>

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL (TEMAS 4 Y 5)
SEMESTRE 2015-2

2015 MAYO 22

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1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE **(sin usar dsolve)**:

a) **(15/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS

b) **(15/100 puntos)** GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA PARA UN INTERVALO DE 0 < t < 3

$$\frac{d^2}{dt^2}\, y(t) + 16\, y(t) = 64\, (t-2)\, \text{Heaviside}(t-2) \cos(3\,t-6)$$

$$y(0) = 0$$

$$D(y)(0) = 1 \qquad (1)$$

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RESPUESTA 1a)

> $Ecuacion := \frac{d^2}{dt^2}\, y(t) + 16\, y(t) = 64\, (t-2)\, \text{Heaviside}(t-2) \cos(3\,t-6)$

$$Ecuacion := \frac{d^2}{dt^2}\, y(t) + 16\, y(t) = 64\, (t-2)\, \text{Heaviside}(t-2) \cos(3\,t-6) \qquad (2)$$

> $Condiciones := y(0) = 0, D(y)(0) = 1$

$$Condiciones := y(0) = 0, D(y)(0) = 1 \qquad (3)$$

> $with(inttrans):$

> $TLE := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TLE := s^2\, laplace(y(t), t, s) - 1 + 16\, laplace(y(t), t, s) = \frac{64\, e^{-2\,s}\, (s^2 - 9)}{(s^2+9)^2} \qquad (4)$$

> $TLS := simplify(isolate(TLE, laplace(y(t), t, s)))$

$$TLS := laplace(y(t), t, s) = \frac{64\, e^{-2\,s}\, s^2 - 576\, e^{-2\,s} + s^4 + 18\, s^2 + 81}{(s^2+9)^2\, (s^2+16)} \qquad (5)$$

> $SolucionParticular := invlaplace(TLS, s, t)$

$$SolucionParticular := y(t) = \frac{1}{4} \sin(4\,t) + \frac{16}{49}\, (24 \sin(3\,t-6) - 25 \sin(4\,t-8) + 28\, (t-2) \cos(3\,t-6))\, \text{Heaviside}(t-2) \qquad (6)$$

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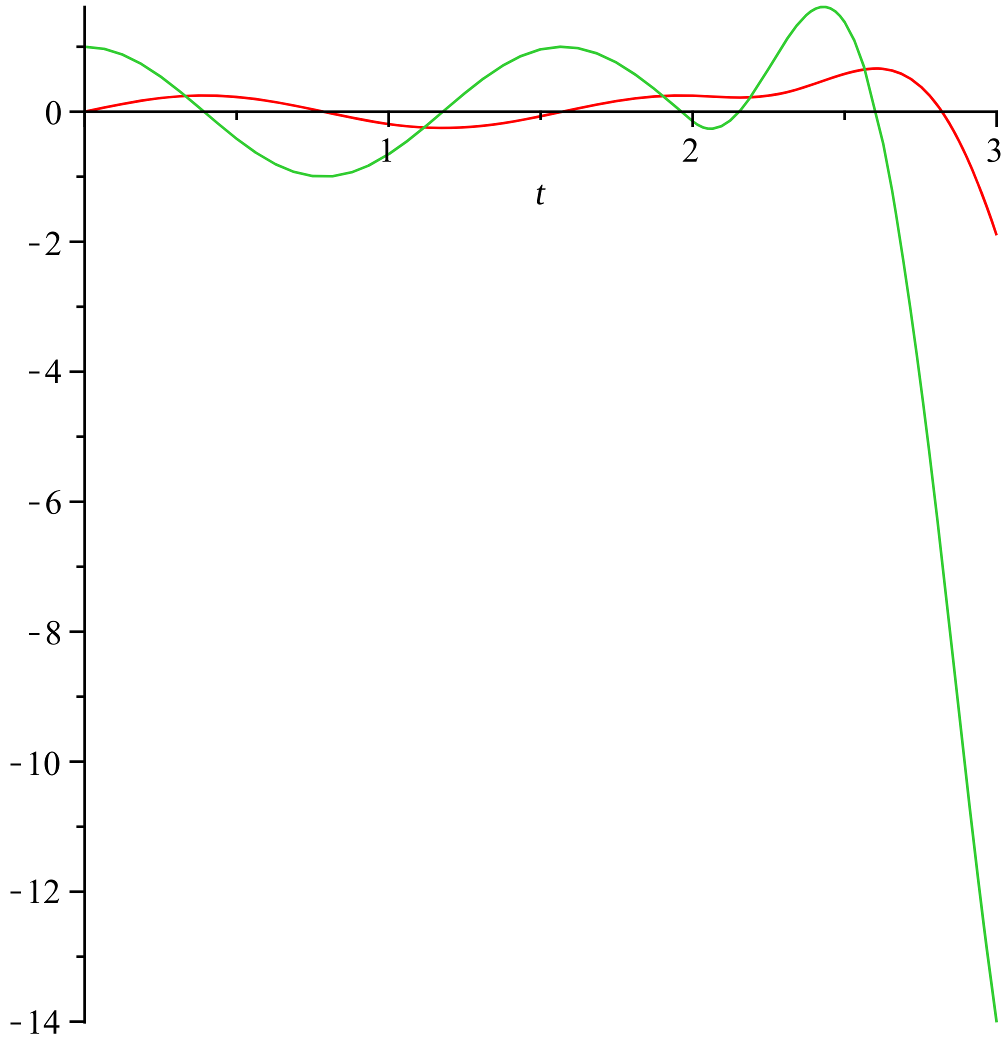
RESPUESTA 1b)

> $plot([rhs(SolucionParticular), rhs(diff(SolucionParticular, t))], t = 0..3)$



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FIN RESPUESTA 1)

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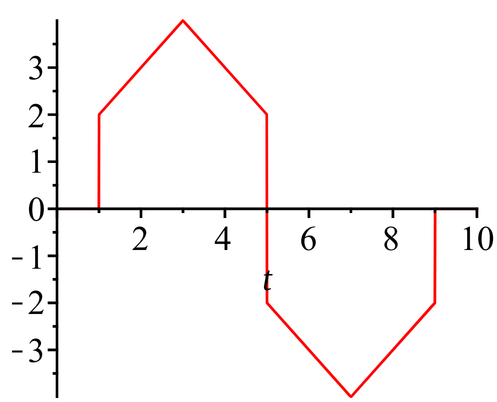
2) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:



a) **(15/100 puntos)** OBTENER SU TRANSFORMADA DE LAPLACE.
b) **(25/100 puntos)** OBTENER Y GRAFICAR SU SERIE COSENO DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

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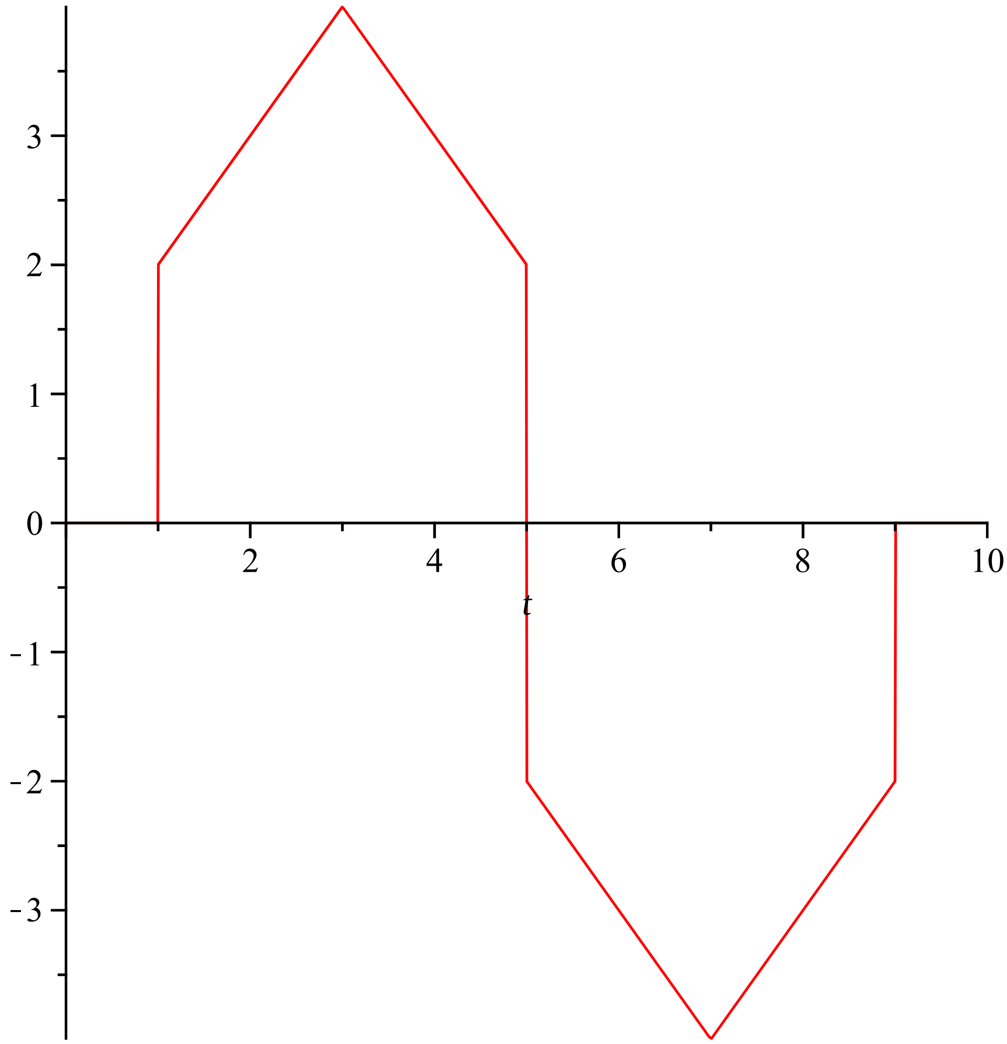
RESPUESTA 2a)

> $f := 2\cdot \text{Heaviside}(t-1) + (t-1)\cdot\text{Heaviside}(t-1) - 2\cdot(t-3)\cdot\text{Heaviside}(t-3) + (t-5)\cdot\text{Heaviside}(t-5) - 4\cdot\text{Heaviside}(t-5) - (t-5)\cdot\text{Heaviside}(t-5) + 2\cdot(t-7)\cdot\text{Heaviside}(t-7) - (t-9)\cdot\text{Heaviside}(t-9) + 2\cdot\text{Heaviside}(t-9) : \mathit{plot}(f, t=0..10)$



> $\mathit{with}(\mathit{inttrans}) :$

> $F := \mathit{laplace}(f, t, s)$

$$F := \frac{e^{-s} - e^{-9s} + 2\,e^{-7s} - 2\,e^{-3s}}{s^2} + \frac{2\left(e^{-s} + e^{-9s} - 2\,e^{-5s}\right)}{s} \qquad \textbf{(7)}$$

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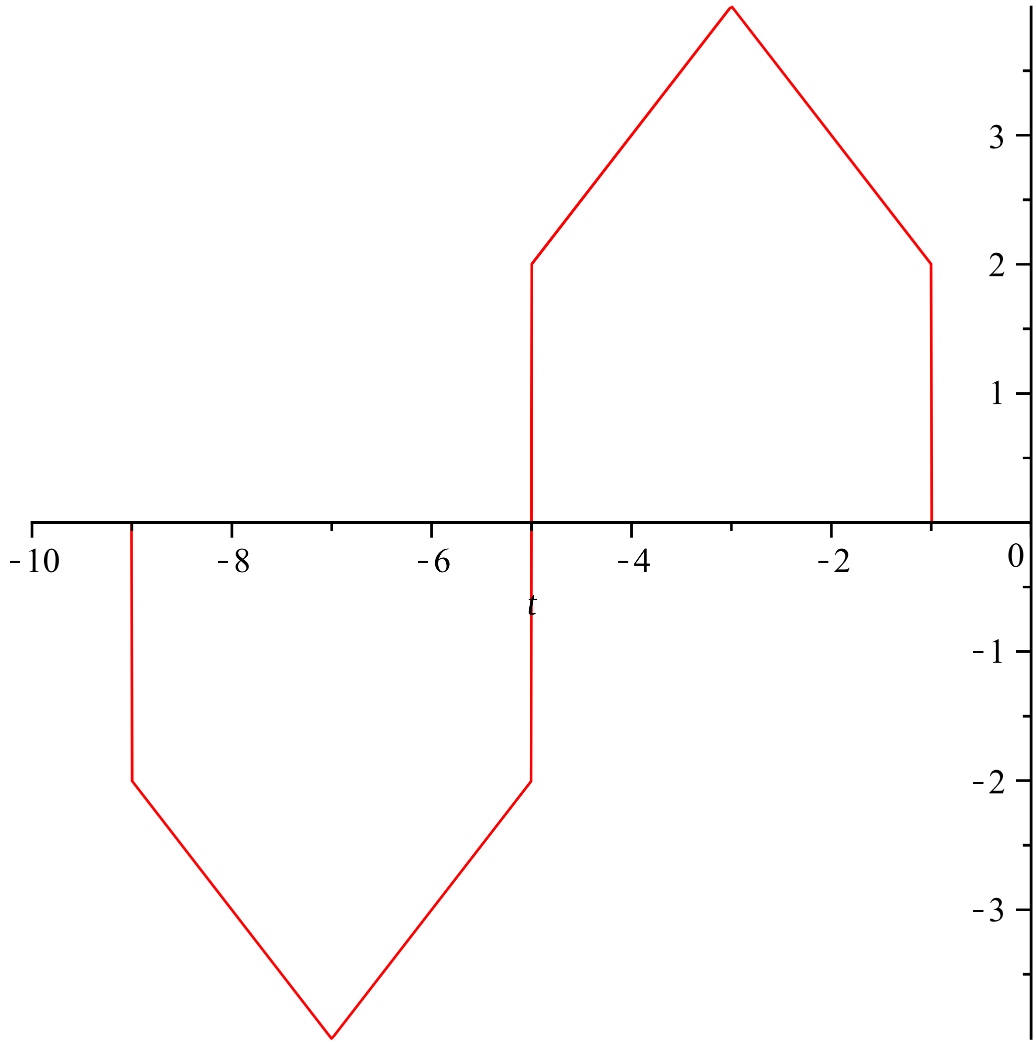
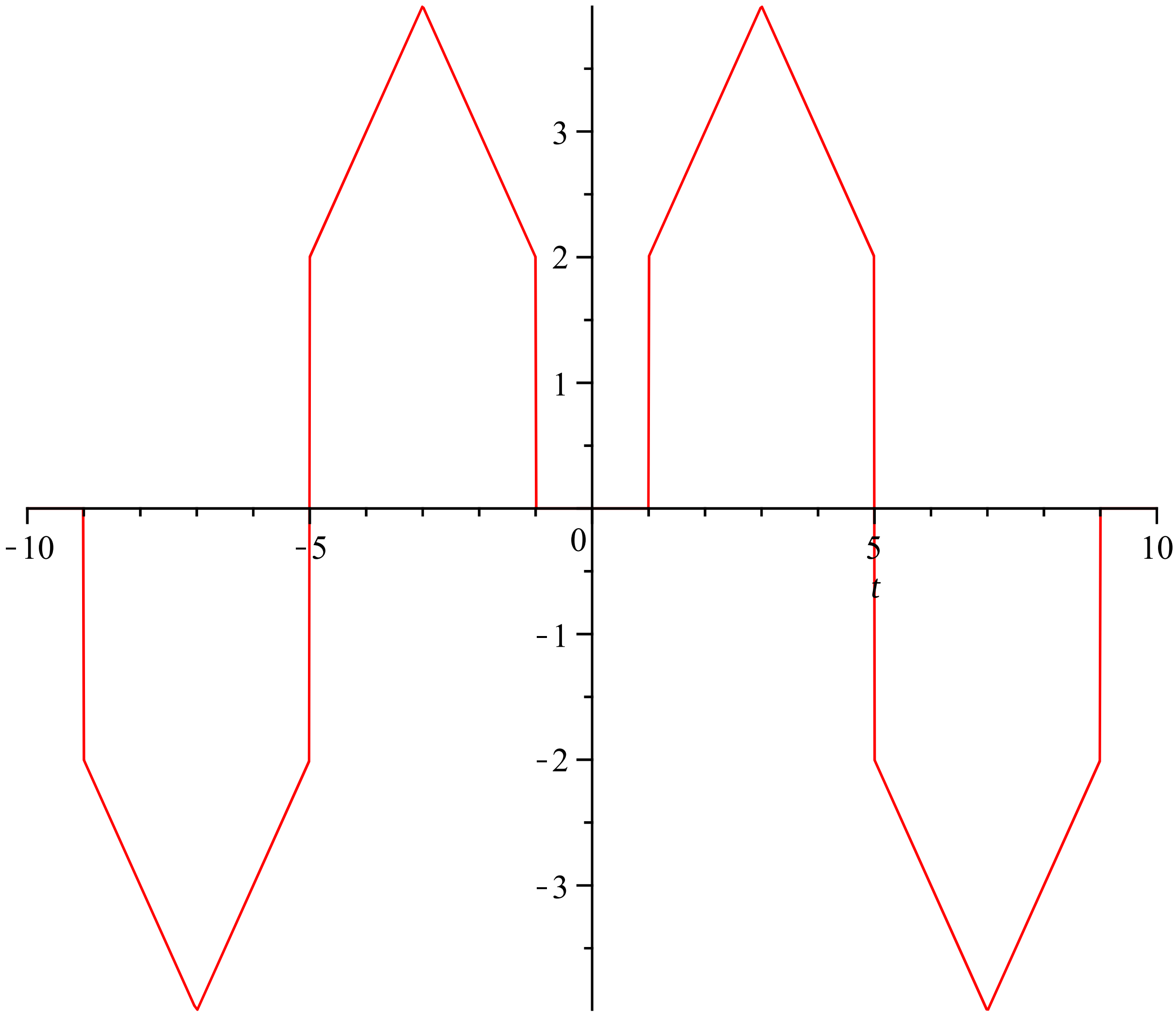
RESPUESTA 2b)

> $g := -2\cdot\text{Heaviside}(t+9) - (t+9)\cdot\text{Heaviside}(t+9) + 2\cdot(t+7)\cdot\text{Heaviside}(t+7) - (t+5)\cdot\text{Heaviside}(t+5) + 4\cdot\text{Heaviside}(t+5) + (t+5)\cdot\text{Heaviside}(t+5) - 2\cdot(t+3)\cdot\text{Heaviside}(t+3) + (t+1)\cdot\text{Heaviside}(t+1) - 2\cdot\text{Heaviside}(t+1) : \mathit{plot}(g, t=-10..0)$



> $h := g + f : plot(h, t = -10 .. 10)$



> $L := 10$

$$L := 10 \tag{8}$$

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> $a_0 := \frac{1}{L} \cdot int(h, t = -L..L)$

$$a_0 := 0 \tag{9}$$

> $a_n := simplify\left(\frac{1}{L} \cdot int\left(h \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

$$a_n := -\frac{1}{n^2 \pi^2} \left(4 \left(n \pi \sin\left(\frac{9}{10} n \pi\right) - 5 \cos\left(\frac{9}{10} n \pi\right) + 10 \cos\left(\frac{7}{10} n \pi\right) - 2 \sin\left(\frac{1}{2} n \pi\right) n \pi - 10 \cos\left(\frac{3}{10} n \pi\right) + 5 \cos\left(\frac{1}{10} n \pi\right) + n \pi \sin\left(\frac{1}{10} n \pi\right)\right)\right) \tag{10}$$

> $b_n := simplify\left(\frac{1}{L} \cdot int\left(h \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

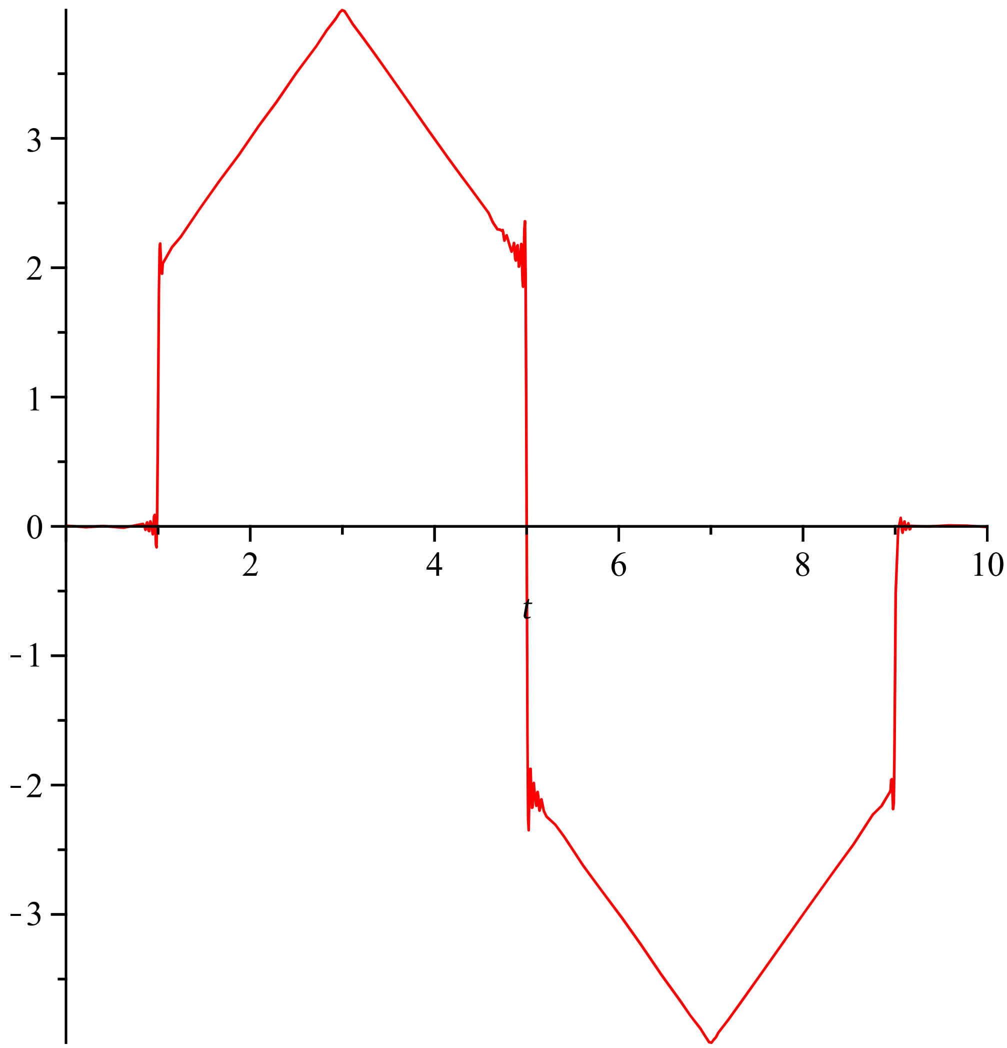
$$b_n := 0 \tag{11}$$

> $STF := Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..infinity\right)$

$$STF := \sum_{n=1}^{\infty} \left(-\frac{1}{n^2\,\pi^2} \left(4 \left(n\,\pi \sin\left(\frac{9}{10}\,n\,\pi \right) - 5 \cos\left(\frac{9}{10}\,n\,\pi \right) + 10 \cos\left(\frac{7}{10}\,n\,\pi \right) - 2 \sin\left(\frac{1}{2}\,n\,\pi \right) n\,\pi - 10 \cos\left(\frac{3}{10}\,n\,\pi \right) + 5 \cos\left(\frac{1}{10}\,n\,\pi \right) + n\,\pi \sin\left(\frac{1}{10}\,n\,\pi \right) \right) \cos\left(\frac{1}{10}\,n\,\pi\,t \right) \right) \right) \tag{12}$$

> $STF_{500} := sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t \right), n = 1 ..500 \right) :$

> $plot\left(STF_{500}, t = 0 ..10 \right)$



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FIN RESPUESTA 2)

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3) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA **CONSTANTE DE SEPARACIÓN NEGATIVA**:

$$\frac{\partial^2}{\partial x^2} z(x,t) + t^2 \left(\frac{\partial}{\partial t} z(x,t) \right) = \frac{\partial}{\partial x} z(x,t) \tag{13}$$

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RESPUESTA 3)

> $Ecuacion := \frac{\partial^2}{\partial x^2} z(x,t) + t^2 \left(\frac{\partial}{\partial t} z(x,t)\right) = \frac{\partial}{\partial x} z(x,t)$

$$Ecuacion := \frac{\partial^2}{\partial x^2} z(x,t) + t^2 \left(\frac{\partial}{\partial t} z(x,t)\right) = \frac{\partial}{\partial x} z(x,t) \quad \textbf{(14)}$$

> $EcuacionDos := simplify(eval(subs(z(x,t) = F(x) \cdot G(t), Ecuacion)))$

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x)\right) G(t) + t^2 F(x) \left(\frac{d}{dt} G(t)\right) = \left(\frac{d}{dx} F(x)\right) G(t) \quad \textbf{(15)}$$

> $EcuacionTres := simplify\left(\frac{\left(lhs(EcuacionDos) - \left(\frac{d}{dx} F(x)\right) G(t) - t^2 F(x)\left(\frac{d}{dt} G(t)\right)\right)}{F(x) \cdot G(t)}\right) = \left(\frac{rhs(EcuacionDos) - \left(\frac{d}{dx} F(x)\right) G(t) - t^2 F(x)\left(\frac{d}{dt} G(t)\right)}{F(x) \cdot G(t)}\right)$

$$EcuacionTres := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} \quad \textbf{(16)}$$

> $EcuacionX := lhs(EcuacionTres) = -\text{beta} \cdot {\cdot}2; EcuacionT := rhs(EcuacionTres) = -\text{beta} \cdot {\cdot}2$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\beta^2$$

$$EcuacionT := -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} = -\beta^2 \quad \textbf{(17)}$$

> $SolucionX := dsolve(EcuacionX); SolucionT := dsolve(EcuacionT)$

$$SolucionX := F(x) = \_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\beta^2}\right)x}$$

$$SolucionT := G(t) = \_C1\, e^{-\frac{\beta^2}{t}} \quad \textbf{(18)}$$

> $SolucionNegativa := z(x,t) = rhs(SolucionX) \cdot subs(\_C1 = 1, rhs(SolucionT))$

$$SolucionNegativa := z(x,t) = \left(\_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\beta^2}\right)x}\right) e^{-\frac{\beta^2}{t}} \quad \textbf{(19)}$$

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FIN RESPUESTA 3)

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FIN DEL EXAMEN