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NOMBRE ALUMNO:

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN PARCIAL (TEMA 3)
SEMESTRE 2019-2

2019 MAYO 2

> *restart*

1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE **(sin usar dsolve)**:
a) **(20/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS
b) **(20/100 puntos)** GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA PARA UN INTERVALO DE 0 < t < 3

> $\frac{d^2}{dt^2} y(t) + 16\, y(t) = 6 \cos(3\, t); y(0) = 0; D(y)(0) = 1$

$$\frac{d^2}{dt^2} y(t) + 16\, y(t) = 6 \cos(3\, t)$$
$$y(0) = 0$$
$$D(y)(0) = 1 \quad (1)$$

inciso a)

> $Ecuacion := \frac{d^2}{dt^2} y(t) + 16\, y(t) = 6 \cos(3\, t)$

$$Ecuacion := \frac{d^2}{dt^2} y(t) + 16\, y(t) = 6 \cos(3\, t) \quad (2)$$

> $Condiciones := y(0) = 0, D(y)(0) = 1$

$$Condiciones := y(0) = 0, D(y)(0) = 1 \quad (3)$$

> $with(inttrans):$

> $EcuaTrans := subs(Condiciones, laplace(Ecuacion, t, s))$

$$EcuaTrans := s^2\, laplace(y(t), t, s) - 1 + 16\, laplace(y(t), t, s) = \frac{6\, s}{s^2 + 9} \quad (4)$$

> $SolTrans := isolate(EcuaTrans, laplace(y(t), t, s))$

$$SolTrans := laplace(y(t), t, s) = \frac{\frac{6\, s}{s^2 + 9} + 1}{s^2 + 16} \quad (5)$$

> $Solucion := invlaplace(SolTrans, s, t)$

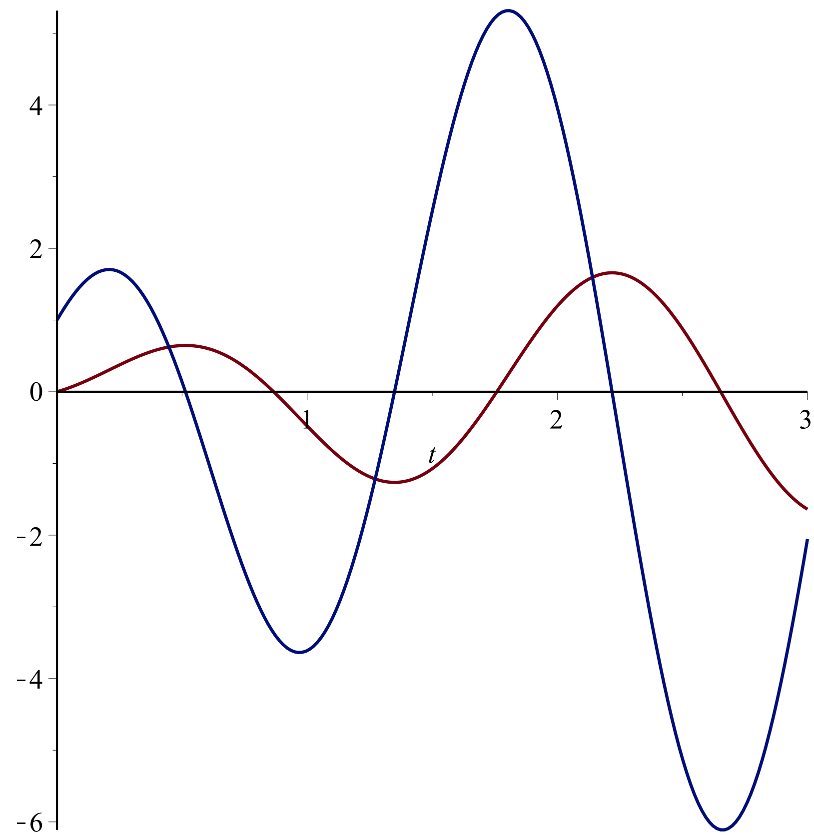
$$Solucion := y(t) = -\frac{6}{7} \cos(4\, t) + \frac{1}{4} \sin(4\, t) + \frac{6}{7} \cos(3\, t) \quad (6)$$

inciso b)

> $DerSolucion := diff(Solucion, t)$

$$DerSolucion := \frac{d}{dt} y(t) = \frac{24}{7} \sin(4\, t) + \cos(4\, t) - \frac{18}{7} \sin(3\, t) \quad (7)$$

> $plot([rhs(Solucion), rhs(DerSolucion)], t = 0 ..3)$



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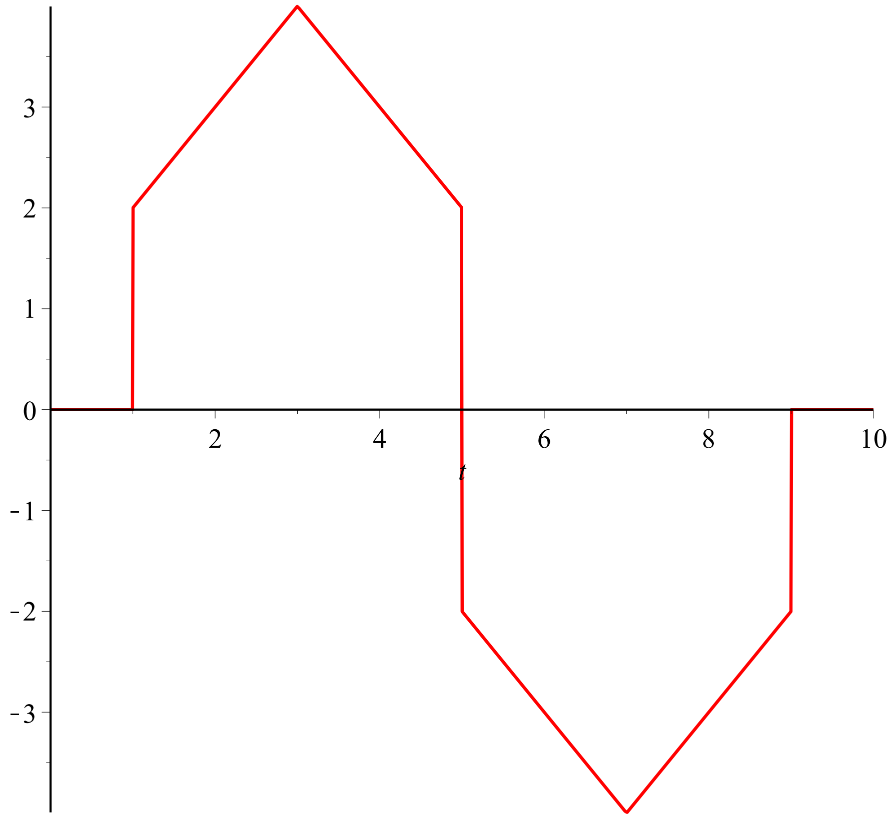
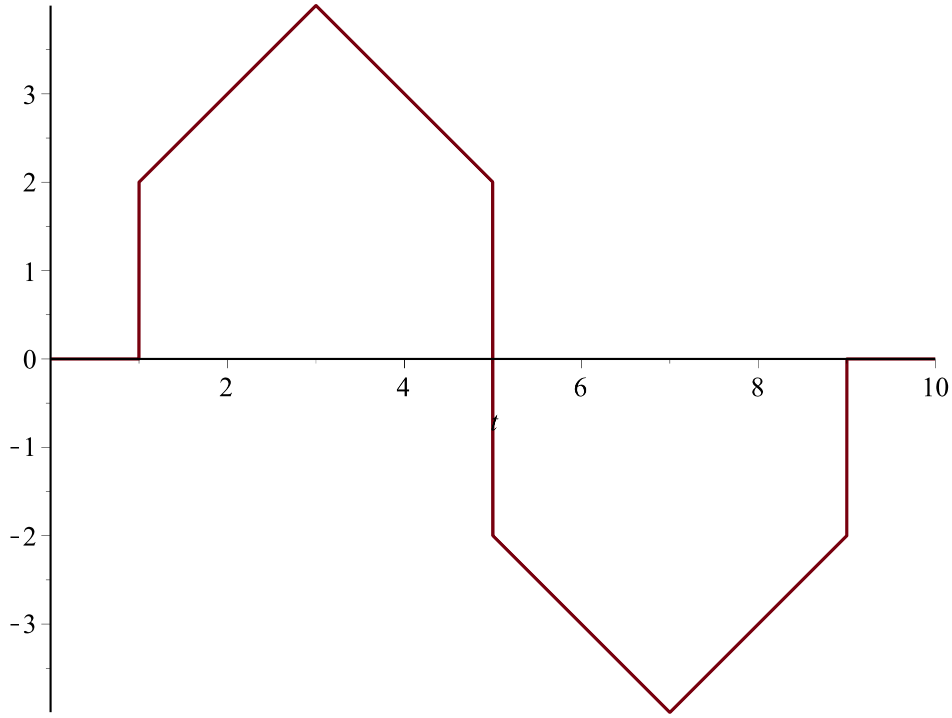
2) (20/100 puntos) OBTENER LA TRANSFORMADA DE LAPLACE,.DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:



> $Curva := 2\cdot \text{Heaviside}(t-1) + (t-1)\cdot\text{Heaviside}(t-1) - 2\cdot(t-3)\cdot\text{Heaviside}(t-3) + (t-5)\cdot\text{Heaviside}(t-5) - 4\cdot\text{Heaviside}(t-5) - (t-5)\cdot\text{Heaviside}(t-5) + 2\cdot(t-7)\cdot\text{Heaviside}(t-7) - (t-9)\cdot\text{Heaviside}(t-9) + 2\cdot \text{Heaviside}(t-9)$; $plot(Curva, t=0..10, scaling=constrained)$

$$Curva := 2\,\text{Heaviside}(t-1) + (t-1)\,\text{Heaviside}(t-1) - 2\,(t-3)\,\text{Heaviside}(t-3) - 4\,\text{Heaviside}(t-5) + 2\,(t-7)\,\text{Heaviside}(t-7) - (t-9)\,\text{Heaviside}(t-9) + 2\,\text{Heaviside}(t-9)$$



> *with*(*inttrans*) :

> *TransCurva* := *laplace*(*Curva*, *t*, *s*)

$$TransCurva := \frac{e^{-s} - e^{-9s} + 2\,e^{-7s} - 2\,e^{-3s}}{s^2} + \frac{2\left(e^{-s} + e^{-9s} - 2\,e^{-5s}\right)}{s} \quad \textbf{(8)}$$

>

> *restart*

3) DADO EL PROBLEMA DE LA ECUACIÓN DIFERENCIAL CON CONDICIONES INICIALES DE LA PREGUNTA 1)

> $\frac{d^2}{dt^2}\,y(t) + 16\,y(t) = 6\,e^{2t}\cos(3\,t); y(0) = 0; D(y)(0) = 1$

$$\frac{d^2}{dt^2}\,y(t) + 16\,y(t) = 6\,e^{2t}\cos(3\,t)$$

$$y(0) = 0$$

$$D(y)(0) = 1 \quad \textbf{(9)}$$

a) **(20/100 puntos)** CONVERTIRLO EN UN SISTEMA DE DOS ECUACIONES CON DOS INCÓGNITAS.

b) **(20/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DEL SISTEMA DADAS SUS CONDICIONES INICIALES.

inciso a)

```
> restart
> Sistema := diff(y[1](t), t) = y[2](t), diff(y[2](t), t) = -16·y[1](t) + 6 cos(3 t) : Sistema[1];
    Sistema[2]
```

$$\frac{d}{dt} y_1(t) = y_2(t)$$

$$\frac{d}{dt} y_2(t) = -16\, y_1(t) + 6 \cos(3\, t) \quad (10)$$

```
> Condiciones := y[1](0) = 0, y[2](0) = 1
```

$$Condiciones := y_1(0) = 0, y_2(0) = 1 \quad (11)$$

```
>
```

inciso b1) Resolviendo por Matriz Exponencial

```
> AA := array([[0, 1], [-16, 0]])
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$$AA := \begin{bmatrix} 0 & 1 \\ -16 & 0 \end{bmatrix} \quad (12)$$

```
> BB := array([0, 6 cos(3 t)])
```

$$BB := \begin{bmatrix} 0 & 6 \cos(3\, t) \end{bmatrix} \quad (13)$$

```
> Xcero := array([0, 1])
```

$$Xcero := \begin{bmatrix} 0 & 1 \end{bmatrix} \quad (14)$$

```
> with(linalg) :
> MatExp := exponential(AA, t)
```

$$MatExp := \begin{bmatrix} \cos(4\, t) & \frac{1}{4} \sin(4\, t) \\ -4 \sin(4\, t) & \cos(4\, t) \end{bmatrix} \quad (15)$$

```
> SolHom := evalm(MatExp &* Xcero)
```

$$SolHom := \begin{bmatrix} \frac{1}{4} \sin(4\, t) & \cos(4\, t) \end{bmatrix} \quad (16)$$

```
> MatExpTau := map(rcurry(eval, t='t − tau'), MatExp)
```

$$MatExpTau := \begin{bmatrix} \cos(4\, t - 4\, \tau) & \frac{1}{4} \sin(4\, t - 4\, \tau) \\ -4 \sin(4\, t - 4\, \tau) & \cos(4\, t - 4\, \tau) \end{bmatrix} \quad (17)$$

```
> BBtau := map(rcurry(eval, t='tau'), BB)
```

$$BBtau := \begin{bmatrix} 0 & 6 \cos(3\, \tau) \end{bmatrix} \quad (18)$$

```
> ProdTau := evalm(MatExpTau &* BBtau)
```

(19)

$$ProdTau := \left[\frac{3}{2} \sin(4t - 4\tau) \cos(3\tau) \quad 6\cos(4t - 4\tau)\cos(3\tau) \right] \tag{19}$$

> SolNoHom := simplify(map(int, ProdTau, tau = 0 ..t)) : SolNoHom[1]; SolNoHom[2]

$$-\frac{48}{7}\cos(t)^4 + \frac{48}{7}\cos(t)^2 - \frac{6}{7} + \frac{24}{7}\cos(t)^3 - \frac{18}{7}\cos(t)$$

$$\frac{24}{7}\sin(4t) - \frac{18}{7}\sin(3t) \tag{20}$$

> Solucion := y[1](t) = SolHom[1] + SolNoHom[1], y[2](t) = SolHom[2] + SolNoHom[2] : Solucion[1]; Solucion[2]

$$y_1(t) = \frac{1}{4}\sin(4t) - \frac{48}{7}\cos(t)^4 + \frac{48}{7}\cos(t)^2 - \frac{6}{7} + \frac{24}{7}\cos(t)^3 - \frac{18}{7}\cos(t)$$

$$y_2(t) = \cos(4t) + \frac{24}{7}\sin(4t) - \frac{18}{7}\sin(3t) \tag{21}$$

> simplify(subs(t = 0, Solucion[1])); simplify(subs(t = 0, Solucion[2]));

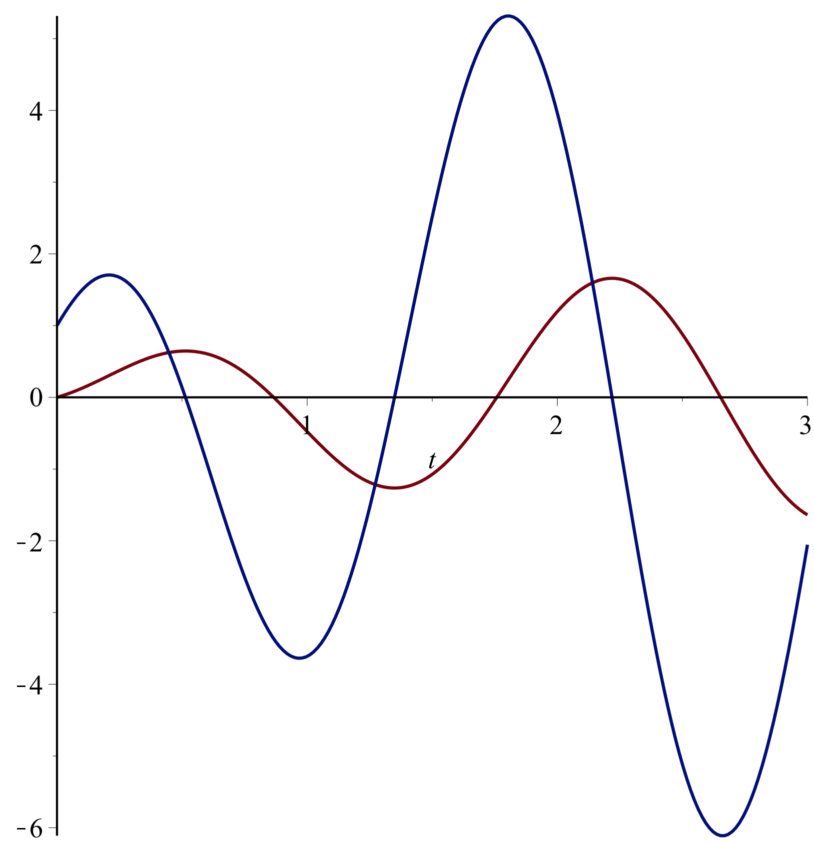
$$y_1(0) = 0$$

$$y_2(0) = 1 \tag{22}$$

>

inciso c1)

> plot([rhs(Solucion[1]), rhs(Solucion[2])], t = 0 ..3)



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> *restart*

inciso b2) utilizando dsolve

> *Sistema* := *diff*(*y*[1](*t*), *t*) = *y*[2](*t*), *diff*(*y*[2](*t*), *t*) = -16·*y*[1](*t*) + 6 cos(3 *t*) : *Sistema*[1]; *Sistema*[2]

$$\frac{d}{dt} y_1(t) = y_2(t)$$

$$\frac{d}{dt} y_2(t) = -16\, y_1(t) + 6 \cos(3\, t) \quad \textbf{(23)}$$

> *Condiciones* := *y*[1](0) = 0, *y*[2](0) = 1

$$Condiciones := y_1(0) = 0,\, y_2(0) = 1 \quad \textbf{(24)}$$

> *Solucion* := *dsolve*({*Sistema*, *Condiciones*}) : *Solucion*[1]; *Solucion*[2]

$$y_1(t) = \frac{1}{4} \sin(4\, t) - \frac{6}{7} \cos(4\, t) + \frac{6}{7} \cos(3\, t)$$

(25)

$$y_2(t) = \cos(4\,t) + \frac{24}{7}\,\sin(4\,t) - \frac{18}{7}\,\sin(3\,t) \tag{25}$$

> *simplify*(*subs*(*t* = 0, *Solucion*[1])); *simplify*(*subs*(*t* = 0, *Solucion*[2]));

$$y_1(0) = 0$$

$$y_2(0) = 1 \tag{26}$$

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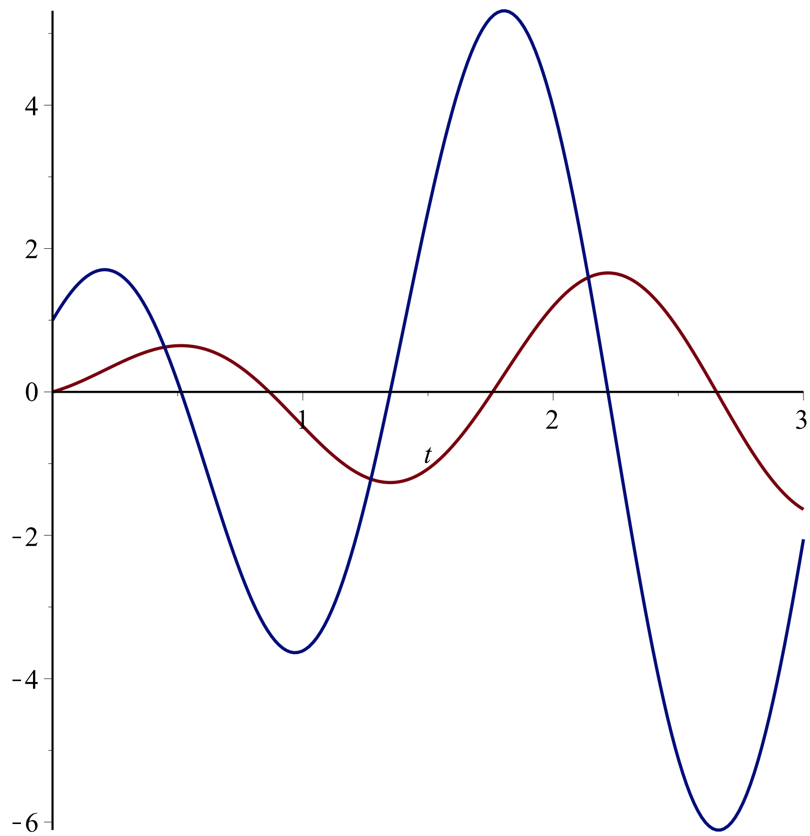
inciso c2)

> *plot*([*rhs*(*Solucion*[1]), *rhs*(*Solucion*[2])], *t* = 0 ..3)



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FIN DEL EXAMEN