>

SERIE GRUPAL ECUACIONES DIFERENCIALES
UNIDAD 2
GRUPO 13 SEMESTRE 2026-1
SOLUCIÓN

> $restart$

1) Resuelva la ecuación diferencial

> $Ecua := y'' + y' - 2\cdot y = 10\cdot\sin(3\,x)$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) - 2\,y(x) = 10\,\sin(3\,x) \quad (1)$$

>

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) - 2\,y(x) = 0 \quad (2)$$

> $Q := rhs(Ecua)$

$$Q := 10\,\sin(3\,x) \quad (3)$$

> $EcuaCarac := m^2 + m - 2 = 0$

$$EcuaCarac := m^2 + m - 2 = 0 \quad (4)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1,\ -2 \quad (5)$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \exp(Raiz[2]\cdot x)$

$$yy_1 := e^{x}$$
$$yy_2 := e^{-2\,x} \quad (6)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{x} & e^{-2\,x} \\ e^{x} & -2\,e^{-2\,x} \end{bmatrix} \quad (7)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 10\,\sin(3\,x) \end{bmatrix} \quad (8)$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} \frac{10\,\sin(3\,x)}{3\,e^{x}} & -\frac{10\,\sin(3\,x)}{3\,e^{-2\,x}} \end{bmatrix} \quad (9)$$

> $Aprima := ParaVar[1]$

$$Aprima := \frac{10\,\sin(3\,x)}{3\,e^{x}} \quad (10)$$

> $Bprima := ParaVar[2]$

$$Bprima := -\frac{10\sin(3x)}{3\,e^{-2x}} \tag{11}$$

> SolGral := y(x) = expand( (int(Aprima, x) + _C1) · yy[1] + (int(Bprima, x) + _C2) · yy[2])

$$SolGral := y(x) = -\frac{12\cos(x)^3}{13} + \frac{9\cos(x)}{13} - \frac{44\sin(x)\cos(x)^2}{13} + \frac{11\sin(x)}{13} + e^x\,\_C1 + \frac{\_C2}{(e^x)^2} \tag{12}$$

> Ecua

$$\frac{d^2}{dx^2}y(x) + \frac{d}{dx}y(x) - 2y(x) = 10\sin(3x) \tag{13}$$

> ComprobarUno := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) − rhs(Ecua) = 0)))

$$ComprobarUno := 0 = 0 \tag{14}$$

>

FIN RESPUESTA 1

> restart

2) Resuelva la ecuación diferencial

> Ecua := y'''' + 10·y''' + 35·y'' + 50·y' + 24·y = exp(x)·sinh(x)

$$Ecua := \frac{d^4}{dx^4}y(x) + 10\frac{d^3}{dx^3}y(x) + 35\frac{d^2}{dx^2}y(x) + 50\frac{d}{dx}y(x) + 24y(x) = e^x\sinh(x) \tag{15}$$

>

RESPUESTA

> EcuaHom := lhs(Ecua) = 0

$$EcuaHom := \frac{d^4}{dx^4}y(x) + 10\frac{d^3}{dx^3}y(x) + 35\frac{d^2}{dx^2}y(x) + 50\frac{d}{dx}y(x) + 24y(x) = 0 \tag{16}$$

> Q := rhs(Ecua)

$$Q := e^x\sinh(x) \tag{17}$$

> EcuaCarac := m^4 + 10·m^3 + 35·m^2 + 50·m + 24 = 0

$$EcuaCarac := m^4 + 10m^3 + 35m^2 + 50m + 24 = 0 \tag{18}$$

> Raiz := solve(EcuaCarac)

$$Raiz := -4, -3, -2, -1 \tag{19}$$

> yy[1] := exp(Raiz[1]·x); yy[2] := exp(Raiz[2]·x); yy[3] := exp(Raiz[3]·x); yy[4] := exp(Raiz[4]·x)

$$yy_1 := e^{-4x}$$
$$yy_2 := e^{-3x}$$
$$yy_3 := e^{-2x}$$
$$yy_4 := e^{-x} \tag{20}$$

> with(linalg) :

> WW := wronskian([yy[1], yy[2], yy[3], yy[4]], x)

$$WW := \begin{bmatrix} e^{-4x} & e^{-3x} & e^{-2x} & e^{-x} \\ -4\,e^{-4x} & -3\,e^{-3x} & -2\,e^{-2x} & -e^{-x} \\ 16\,e^{-4x} & 9\,e^{-3x} & 4\,e^{-2x} & e^{-x} \\ -64\,e^{-4x} & -27\,e^{-3x} & -8\,e^{-2x} & -e^{-x} \end{bmatrix} \quad \textbf{(21)}$$

> $BB := array([0, 0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 0 & e^{x}\sinh(x) \end{bmatrix} \quad \textbf{(22)}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{\sinh(x)\,e^{5x}}{6} & \frac{\sinh(x)\,e^{4x}}{2} & -\frac{\sinh(x)\,e^{3x}}{2} & \frac{\sinh(x)\,e^{2x}}{6} \end{bmatrix} \quad \textbf{(23)}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]; Dprima := ParaVar[3]; Eprima := ParaVar[4]$

$$Aprima := -\frac{\sinh(x)\,e^{5x}}{6}$$

$$Bprima := \frac{\sinh(x)\,e^{4x}}{2}$$

$$Dprima := -\frac{\sinh(x)\,e^{3x}}{2}$$

$$Eprima := \frac{\sinh(x)\,e^{2x}}{6} \quad \textbf{(24)}$$

> $SolGral := y(x) = expand(simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2] + (int(Dprima, x) + \_C3)\cdot yy[3] + (int(Eprima, x) + \_C4)\cdot yy[4]))$

$$SolGral := y(x) = \frac{(e^{x})^{2}}{720} - \frac{1}{48} + \frac{\_C1}{(e^{x})^{4}} + \frac{\_C4}{e^{x}} + \frac{\_C3}{(e^{x})^{2}} + \frac{\_C2}{(e^{x})^{3}} \quad \textbf{(25)}$$

> $Ecua$

$$\frac{d^4}{dx^4}\,y(x) + 10\,\frac{d^3}{dx^3}\,y(x) + 35\,\frac{d^2}{dx^2}\,y(x) + 50\,\frac{d}{dx}\,y(x) + 24\,y(x) = e^{x}\sinh(x) \quad \textbf{(26)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad \textbf{(27)}$$

>

FIN RESPUESTA 2

> $restart$

3) Determine la solución general

> $Ecua := y'' + 4\cdot y = 8\cdot\sin(2\,x)$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + 4\,y(x) = 8\,\sin(2\,x) \quad \textbf{(28)}$$

>

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) + 4\, y(x) = 0 \quad (29)$$

> $Q := rhs(Ecua)$

$$Q := 8 \sin(2\, x) \quad (30)$$

> $EcuaCarac := m^2 + 4 = 0$

$$EcuaCarac := m^2 + 4 = 0 \quad (31)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2\, \mathrm{I},\ -2\, \mathrm{I} \quad (32)$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x);\ yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(2\, x)$$

$$yy_2 := \sin(2\, x) \quad (33)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(2\, x) & \sin(2\, x) \\ -2 \sin(2\, x) & 2 \cos(2\, x) \end{bmatrix} \quad (34)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 8 \sin(2\, x) \end{bmatrix} \quad (35)$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -4 \sin(2\, x)^2 & 4 \cos(2\, x) \sin(2\, x) \end{bmatrix} \quad (36)$$

> $Aprima := ParaVar[1];\ Bprima := ParaVar[2]$

$$Aprima := -4 \sin(2\, x)^2$$

$$Bprima := 4 \cos(2\, x) \sin(2\, x) \quad (37)$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = (\_C1 - 2\, x) \cos(2\, x) + \sin(2\, x)\, \_C2 \quad (38)$$

> $Ecua$

$$\frac{d^2}{dx^2}\, y(x) + 4\, y(x) = 8 \sin(2\, x) \quad (39)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (40)$$

FIN RESPUESTA 3

> $restart$

4) Obtener la solución general

> $Ecua := (x^2 \cdot y'' - x^2 \cdot y') = x^2 \cdot \sin(x) - x^2 + x^2 \cdot \cos(x)$

$$Ecua := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - x^2 \left(\frac{d}{dx}\, y(x)\right) = x^2 \sin(x) - x^2 + x^2 \cos(x) \quad (41)$$

> $EcuaDos := simplify\left(\frac{lhs(Ecua)}{x^2} = \frac{rhs(Ecua)}{x^2}\right)$

$$EcuaDos := \frac{d^2}{dx^2}\, y(x) - \frac{d}{dx}\, y(x) = \sin(x) + \cos(x) - 1 \quad \textbf{(42)}$$

>

RESPUESTA

> $EcuaHom := lhs(EcuaDos) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) - \frac{d}{dx}\, y(x) = 0 \quad \textbf{(43)}$$

> $Q := rhs(EcuaDos)$

$$Q := \sin(x) + \cos(x) - 1 \quad \textbf{(44)}$$

> $EcuaCarac := m^2 - m = 0$

$$EcuaCarac := m^2 - m = 0 \quad \textbf{(45)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 0, 1 \quad \textbf{(46)}$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \exp(Raiz[2]\cdot x)$

$$yy_1 := 1$$

$$yy_2 := e^x \quad \textbf{(47)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} 1 & e^x \\ 0 & e^x \end{bmatrix} \quad \textbf{(48)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \sin(x) + \cos(x) - 1 \end{bmatrix} \quad \textbf{(49)}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} -\sin(x) - \cos(x) + 1 & \frac{\sin(x) + \cos(x) - 1}{e^x} \end{bmatrix} \quad \textbf{(50)}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\sin(x) - \cos(x) + 1$$

$$Bprima := \frac{\sin(x) + \cos(x) - 1}{e^x} \quad \textbf{(51)}$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGral := y(x) = \_C2\, e^x - \sin(x) + \_C1 + x + 1 \quad \textbf{(52)}$$

> $Ecua$

$$x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - x^2 \left(\frac{d}{dx}\, y(x)\right) = x^2 \sin(x) - x^2 + x^2 \cos(x) \quad \textbf{(53)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

**(54)**

$$Comprobar := 0 = 0 \quad (54)$$

>

FIN RESPUESTA 4

> *restart*

5) Obtener la solución general

> $Ecua := x \cdot y'' + (1 - 2x) \cdot y' + (x - 1) \cdot y = x \cdot \exp(x)$

$$Ecua := x \left( \frac{d^2}{dx^2} y(x) \right) + (1 - 2x) \left( \frac{d}{dx} y(x) \right) + (x - 1)\, y(x) = x\, e^x \quad (55)$$

> $yy[1] := \exp(x); yy[2] := \exp(x) \cdot \log(x)$

$$yy_1 := e^x$$

$$yy_2 := e^x \ln(x) \quad (56)$$

> $EcuaDos := expand\left( \frac{lhs(Ecua)}{x} \right) = \frac{rhs(Ecua)}{x}$

$$EcuaDos := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} - 2 \frac{d}{dx} y(x) + y(x) - \frac{y(x)}{x} = e^x \quad (57)$$

> $Q := rhs(EcuaDos)$

$$Q := e^x \quad (58)$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & e^x \ln(x) \\ e^x & e^x \ln(x) + \frac{e^x}{x} \end{bmatrix} \quad (59)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^x \end{bmatrix} \quad (60)$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} -\ln(x)\, x & x \end{bmatrix} \quad (61)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\ln(x)\, x$$

$$Bprima := x \quad (62)$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = e^x \left( \frac{x^2}{4} + \_C1 + \ln(x)\, \_C2 \right) \quad (63)$$

> $Ecua$

$$x \left( \frac{d^2}{dx^2} y(x) \right) + (1 - 2x) \left( \frac{d}{dx} y(x) \right) + (x - 1)\, y(x) = x\, e^x \quad (64)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (65)$$

>

FIN RESPUESTA 5

> *restart*

6) Obtenga la solución general

> $Ecua := y'' - y = \exp(x) + \cos(2\,x)$

$$Ecua := \frac{d^2}{dx^2}\,y(x) - y(x) = e^x + \cos(2\,x) \tag{66}$$

>

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) - y(x) = 0 \tag{67}$$

> $Q := rhs(Ecua)$

$$Q := e^x + \cos(2\,x) \tag{68}$$

> $EcuaCarac := m^2 - 1 = 0$

$$EcuaCarac := m^2 - 1 = 0 \tag{69}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -1 \tag{70}$$

> $yy[1] := \exp(Raiz[1] \cdot x);\ yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_1 := e^x$$

$$yy_2 := e^{-x} \tag{71}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{bmatrix} \tag{72}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^x + \cos(2\,x) \end{bmatrix} \tag{73}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} \frac{e^x + \cos(2\,x)}{2\,e^x} & -\frac{e^x + \cos(2\,x)}{2\,e^{-x}} \end{bmatrix} \tag{74}$$

> $Aprima := ParaVar[1]$

$$Aprima := \frac{e^x + \cos(2\,x)}{2\,e^x} \tag{75}$$

> $Bprima := ParaVar[2]$

$$Bprima := -\frac{e^x + \cos(2\,x)}{2\,e^{-x}} \tag{76}$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = \frac{1}{5} + e^{-x}\_C2 + \frac{(-1+2x+4\_C1)e^{x}}{4} - \frac{2\cos(x)^2}{5} \quad (77)$$

> *Ecua*

$$\frac{d^2}{dx^2} y(x) - y(x) = e^{x} + \cos(2x) \quad (78)$$

> *Comprobar* := *simplify*(*eval*(*subs*($y(x) = rhs(SolGral)$, $lhs(Ecua) - rhs(Ecua) = 0$)))

$$Comprobar := 0 = 0 \quad (79)$$

>

FIN RESPUESTA 6

> *restart*

7) Resuelva el problema del valor inicial

> $Ecua := x^2 \cdot y'' + x \cdot y' + y = \log(x)$

$$Ecua := x^2 \left(\frac{d^2}{dx^2} y(x)\right) + x\left(\frac{d}{dx} y(x)\right) + y(x) = \ln(x) \quad (80)$$

> $EcuaBis := expand\left(\frac{lhs(Ecua)}{x^2}\right) = \frac{rhs(Ecua)}{x^2}$

$$EcuaBis := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} + \frac{y(x)}{x^2} = \frac{\ln(x)}{x^2} \quad (81)$$

> $CondIni := y(1) = 4, D(y)(1) = 10$

$$CondIni := y(1) = 4, D(y)(1) = 10 \quad (82)$$

> $Q := rhs(EcuaBis)$

$$Q := \frac{\ln(x)}{x^2} \quad (83)$$

sabiendo que

> $SolHom := y(x) = 4 \cdot \cos(\log(x)) + 10 \cdot \sin(\log(x))$

$$SolHom := y(x) = 4\cos(\ln(x)) + 10\sin(\ln(x)) \quad (84)$$

satisface a la ecuación

> $EcuaHom := lhs(EcuaBis) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} + \frac{y(x)}{x^2} = 0 \quad (85)$$

y satisface a las condiciones

> *CondIni*

$$y(1) = 4, D(y)(1) = 10 \quad (86)$$

>

RESPUESTA

> *ComprobarUno* := *simplify*(*eval*(*subs*($y(x) = rhs(SolHom)$, $lhs(EcuaHom) - rhs(EcuaHom) = 0$)))

$$ComprobarUno := 0 = 0 \quad (87)$$

> $ComprobarDos := simplify(subs(x=1, SolHom))$

$$ComprobarDos := y(1) = 4 \tag{88}$$

> $ComprobarTres := \mathrm{D}(y)(1) = simplify(subs(x=1, rhs(diff(SolHom, x))))$

$$ComprobarTres := \mathrm{D}(y)(1) = 10 \tag{89}$$

> $yy[1] := \cos(\ln(x)); yy[2] := \sin(\ln(x))$

$$yy_1 := \cos(\ln(x))$$

$$yy_2 := \sin(\ln(x)) \tag{90}$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGralHom := y(x) = \_C1\cos(\ln(x)) + \_C2\sin(\ln(x)) \tag{91}$$

> $EcuaHom$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + \frac{\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)}{x} + \frac{y(x)}{x^2} = 0 \tag{92}$$

> $ComprobarCero := simplify(eval(subs(y(x) = rhs(SolGralHom), lhs(EcuaHom) - rhs(EcuaHom) = 0)))$

$$ComprobarCero := 0 = 0 \tag{93}$$

>

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(\ln(x)) & \sin(\ln(x)) \\ -\frac{\sin(\ln(x))}{x} & \frac{\cos(\ln(x))}{x} \end{bmatrix} \tag{94}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{\ln(x)}{x^2} \end{bmatrix} \tag{95}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{\ln(x)\sin(\ln(x))}{x} & \frac{\ln(x)\cos(\ln(x))}{x} \end{bmatrix} \tag{96}$$

> $Aprima := ParaVar[1]$

$$Aprima := -\frac{\ln(x)\sin(\ln(x))}{x} \tag{97}$$

> $Bprima := ParaVar[2]$

$$Bprima := \frac{\ln(x)\cos(\ln(x))}{x} \tag{98}$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = \_C1\cos(\ln(x)) + \_C2\sin(\ln(x)) + \ln(x) \tag{99}$$

> $SolGralHom := y(x) = \_C1\cos(\ln(x)) + \_C2\sin(\ln(x))$

$$SolGralHom := y(x) = \_C1\cos(\ln(x)) + \_C2\sin(\ln(x)) \tag{100}$$

> $ComprobarCeroCeroCero := simplify(eval(subs(y(x) = rhs(SolGralHom), lhs(EcuaHom)$

$- rhs(EcuaHom) = 0)))$

$$ComprobarCeroCeroCero := 0 = 0 \tag{101}$$

> $SolPartNoHom := y(x) = \ln(x)$

$$SolPartNoHom := y(x) = \ln(x) \tag{102}$$

>

> $ComprobarCeroCeroCeroCero := simplify(eval(subs(y(x) = rhs(SolPartNoHom), EcuaBis)))$

$$ComprobarCeroCeroCeroCero := \frac{\ln(x)}{x^2} = \frac{\ln(x)}{x^2} \tag{103}$$

> $EcuaBis$

$$\frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} + \frac{y(x)}{x^2} = \frac{\ln(x)}{x^2} \tag{104}$$

> $ComprobarCeroCero := simplify(eval(subs(y(x) = rhs(SolGral), lhs(EcuaBis)$
> $- rhs(EcuaBis) = 0)))$

$$ComprobarCeroCero := 0 = 0 \tag{105}$$

>

> $CondIni$

$$y(1) = 4, D(y)(1) = 10 \tag{106}$$

> $EcuaUno := simplify(subs(x = 1, rhs(SolGral) = 4))$

$$EcuaUno := \_C1 = 4 \tag{107}$$

> $EcuaDos := simplify(subs(x = 1, rhs(diff(SolGral, x)) = 10))$

$$EcuaDos := 1 + \_C2 = 10 \tag{108}$$

> $Constante := solve(\{EcuaUno, EcuaDos\}, \{\_C1, \_C2\})$

$$Constante := \{\_C1 = 4, \_C2 = 9\} \tag{109}$$

> $SolPart := expand(simplify(subs(\_C1 = rhs(Constante[1]), \_C2 = rhs(Constante[2]),$
> $SolGral)))$

$$SolPart := y(x) = 4\cos(\ln(x)) + 9\sin(\ln(x)) + \ln(x) \tag{110}$$

> $SolHom$

$$y(x) = 4\cos(\ln(x)) + 10\sin(\ln(x)) \tag{111}$$

> $EcuaBis$

$$\frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} + \frac{y(x)}{x^2} = \frac{\ln(x)}{x^2} \tag{112}$$

> $ComprobarCinco := simplify(eval(subs(y(x) = rhs(SolGral), EcuaBis)))$

$$ComprobarCinco := \frac{\ln(x)}{x^2} = \frac{\ln(x)}{x^2} \tag{113}$$

> $ComprobarSeis := simplify(eval(subs(y(x) = rhs(SolPart), EcuaBis)))$

$$ComprobarSeis := \frac{\ln(x)}{x^2} = \frac{\ln(x)}{x^2} \tag{114}$$

> $ComprobarSiete := simplify(subs(x = 1, SolPart))$

(115)

$$ComprobarSiete := y(1) = 4 \quad (115)$$

> $ComprobarOcho := \mathrm{D}(y)(1) = simplify(subs(x=1, rhs(diff(SolPart, x))))$

$$ComprobarOcho := \mathrm{D}(y)(1) = 10 \quad (116)$$

FIN RESPUESTA 7

> $restart$

8) Obtenga la solución general

> $Ecua := diff(y(\text{theta}), \text{theta}\$3) + diff(y(\text{theta}), \text{theta}) = \csc(\text{theta}) \cdot \cot(\text{theta})$

$$Ecua := \frac{d^3}{d\theta^3} y(\theta) + \frac{d}{d\theta} y(\theta) = \csc(\theta) \cot(\theta) \quad (117)$$

>

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^3}{d\theta^3} y(\theta) + \frac{d}{d\theta} y(\theta) = 0 \quad (118)$$

> $Q := rhs(Ecua)$

$$Q := \csc(\theta) \cot(\theta) \quad (119)$$

> $EcuaCarac := m^3 + m = 0$

$$EcuaCarac := m^3 + m = 0 \quad (120)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 0, \mathrm{I}, -\mathrm{I} \quad (121)$$

> $yy[1] := \exp(Raiz[1] \cdot \text{theta}); yy[2] := \cos(\mathrm{Im}(Raiz[2]) \cdot \text{theta}); yy[3] := \sin(\mathrm{Im}(Raiz[2]) \cdot \text{theta})$

$$yy_1 := 1$$

$$yy_2 := \cos(\theta)$$

$$yy_3 := \sin(\theta) \quad (122)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2], yy[3]], \text{theta})$

$$WW := \begin{bmatrix} 1 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \\ 0 & -\cos(\theta) & -\sin(\theta) \end{bmatrix} \quad (123)$$

> $BB := array([0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & \csc(\theta)\cot(\theta) \end{bmatrix} \quad (124)$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} \csc(\theta)\cot(\theta) & -\cot(\theta)^2 & -\cot(\theta) \end{bmatrix} \quad (125)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]; Dprima := ParaVar[3]$

$$Aprima := \csc(\theta) \cot(\theta)$$

$$Bprima := -\cot(\theta)^2$$

$$Dprima := -\cot(\theta) \tag{126}$$

> $SolGral := y(\text{theta}) = simplify((int(Aprima, \text{theta}) + \_C1) \cdot yy[1] + (int(Bprima, \text{theta}) + \_C2) \cdot yy[2] + (int(Dprima, \text{theta}) + \_C3) \cdot yy[3])$

$$SolGral := y(\theta) = -\sin(\theta)\ln(\sin(\theta)) + \frac{(-\pi + 2\_C2 + 2\theta)\cos(\theta)}{2} + (-1 + \_C3)\sin(\theta) + \_C1 \tag{127}$$

> $Ecua$

$$\frac{d^3}{d\theta^3} y(\theta) + \frac{d}{d\theta} y(\theta) = \csc(\theta)\cot(\theta) \tag{128}$$

> $Comprobar := simplify(eval(subs(y(\text{theta}) = rhs(SolGral), Ecua)))$

$$Comprobar := \csc(\theta)\cot(\theta) = \csc(\theta)\cot(\theta) \tag{129}$$

FIN RESPUESTA 8

> *restart*

9) Obtenga la solución

> $Ecua := x \cdot (y'' + 6 \cdot y' + 9 \cdot y) = -x^2 \cdot \exp(4 \cdot x)$

$$Ecua := x\left(\frac{d^2}{dx^2} y(x) + 6\frac{d}{dx} y(x) + 9y(x)\right) = -x^2 e^{4x} \tag{130}$$

> $CondIni := y(0) = \text{Pi}, D(y)(0) = 1$

$$CondIni := y(0) = \pi, D(y)(0) = 1 \tag{131}$$

>

RESPUESTA

> $EcuaDos := \frac{lhs(Ecua)}{x} = \frac{rhs(Ecua)}{x}$

$$EcuaDos := \frac{d^2}{dx^2} y(x) + 6\frac{d}{dx} y(x) + 9y(x) = -x e^{4x} \tag{132}$$

> $EcuaHom := lhs(EcuaDos) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 6\frac{d}{dx} y(x) + 9y(x) = 0 \tag{133}$$

> $Q := rhs(EcuaDos)$

$$Q := -x e^{4x} \tag{134}$$

> $EcuaCarac := m^2 + 6 \cdot m + 9 = 0$

$$EcuaCarac := m^2 + 6m + 9 = 0 \tag{135}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -3, -3 \tag{136}$$

> $yy[1] := \exp(Raiz[1] \cdot x); yy[2] := x \cdot \exp(Raiz[1] \cdot x)$

$$yy_1 := e^{-3x}$$

$$yy_2 := x e^{-3x} \tag{137}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-3x} & x\,e^{-3x} \\ -3\,e^{-3x} & e^{-3x} - 3\,x\,e^{-3x} \end{bmatrix} \tag{138}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & -x\,e^{4x} \end{bmatrix} \tag{139}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} x^2\,e^{7x} & -x\,e^{7x} \end{bmatrix} \tag{140}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := x^2\,e^{7x}$$

$$Bprima := -x\,e^{7x} \tag{141}$$

> $SolGral := y(x) = expand(simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2]))$

$$SolGral := y(x) = -\frac{x\,(e^x)^4}{49} + \frac{2\,(e^x)^4}{343} + \frac{\_C1}{(e^x)^3} + \frac{x\,\_C2}{(e^x)^3} \tag{142}$$

FIN RESPUESTA 9

> $restart$

>

>

>