> *restart*

SOLUCION PRIMER FINAL

>

1)

> $Ecuacion := y(x) = (x + \mathrm{sqrt}(y(x)\cdot\cdot 2 - x\cdot\cdot 2))\cdot diff(y(x), x)$

$$Ecuacion := y(x) = \left(x + \sqrt{y(x)^2 - x^2}\right)\left(\frac{d}{dx}\, y(x)\right) \quad (1)$$

> $Solucion := simplify(dsolve(Ecuacion))$

$$Solucion := -\frac{\_C1\, y(x) - 2^{\frac{x}{\sqrt{-x^2}}} \left(\frac{\sqrt{-x^2}\,\sqrt{y(x)^2 - x^2} - x^2}{y(x)}\right)^{\frac{x}{\sqrt{-x^2}}} x^{\frac{\sqrt{-x^2}}{x}}}{y(x)} = 0 \quad (2)$$

> $EcuacionDos := lhs(Ecuacion) - rhs(Ecuacion) = 0$

$$EcuacionDos := y(x) - \left(x + \sqrt{y(x)^2 - x^2}\right)\left(\frac{d}{dx}\, y(x)\right) = 0 \quad (3)$$

>

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert] \quad (4)$$

> $EcuacionTres := simplify(isolate(simplify(eval(subs(y(x) = u(x)\cdot x, EcuacionDos))), diff(u(x), x)))$

$$EcuacionTres := \frac{d}{dx}\, u(x) = -\frac{\sqrt{x^2\left(u(x)^2 - 1\right)}\; u(x)}{x\left(x + \sqrt{x^2\left(u(x)^2 - 1\right)}\right)} \quad (5)$$

> $EcuacionCuatro := \frac{d}{dx}\, u(x) = -\frac{\left(x\cdot\sqrt{\left(u(x)^2 - 1\right)}\; u(x)\right)}{x\left(x + x\cdot\sqrt{\left(u(x)^2 - 1\right)}\right)}$

$$EcuacionCuatro := \frac{d}{dx}\, u(x) = -\frac{\sqrt{u(x)^2 - 1}\; u(x)}{x + x\sqrt{u(x)^2 - 1}} \quad (6)$$

> $SolucionCuatro := int\left(\frac{1}{x}, x\right) + int\left(\frac{1}{-\frac{\sqrt{u^2 - 1}\; u}{1 + \sqrt{u^2 - 1}}}, u\right) = C1$

$$SolucionCuatro := \ln(x) - \ln(u) + \arctan\left(\frac{1}{\sqrt{u^2 - 1}}\right) = C1 \quad (7)$$

> $SolucionFinal := subs\left(u = \frac{y}{x}, SolucionCuatro\right)$

$$SolucionFinal := \ln(x) - \ln\left(\frac{y}{x}\right) + \arctan\left(\frac{1}{\sqrt{\frac{y^2}{x^2} - 1}}\right) = C1 \quad (8)$$

>

> $EcuacionCinco := y\cdot diff(x(y), y) = x(y) + \mathrm{sqrt}(y\cdot\cdot 2 - x(y)\cdot\cdot 2)$

$$EcuacionCinco := y\left(\frac{d}{dy}x(y)\right) = x(y) + \sqrt{y^2 - x(y)^2} \quad (9)$$

> $SolucionCinco := dsolve(EcuacionCinco)$

$$SolucionCinco := -\arctan\left(\frac{x(y)}{\sqrt{y^2 - x(y)^2}}\right) + \ln(y) - \_C1 = 0 \quad (10)$$

> $EcuacionSeis := simplify(isolate(eval(subs(x(y) = v(y) \cdot y, EcuacionCinco)), diff(v(y), y)))$

$$EcuacionSeis := \frac{d}{dy}v(y) = \frac{\sqrt{-y^2\left(v(y)^2 - 1\right)}}{y^2} \quad (11)$$

> $EcuacionSiete := \frac{d}{dy}v(y) = \frac{y \cdot \sqrt{-\left(v(y)^2 - 1\right)}}{y^2}$

$$EcuacionSiete := \frac{d}{dy}v(y) = \frac{\sqrt{-v(y)^2 + 1}}{y} \quad (12)$$

> $SolucionSiete := int\left(\frac{1}{y}, y\right) - int\left(\frac{1}{\sqrt{-v^2 + 1}}, v\right) = C1$

$$SolucionSiete := \ln(y) - \arcsin(v) = C1 \quad (13)$$

> $SolucionFinalDos := subs\left(v = \frac{x}{y}, SolucionSiete\right)$

$$SolucionFinalDos := \ln(y) - \arcsin\left(\frac{x}{y}\right) = C1 \quad (14)$$

>

> *restart*

2)

> $Ecuacion := diff(y(t), t\$3) - diff(y(t), t) = \exp(t)$

$$Ecuacion := \frac{d^3}{dt^3}y(t) - \left(\frac{d}{dt}y(t)\right) = e^t \quad (15)$$

> $Solucion := dsolve(Ecuacion)$

$$Solucion := y(t) = e^t\,\_C2 - e^{-t}\,\_C1 + \frac{1}{2}\,t\,e^t - \frac{1}{2}\,e^t + \_C3 \quad (16)$$

> $SolucionDos := simplify\left(subs\left(\_C3 = C1, \_C1 = -C2, \_C2 = C3 + \frac{1}{2}, Solucion\right)\right)$

$$SolucionDos := y(t) = e^t\,C3 + e^{-t}\,C2 + \frac{1}{2}\,t\,e^t + C1 \quad (17)$$

> *restart*

3)

> $Sistema := diff(x(t), t) + 2 \cdot diff(y(t), t) = \exp(t), 2 \cdot diff(x(t), t) + diff(y(t), t) = \sin(t) :$
> $Sistema[1]; Sistema[2]$

$$\frac{d}{dt}x(t) + 2\left(\frac{d}{dt}y(t)\right) = e^t$$

$$2\left(\frac{d}{dt}x(t)\right) + \frac{d}{dt}y(t) = \sin(t) \quad (18)$$

> $Cond := x(0) = 1, y(0) = -1$

$$Cond := x(0) = 1, y(0) = -1 \quad (19)$$

> $Solucion := dsolve(\{Sistema, Cond\}) : Solucion[1]; Solucion[2]$

$$x(t) = -\frac{1}{3}\,e^{t} - \frac{2}{3}\cos(t) + 2$$

$$y(t) = \frac{2}{3}\,e^{t} + \frac{1}{3}\cos(t) - 2 \qquad \textbf{(20)}$$

CON TRANSFORMADA DE LAPLACE

> $with(inttrans) :$

> $SistLap := subs(Cond, laplace(\{Sistema\}, t, s)) : SistLap[1]; SistLap[2]$

$$s\,laplace(x(t), t, s) + 1 + 2\,s\,laplace(y(t), t, s) = \frac{1}{s-1}$$

$$2\,s\,laplace(x(t), t, s) - 1 + s\,laplace(y(t), t, s) = \frac{1}{s^2+1} \qquad \textbf{(21)}$$

> $VarTransUno := isolate(SistLap[1], laplace(x(t), t, s))$

$$VarTransUno := laplace(x(t), t, s) = \frac{\frac{1}{s-1} - 1 - 2\,s\,laplace(y(t), t, s)}{s} \qquad \textbf{(22)}$$

> $SolTransY := isolate(subs(laplace(x(t), t, s) = rhs(VarTransUno), SistLap[2]), laplace(y(t), t, s))$

$$SolTransY := laplace(y(t), t, s) = -\frac{1}{3}\,\frac{\frac{1}{s^2+1} - \frac{2}{s-1} + 3}{s} \qquad \textbf{(23)}$$

> $SolY := invlaplace(SolTransY, s, t)$

$$SolY := y(t) = \frac{2}{3}\,e^{t} + \frac{1}{3}\cos(t) - 2 \qquad \textbf{(24)}$$

> $SolTransX := subs(laplace(y(t), t, s) = rhs(SolTransY), VarTransUno)$

$$SolTransX := laplace(x(t), t, s) = \frac{-\frac{1}{3\,(s-1)} + 1 + \frac{2}{3\,(s^2+1)}}{s} \qquad \textbf{(25)}$$

> $SolX := invlaplace(SolTransX, s, t) : SolX; SolY$

$$x(t) = -\frac{1}{3}\,e^{t} - \frac{2}{3}\cos(t) + 2$$

$$y(t) = \frac{2}{3}\,e^{t} + \frac{1}{3}\cos(t) - 2 \qquad \textbf{(26)}$$

> $restart$

4)

> $Ecuacion := diff(y(t), t) - 4\cdot y(t) = 4\cdot \mathrm{Heaviside}(t-2)$

$$Ecuacion := \frac{d}{dt}\,y(t) - 4\,y(t) = 4\,\mathrm{Heaviside}(t-2) \qquad \textbf{(27)}$$

> $Cond := y(0) = 0$

$$Cond := y(0) = 0 \qquad \textbf{(28)}$$

> $with(inttrans) :$

> $EcuaLap := subs(Cond, laplace(Ecuacion, t, s))$

$$EcuaLap := s\,laplace(y(t), t, s) - 4\,laplace(y(t), t, s) = \frac{4\,e^{-2s}}{s} \qquad \textbf{(29)}$$

> $SolLap := isolate(EcuaLap, laplace(y(t), t, s))$

$$SolLap := laplace(y(t), t, s) = \frac{4\,e^{-2s}}{s\,(s-4)} \quad \textbf{(30)}$$

> $Solucion := invlaplace(SolLap, s, t)$

$$Solucion := y(t) = \mathrm{Heaviside}(t-2)\,\left(-1 + e^{4t-8}\right) \quad \textbf{(31)}$$

> restart

5)

> $f := x + 1$

$$f := x + 1 \quad \textbf{(32)}$$

> $L := \frac{1}{2}$

$$L := \frac{1}{2} \quad \textbf{(33)}$$

> $a[0] := \frac{1}{L} \cdot int(f, x = 0..1)$

$$a_0 := 3 \quad \textbf{(34)}$$

> $C := \frac{a[0]}{2}$

$$C := \frac{3}{2} \quad \textbf{(35)}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0..1\right)$

$$a_n := \frac{4\,n\,\pi \sin(n\,\pi)\,\cos(n\,\pi) + \cos(n\,\pi)^2 - 1}{n^2\,\pi^2} \quad \textbf{(36)}$$

> $b[n] := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0..1\right)$

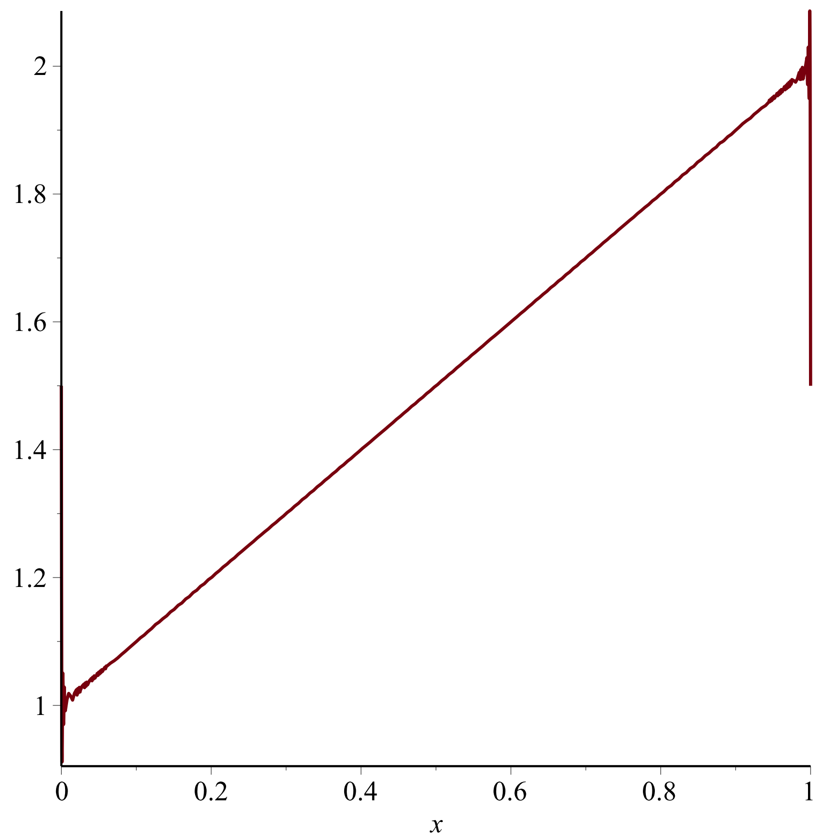
$$b_n := -\frac{4\,n\,\pi \cos(n\,\pi)^2 - \sin(n\,\pi)\,\cos(n\,\pi) - 3\,n\,\pi}{n^2\,\pi^2} \quad \textbf{(37)}$$

> $STF := C + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..infinity\right)$

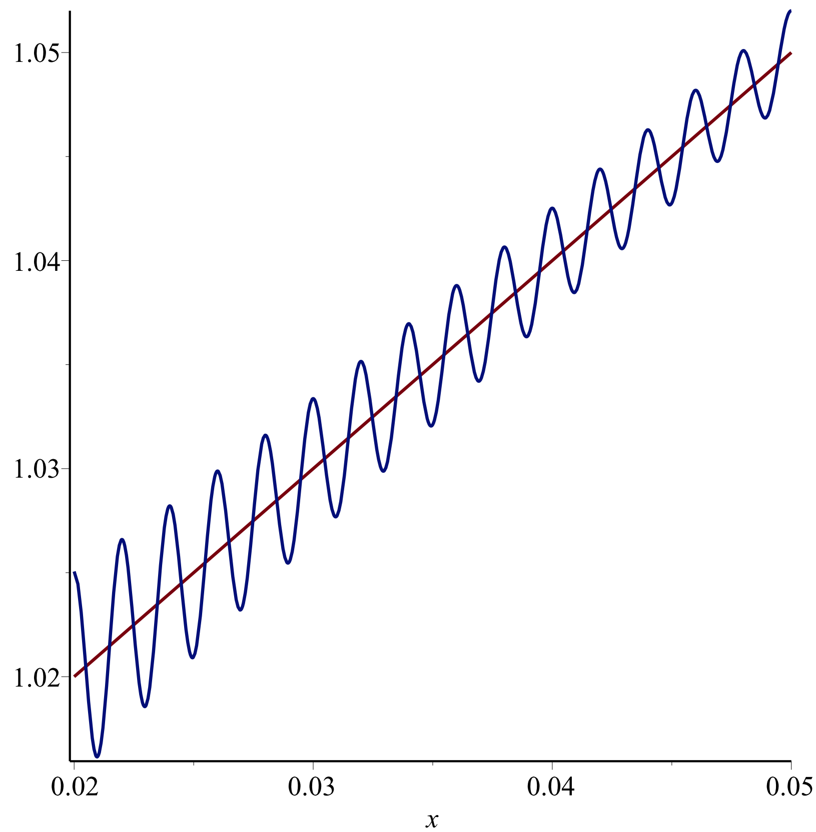
$$STF := \frac{3}{2} + \sum_{n=1}^{\infty} \left(\frac{\left(4\,n\,\pi \sin(n\,\pi)\,\cos(n\,\pi) + \cos(n\,\pi)^2 - 1\right) \cos(2\,n\,\pi\,x)}{n^2\,\pi^2} - \frac{\left(4\,n\,\pi \cos(n\,\pi)^2 - \sin(n\,\pi)\,\cos(n\,\pi) - 3\,n\,\pi\right) \sin(2\,n\,\pi\,x)}{n^2\,\pi^2} \right) \quad \textbf{(38)}$$

> $STF[500] := C + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..500\right) :$

> $plot(STF[500], x = 0..1)$



> $plot([f, STF[500]], x = 0.02..0.05)$



> $LL := 1; a[0] := 0; a[n] := 0$

$$LL := 1$$
$$a_0 := 0$$
$$a_n := 0 \qquad (39)$$

> $f;$

$$x + 1 \qquad (40)$$

> $b[n] := \frac{2}{LL} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{LL}\right), x = 0..1\right)$

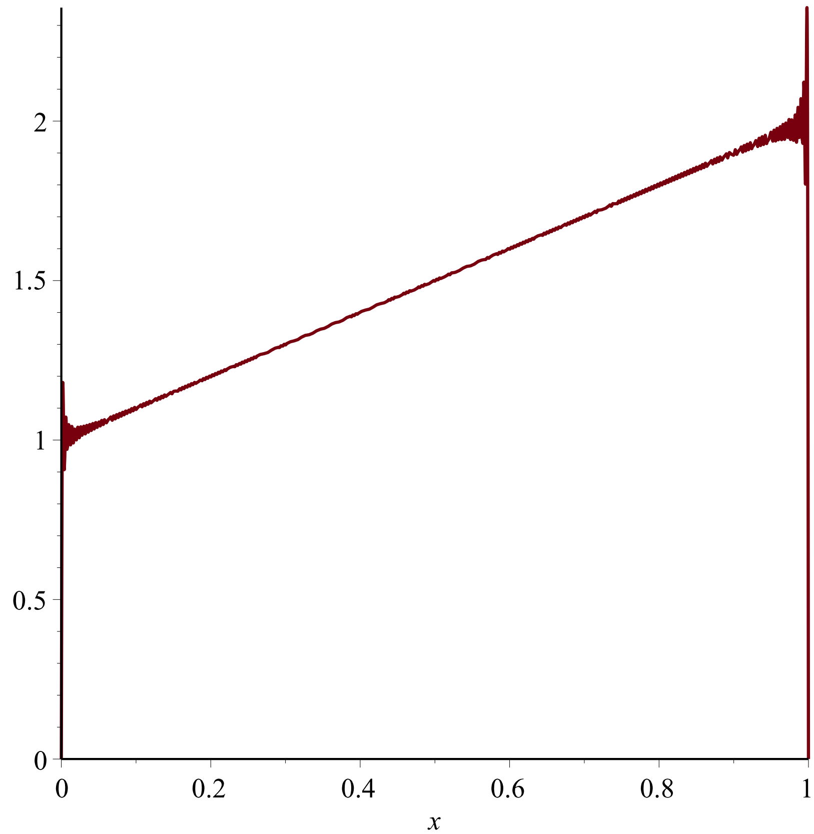
$$b_n := \frac{2\left(-2\cos(n\pi)\, n\pi + n\pi + \sin(n\pi)\right)}{n^2 \pi^2} \qquad (41)$$

> $STF2 := Sum\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{LL}\right), n = 1..infinity\right)$

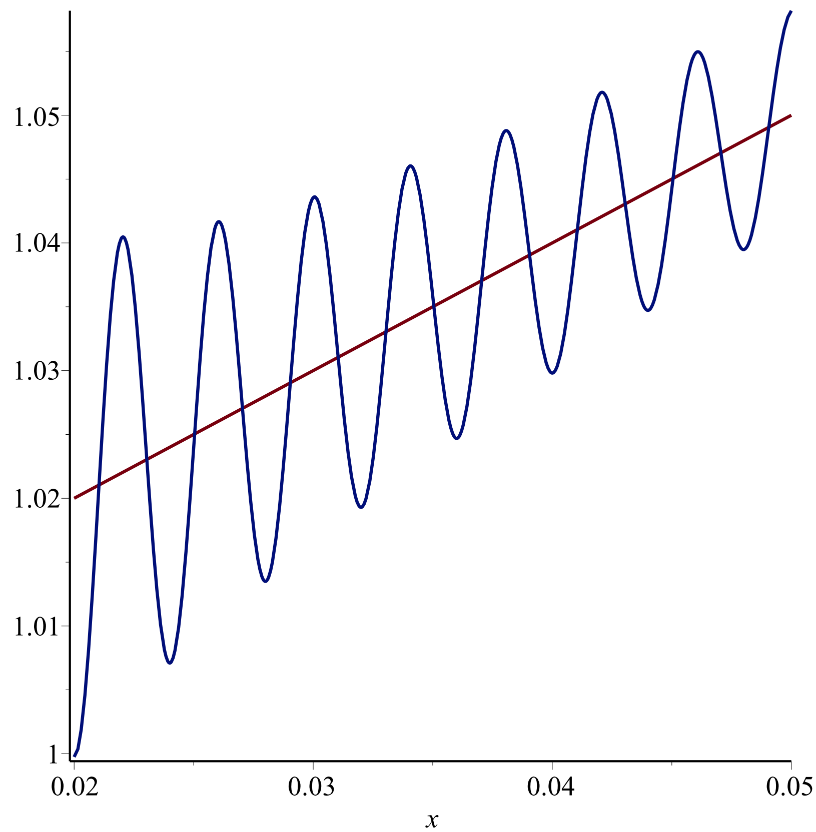
$$STF2 := \sum_{n=1}^{\infty} \frac{2\left(-2\cos(n\pi)\, n\pi + n\pi + \sin(n\pi)\right)\sin(n\pi x)}{n^2 \pi^2} \qquad (42)$$

> $STF2[500] := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{LL}\right), n = 1..500\right):$

> $plot(STF2[500], x = 0..1)$



> $plot([f, STF2[500]], x = 0.02..0.05)$



>

> *restart*

FIN EXAMEN

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