

> restart

SOLUCIÓN TIPO A

PRIMER EXAMEN FINAL COLEGIADO
ECUACIONES DIFERENCIALES

> restart

RESPUESTA 1)

> *CondicionInicial* := $T(0) = 900$; *CondicionFinal* := $T(60) = 50$

$$\text{CondicionInicial} := T(0) = 900$$

$$\text{CondicionFinal} := T(60) = 50$$

(1)

> *Ecuacion* := $\text{diff}(T(t), t) = K \cdot (T(t) - 20)$

$$\text{Ecuacion} := \frac{d}{dt} T(t) = K (T(t) - 20)$$

(2)

> *Solucion* := $\text{dsolve}(\{\text{Ecuacion}, \text{CondicionInicial}\})$

$$\text{Solucion} := T(t) = 20 + 880 e^{Kt}$$

(3)

> *Parametro* := $\text{isolate}(\text{subs}(t = 60, \text{rhs}(\text{Solucion}) = \text{rhs}(\text{CondicionFinal})), K)$; $\text{evalf}(\%)$

$$\text{Parametro} := K = \frac{1}{60} \ln\left(\frac{3}{88}\right)$$

$$K = -0.05631207544$$

(4)

> *SolucionParticular* := $\text{subs}(K = \text{rhs}(\text{Parametro}), \text{Solucion})$

$$\text{SolucionParticular} := T(t) = 20 + 880 e^{\frac{1}{60} \ln\left(\frac{3}{88}\right)t}$$

(5)

> *Tiempo₃₅* := $\text{isolate}(\text{rhs}(\text{SolucionParticular}) = 35, t)$; $\text{evalf}(\%)$

$$\text{Tiempo}_{35} := t = \frac{60 \ln\left(\frac{3}{176}\right)}{\ln\left(\frac{3}{88}\right)}$$

$$t = 72.30903276$$

(6)

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FIN RESPUESTA 1)

> restart

RESPUESTA 2)

> *Ecuacion* := $\text{diff}(y(x), x^2) + 3 \cdot \text{diff}(y(x), x) + 2 \cdot y(x) = \sin(\exp(x))$

$$\text{Ecuacion} := \frac{d^2}{dx^2} y(x) + 3 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = \sin(e^x)$$

(7)

> *SolucionUno* := $\text{expand}(\text{dsolve}(\text{Ecuacion}))$

$$\text{SolucionUno} := y(x) = -\frac{2 \tan\left(\frac{1}{2} e^x\right)}{(e^x)^2 \left(1 + \tan\left(\frac{1}{2} e^x\right)^2\right)} - \frac{-C1}{(e^x)^2} + \frac{-C2}{e^x}$$

(8)

> *SolucionDos* := $\text{simplify}(\text{dsolve}(\text{Ecuacion}))$

SolucionDos := $y(x) =$

(9)

$$\frac{\left(2 \tan\left(\frac{1}{2} e^x\right) e^{-x} + e^{-x} _C1 + e^{-x} _C1 \tan\left(\frac{1}{2} e^x\right)^2 - _C2 - \tan\left(\frac{1}{2} e^x\right)^2 _C2\right) e^{-x}}{1 + \tan\left(\frac{1}{2} e^x\right)^2}$$

> comprobacion₁ := simplify(eval(subs(y(x) = rhs(SolucionUno), Ecuacion)))

$$\text{comprobacion}_1 := 2 \sin\left(\frac{1}{2} e^x\right) \cos\left(\frac{1}{2} e^x\right) = \sin(e^x) \quad (10)$$

> comprobacion₂ := simplify(eval(subs(y(x) = rhs(SolucionDos), Ecuacion)))

$$\text{comprobacion}_2 := 2 \sin\left(\frac{1}{2} e^x\right) \cos\left(\frac{1}{2} e^x\right) = \sin(e^x) \quad (11)$$

> SolucionTres := y(x) = C₁·exp(-2x) + C₂·exp(-x) - exp(-2x)·sin(exp(x))

$$\text{SolucionTres} := y(x) = C_1 e^{-2x} + C_2 e^{-x} - e^{-2x} \sin(e^x) \quad (12)$$

> comprobacion₃ := simplify(eval(subs(y(x) = rhs(SolucionTres), Ecuacion)))

$$\text{comprobacion}_3 := \sin(e^x) = \sin(e^x) \quad (13)$$

FIN RESPUESTA 2)

> restart

RESPUESTA 3)

> Ecuacion := diff(y(t), t\$4) - y(t) = 0

$$\text{Ecuacion} := \frac{d^4}{dt^4} y(t) - y(t) = 0 \quad (14)$$

> EcuacionCaracteristica := m·4 - 1 = 0

$$\text{EcuacionCaracteristica} := m^4 - 1 = 0 \quad (15)$$

> Raiz := solve(EcuacionCaracteristica, m)

$$\text{Raiz} := 1, -1, I, -I \quad (16)$$

> Sol₁ := y(t) = exp(Raiz₁·t); Sol₂ := y(t) = exp(Raiz₂·t); Sol₃ := y(t) = exp(Re(Raiz₃)·t)·cos(Im(Raiz₃)·t); Sol₄ := y(t) = exp(Re(Raiz₃)·t)·sin(Im(Raiz₃)·t)

$$\text{Sol}_1 := y(t) = e^t$$

$$\text{Sol}_2 := y(t) = e^{-t}$$

$$\text{Sol}_3 := y(t) = \cos(t)$$

$$\text{Sol}_4 := y(t) = \sin(t)$$

(17)

> SolucionGeneral := y(t) = C₁·rhs(Sol₁) + C₂·rhs(Sol₂) + C₃·rhs(Sol₃) + C₄·rhs(Sol₄)

$$\text{SolucionGeneral} := y(t) = C_1 e^t + C_2 e^{-t} + C_3 \cos(t) + C_4 \sin(t) \quad (18)$$

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FIN RESPUESTA 3)

> restart

RESPUESTA 4)

> Sistema := diff(x(t), t) = x(t) - y(t), diff(y(t), t) = -x(t) + 2y(t) + exp(-t) : Sistema₁;
Sistema₂;

$$\frac{d}{dt} x(t) = x(t) - y(t)$$

$$\frac{d}{dt} y(t) = -x(t) + 2y(t) + e^{-t} \quad (19)$$

> Variable := isolate(Sistema₁, y(t))

$$\text{Variable} := y(t) = -\left(\frac{d}{dt} x(t)\right) + x(t) \quad (20)$$

> Ecuacion := eval(subs(y(t) = rhs(Variable), Sistema₂))

$$\text{Ecuacion} := -\left(\frac{d^2}{dt^2} x(t)\right) + \frac{d}{dt} x(t) = x(t) - 2\left(\frac{d}{dt} x(t)\right) + e^{-t} \quad (21)$$

> NoHom := isolate(Ecuacion, exp(-t))

$$\text{NoHom} := e^{-t} = -\left(\frac{d^2}{dt^2} x(t)\right) + 3\left(\frac{d}{dt} x(t)\right) - x(t) \quad (22)$$

> EcuacionFinal := -rhs(NoHom) = -exp(-t)

$$\text{EcuacionFinal} := \frac{d^2}{dt^2} x(t) - 3\left(\frac{d}{dt} x(t)\right) + x(t) = -e^{-t} \quad (23)$$

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FIN RESPUESTA 4)

> restart

RESPUESTA 5)

> Sistema := diff(x(t), t\$2) + diff(y(t), t\$2) = Heaviside(t), diff(x(t), t\$2) - diff(y(t), t\$2) = 4 t : Sistema₁; Sistema₂

$$\frac{d^2}{dt^2} x(t) + \frac{d^2}{dt^2} y(t) = \text{Heaviside}(t)$$

$$\frac{d^2}{dt^2} x(t) - \left(\frac{d^2}{dt^2} y(t)\right) = 4 t \quad (24)$$

> Condiciones := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0

$$\text{Condiciones} := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0 \quad (25)$$

> with(inttrans) :

> TransLapSisUno := laplace(Sistema₁, t, s)

$$\begin{aligned} \text{TransLapSisUno} &:= s^2 \text{laplace}(x(t), t, s) - D(x)(0) - s x(0) + s^2 \text{laplace}(y(t), t, s) \\ &\quad - D(y)(0) - s y(0) = \frac{1}{s} \end{aligned} \quad (26)$$

> TransLapSisDos := laplace(Sistema₂, t, s)

$$\begin{aligned} \text{TransLapSisDos} &:= s^2 \text{laplace}(x(t), t, s) - D(x)(0) - s x(0) - s^2 \text{laplace}(y(t), t, s) \\ &\quad + D(y)(0) + s y(0) = \frac{4}{s^2} \end{aligned} \quad (27)$$

> TransLapVariableUno := isolate(TransLapSisUno, laplace(y(t), t, s))

$$\text{TransLapVariableUno} := \text{laplace}(y(t), t, s) \quad (28)$$

$$= \frac{\frac{1}{s} - s^2 \text{laplace}(x(t), t, s) + D(x)(0) + s x(0) + D(y)(0) + s y(0)}{s^2}$$

> *TransLapEcuacionDos* := subs(laplace(y(t), t, s) = rhs(*TransLapVariableUno*),
TransLapSisDos)

$$\text{TransLapEcuacionDos} := 2 s^2 \text{laplace}(x(t), t, s) - 2 D(x)(0) - 2 s x(0) - \frac{1}{s} = \frac{4}{s^2} \quad (29)$$

> *TransLapSolucionDos* := isolate(*TransLapEcuacionDos*, laplace(x(t), t, s))

$$\text{TransLapSolucionDos} := \text{laplace}(x(t), t, s) = \frac{1}{2} \frac{\frac{4}{s^2} + 2 D(x)(0) + 2 s x(0) + \frac{1}{s}}{s^2} \quad (30)$$

> *TransLapSolucionUno* := subs(laplace(x(t), t, s) = rhs(*TransLapSolucionDos*),
TransLapVariableUno)

$$\text{TransLapSolucionUno} := \text{laplace}(y(t), t, s) = \frac{\frac{1}{2s} - \frac{2}{s^2} + D(y)(0) + s y(0)}{s^2} \quad (31)$$

> *SolucionUno* := expand(subs(Condiciones, invlaplace(*TransLapSolucionUno*, s, t)))

$$\text{SolucionUno} := y(t) = \frac{1}{4} t^2 - \frac{1}{3} t^3 \quad (32)$$

> *SolucionDos* := expand(subs(Condiciones, invlaplace(*TransLapSolucionDos*, s, t)))

$$\text{SolucionDos} := x(t) = \frac{1}{3} t^3 + \frac{1}{4} t^2 \quad (33)$$

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FIN RESPUESTA 5)

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RESPUESTA 6)

> *Ecuacion* := diff(z(x, y), x, x, y) = diff(z(x, y), x)

$$\text{Ecuacion} := \frac{\partial^3}{\partial y \partial x^2} z(x, y) = \frac{\partial}{\partial x} z(x, y) \quad (34)$$

> *EcuacionDos* := eval(subs(z(x, y) = F(x) · G(y), *Ecuacion*))

$$\text{EcuacionDos} := \left(\frac{d^2}{dx^2} F(x) \right) \left(\frac{d}{dy} G(y) \right) = \left(\frac{d}{dx} F(x) \right) G(y) \quad (35)$$

> *EcuacionSeparada* := $\frac{\text{lhs}(\text{EcuacionDos})}{\text{diff}(G(y), y) \cdot (\text{diff}(F(x), x))} = \frac{\text{rhs}(\text{EcuacionDos})}{\text{diff}(G(y), y) \cdot (\text{diff}(F(x), x))}$

$$\text{EcuacionSeparada} := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \frac{G(y)}{\frac{d}{dy} G(y)} \quad (36)$$

> *EcuacionX* := lhs(*EcuacionSeparada*) = alpha; *EcuacionY* := rhs(*EcuacionSeparada*) = alpha

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \alpha$$

$$EcuacionY := \frac{G(y)}{\frac{d}{dy} G(y)} = \alpha \quad (37)$$

> *SolucionX* := dsolve(subs(alpha = beta·2, EcuacionX))

$$SolucionX := F(x) = _C1 + _C2 e^{\beta^2 x} \quad (38)$$

> *SolucionY* := dsolve(subs(alpha = beta·2, EcuacionY))

$$SolucionY := G(y) = _C1 e^{\frac{y}{\beta^2}} \quad (39)$$

> *SolucionGeneral* := z(x, y) = rhs(*SolucionX*) · subs(_C1 = 1, rhs(*SolucionY*))

$$SolucionGeneral := z(x, y) = (_C1 + _C2 e^{\beta^2 x}) e^{\frac{y}{\beta^2}} \quad (40)$$

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FIN RESPUESTA 6)

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FIN EXAMEN

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