C1 \, e^{2x}} - \frac{8 \_C1^2 \left(e^{2x}\right)^2}{\left(1 + 2 \_C1 \, e^{2x}\right)^2} \quad (67)$$

> $EcuaDif := diff(y(x), x) = 2 \cdot y(x) - 2 \cdot y(x)^2$

$$EcuaDif := \frac{d}{dx} y(x) = 2 y(x) - 2 y(x)^2 \quad (68)$$

> $with(DEtools) :$

> $odeadvisor(EcuaDif)$

$$[\_quadrature] \quad (69)$$

> $intfactor(EcuaDif)$

$$\frac{1}{y(x) \, (y(x) - 1)} \quad (70)$$

> $IntFact := \frac{1}{y \, (y - 1)}$

$$IntFact := \frac{1}{y \, (y - 1)} \quad (71)$$

> $M := -(2y - 2y^2)$

$$M := 2y^2 - 2y \quad (72)$$

> $N := 1$

$$N := 1 \quad (73)$$

> $MM := simplify(IntFact \cdot M)$

$$MM := 2 \quad (74)$$

> $NN := IntFact \cdot N$

$$NN := \frac{1}{y(y-1)} \quad (75)$$

> $diff(MM, y) = diff(NN, x)$

$$0 = 0 \quad (76)$$

> $IntMMx := int(MM, x)$

$$IntMMx := 2x \quad (77)$$

> $SolGralDos := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGralDos := 2x - \ln(y) + \ln(y-1) = \_C1 \quad (78)$$

> $ParaPara := \_C1 = simplify(\exp(lhs(SolGralDos)))$

$$ParaPara := \_C1 = \frac{(y-1)\,e^{2x}}{y} \quad (79)$$

> $Para := isolate(SolGral, \_C1)$

$$Para := \_C1 = -\frac{y(x)}{2\,e^{2x}\,(y(x)-1)} \quad (80)$$

Fin respuesta 6

> $restart$

7)

> $Ecua := 2 \cdot x + 1 + 6 \cdot y + 3 \cdot y' = 0$

$$Ecua := 2x + 1 + 6\,y(x) + 3\,\frac{d}{dx}\,y(x) = 0 \quad (81)$$

Respuesta

> $with(DEtools)$:

> $odeadvisor(Ecua)$

$$[[\_linear, class\ A]] \quad (82)$$

> $EcuaDos := \frac{(lhs(Ecua) - 2 \cdot x - 1)}{3} = \frac{(rhs(Ecua) - 2 \cdot x - 1)}{3}$

$$EcuaDos := 2\,y(x) + \frac{d}{dx}\,y(x) = -\frac{1}{3} - \frac{2x}{3} \quad (83)$$

> $p := 2$

$$p := 2 \quad (84)$$

> $q := rhs(EcuaDos)$

$$q := -\frac{1}{3} - \frac{2x}{3} \quad (85)$$

```
> SolGral := y(x) = simplify(_C1·exp(−int(p, x)) + exp(−int(p, x))·int(exp(int(p, x))·q,
      x))
```

$$SolGral := y(x) = \_C1\, e^{-2x} - \frac{x}{3} \tag{86}$$

```
> Ecua
```

$$2x + 1 + 6\,y(x) + 3\,\frac{d}{dx}\,y(x) = 0 \tag{87}$$

```
> Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))
```

$$Comprobar := 0 = 0 \tag{88}$$

Fin respuesta 7

```
> restart
```

8) Determine la ecuación diferencial de la siguiente

```
> SolGral := y(x) = _C1·x
```

$$SolGral := y(x) = \_C1\, x \tag{89}$$

```
> DerSolGral := diff(SolGral, x)
```

$$DerSolGral := \frac{d}{dx}\,y(x) = \_C1 \tag{90}$$

```
> Ecua := isolate(subs(_C1 = lhs(DerSolGral), SolGral), diff(y(x), x))
```

$$Ecua := \frac{d}{dx}\,y(x) = \frac{y(x)}{x} \tag{91}$$

```
> EcuaFinal := lhs(Ecua) − rhs(Ecua) = 0
```

$$EcuaFinal := \frac{d}{dx}\,y(x) - \frac{y(x)}{x} = 0 \tag{92}$$

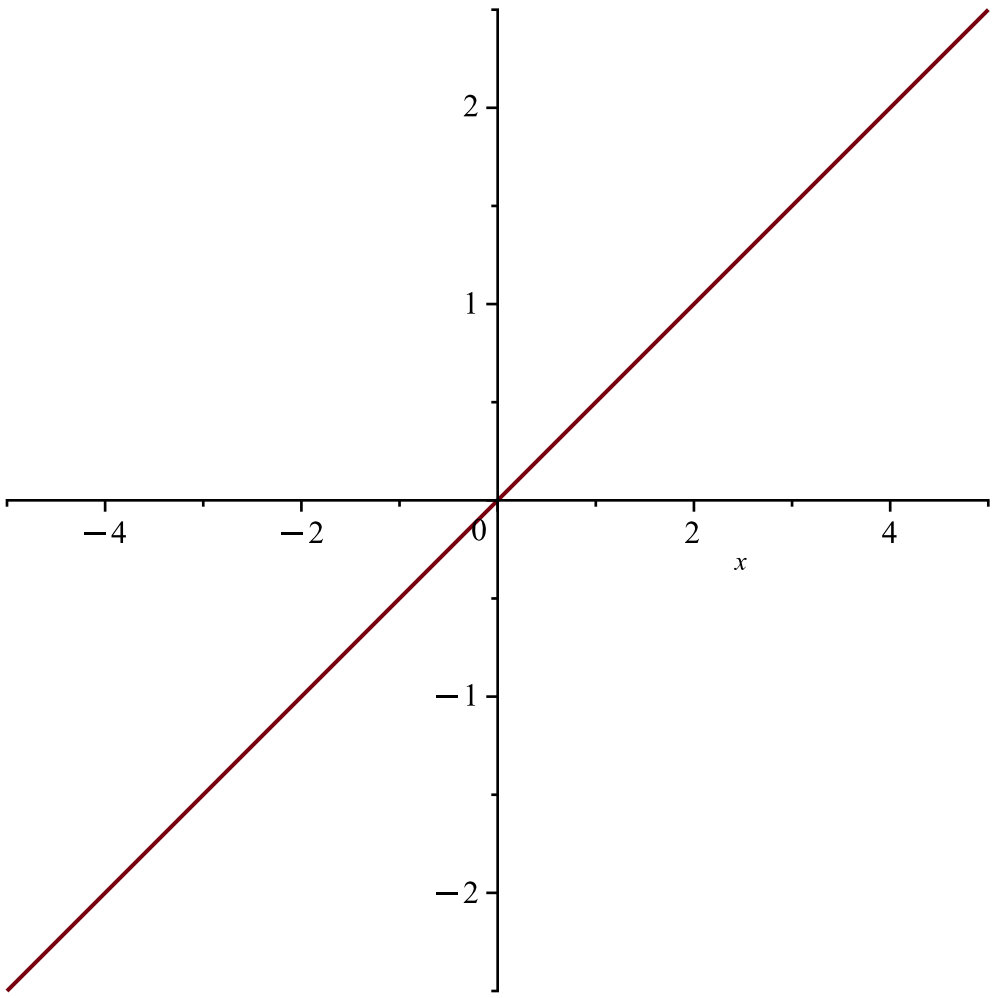
```
> ParaUno := isolate(subs(x = 2, y(2) = 1, SolGral), _C1)
```

$$ParaUno := \_C1 = \frac{1}{2} \tag{93}$$

```
> SolPartUno := subs(_C1 = rhs(ParaUno), SolGral)
```

$$SolPartUno := y(x) = \frac{x}{2} \tag{94}$$

```
> plot(rhs(SolPartUno), x = −5..5)
```



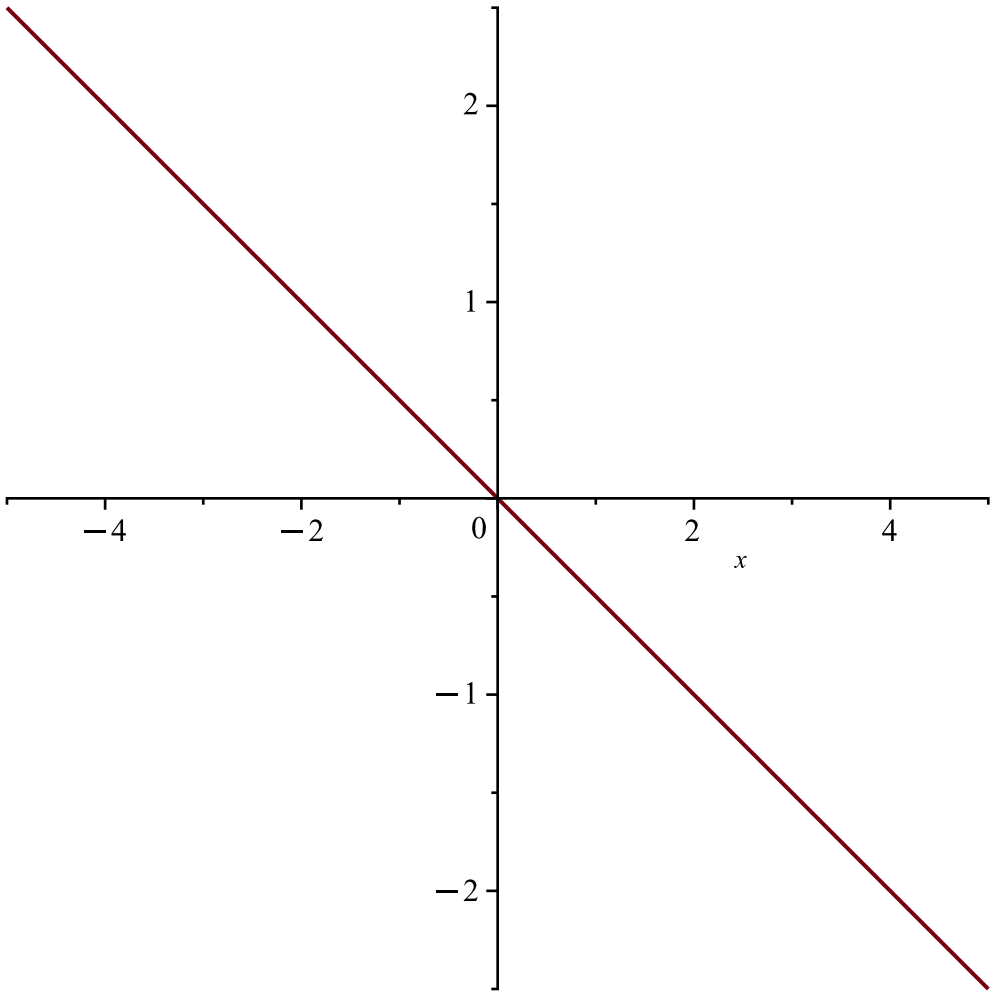
> $ParaDos := isolate(subs(x = -2, y(-2) = 1, SolGral), \_C1)$

$$ParaDos := \_C1 = -\frac{1}{2} \quad \textbf{(95)}$$

> $SolPartDos := subs(\_C1 = rhs(ParaDos), SolGral)$

$$SolPartDos := y(x) = -\frac{x}{2} \quad \textbf{(96)}$$

> $plot(rhs(SolPartDos), x = -5..5)$



Fin respuesta 8

```
> restart
```

9) resolver un problema de temperatura

```
> Condiciones := T(0) = 900, T(60) = 50
```

$$Condiciones := T(0) = 900,\ T(60) = 50 \quad \textbf{(97)}$$

```
> Ecua := diff(T(t), t) = K·(T(t) − 20)
```

$$Ecua := \frac{d}{dt} T(t) = K\,(T(t) - 20) \quad \textbf{(98)}$$

```
> SolGral := dsolve(Ecua)
```

$$SolGral := T(t) = 20 + e^{K t}\, c_1 \quad \textbf{(99)}$$

```
> SolPart := simplify(isolate(subs(t = 0, T(0) = 900, SolGral), c[1]))
```

$$SolPart := c_1 = 880 \quad \textbf{(100)}$$

```
> SolPart := subs(c[1] = rhs(SolPart), SolGral)
```

$$SolPart := T(t) = 20 + 880\, e^{K t} \quad \textbf{(101)}$$

```
> Para := simplify(isolate(subs(t = 60, rhs(SolPart) = 50), K)) : evalf(%)
```

$$K = -0.05631207543 \quad \textbf{(102)}$$

```
> SolFinal := subs(K = rhs(Para), SolPart)
```

$$SolFinal := T(t) = 20 + 880\, e^{\left(\frac{\ln(3)}{60} - \frac{\ln(2)}{20} - \frac{\ln(11)}{60}\right) t} \quad \textbf{(103)}$$

```
> tiempo := simplify(isolate(rhs(SolFinal) = 35, t)) : evalf(%)
```

$$t = 72.30903275 \quad \textbf{(104)}$$

```
> restart
>
```