>

SOLUCIÓN

ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL
SEMESTRE 2019-2

5 DE JUNIO DE 2019

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1) resuelva el siguiente problema de valores iniciales

> $Ecuacion := x \cdot diff(y(x), x) + (x \cdot y(x) + y(x) - x \cdot \cdot 2 - 2 \cdot x) = 0$

$$Ecuacion := x \left(\frac{d}{dx} y(x) \right) + x\, y(x) + y(x) - x^2 - 2\, x = 0 \quad \textbf{(1)}$$

> $CondicionInicial := y(1) = 0$

$$CondicionInicial := y(1) = 0 \quad \textbf{(2)}$$

> $Solucion := dsolve(\{Ecuacion, CondicionInicial\})$

$$Solucion := y(x) = x - \frac{e^{-x}}{x\, e^{-1}} \quad \textbf{(3)}$$

> *restart*

2)

> $Ecua := diff(y(\text{theta}), \text{theta\$2}) + y(\text{theta}) = \sec(\text{theta}) \cdot \tan(\text{theta})$

$$Ecua := \frac{d^2}{d\theta^2} y(\theta) + y(\theta) = \sec(\theta) \tan(\theta) \quad \textbf{(4)}$$

> $Sol := dsolve(Ecua)$

$$Sol := y(\theta) = \sin(\theta)\ \_C2 + \cos(\theta)\ \_C1 - \ln(\cos(\theta)) \sin(\theta) - \sin(\theta) + \cos(\theta)\ \theta \quad \textbf{(5)}$$

> *restart*

3) resuelva el sistema

> $Sist := 2 \cdot diff(x(t), t) + diff(y(t), t) - 2 \cdot x(t) = 1, diff(x(t), t) + diff(y(t), t) - 3 \cdot x(t) - 3 \cdot y(t) = 2 : Sist[1]; Sist[2]$

$$2 \left(\frac{d}{dt} x(t) \right) + \frac{d}{dt} y(t) - 2\, x(t) = 1$$

$$\frac{d}{dt} x(t) + \frac{d}{dt} y(t) - 3\, x(t) - 3\, y(t) = 2 \quad \textbf{(6)}$$

> $Cond := x(0) = 0, y(0) = 0$

$$Cond := x(0) = 0, y(0) = 0 \quad \textbf{(7)}$$

> $Sol := dsolve(\{Sist, Cond\}) : Sol[1]; Sol[2]$

$$x(t) = \frac{5}{2} e^{2t} - 2\, e^{3t} - \frac{1}{2}$$

$$y(t) = -\frac{5}{2} e^{2t} + \frac{8}{3} e^{3t} - \frac{1}{6} \quad \textbf{(8)}$$

> $with(inttrans) :$

> $SistLap := subs(Cond, laplace(\{Sist\}, t, s)) : SistLap[1]; SistLap[2]$

$$2\, s\, laplace(x(t), t, s) + s\, laplace(y(t), t, s) - 2\, laplace(x(t), t, s) = \frac{1}{s}$$

$$s\, laplace(x(t), t, s) + s\, laplace(y(t), t, s) - 3\, laplace(x(t), t, s) - 3\, laplace(y(t), t, s) = \frac{2}{s} \quad \textbf{(9)}$$

> $VarY := isolate(SistLap[1], laplace(y(t), t, s))$

$$VarY := laplace(y(t), t, s) = \frac{\frac{1}{s} - 2\, s\, laplace(x(t), t, s) + 2\, laplace(x(t), t, s)}{s} \quad \textbf{(10)}$$

> $VarX := isolate(subs(laplace(y(t), t, s) = rhs(VarY), SistLap[2]), laplace(x(t), t, s))$

$$VarX := laplace(x(t), t, s) = \frac{-s - 3}{s^3 - 5\, s^2 + 6\, s} \quad \textbf{(11)}$$

> $VarYY := subs(laplace(x(t), t, s) = rhs(VarX), VarY)$

$$VarYY := laplace(y(t), t, s) = \frac{\frac{1}{s} - \frac{2\, s\, (-s - 3)}{s^3 - 5\, s^2 + 6\, s} + \frac{2\, (-s - 3)}{s^3 - 5\, s^2 + 6\, s}}{s} \quad \textbf{(12)}$$

> $SolX := invlaplace(VarX, s, t)$

$$SolX := x(t) = \frac{5}{2}\, e^{2\, t} - 2\, e^{3\, t} - \frac{1}{2} \quad \textbf{(13)}$$

> $SolY := invlaplace(VarYY, s, t)$

$$SolY := y(t) = -\frac{5}{2}\, e^{2\, t} + \frac{8}{3}\, e^{3\, t} - \frac{1}{6} \quad \textbf{(14)}$$

> *restart*

4) con Transformada de Laplace

> $Ecua := diff(y(t), t\$2) + 6 \cdot diff(y(t), t) + 5 \cdot y(t) = \exp(t) \cdot \mathrm{Dirac}(t - 1)$

$$Ecua := \frac{d^2}{dt^2}\, y(t) + 6\, \left(\frac{d}{dt}\, y(t)\right) + 5\, y(t) = e^t\, \mathrm{Dirac}(t - 1) \quad \textbf{(15)}$$

> $Cond := y(0) = 0, \mathrm{D}(y)(0) = 4$

$$Cond := y(0) = 0, \mathrm{D}(y)(0) = 4 \quad \textbf{(16)}$$

> $with(inttrans) :$

> $EcuaLap := subs(Cond, laplace(Ecua, t, s))$

$$EcuaLap := s^2\, laplace(y(t), t, s) - 4 + 6\, s\, laplace(y(t), t, s) + 5\, laplace(y(t), t, s) = e^{1 - s} \quad \textbf{(17)}$$

> $SolLap := isolate(EcuaLap, laplace(y(t), t, s))$

$$SolLap := laplace(y(t), t, s) = \frac{e^{1 - s} + 4}{s^2 + 6\, s + 5} \quad \textbf{(18)}$$

> $SolPart := invlaplace(SolLap, s, t)$

$$SolPart := y(t) = \frac{1}{2}\, \mathrm{Heaviside}(t - 1)\, \sinh(2\, t - 2)\, e^{4 - 3\, t} + 2\, e^{-3\, t} \sinh(2\, t) \quad \textbf{(19)}$$

> $SolucionPart := expand(convert(SolPart, \exp))$

$$SolucionPart := y(t) = \frac{1}{4}\, \frac{\mathrm{Heaviside}(t - 1)\, e^2}{e^t} + \frac{1}{e^t} - \frac{1}{4}\, \frac{\mathrm{Heaviside}(t - 1)\, e^6}{(e^t)^5} - \frac{1}{(e^t)^5} \quad \textbf{(20)}$$

> *restart*

5)

> $Ecua := diff(u(x, y), x\$2) + diff(u(x, y), y\$2) = u(x, y)$

$$Ecua := \frac{\partial^2}{\partial x^2}\, u(x, y) + \frac{\partial^2}{\partial y^2}\, u(x, y) = u(x, y) \quad \textbf{(21)}$$

> $SolUno := pdsolve(Ecua)$

$$SolUno := (u(x,y) = \_F1(x)\,\_F2(y)) \;\&where\; \left[\left\{\frac{d^2}{dx^2}\_F1(x) = \_c_1\,\_F1(x),\ \frac{d^2}{dy^2}\_F2(y) = -\_c_1\,\_F2(y) + \_F2(y)\right\}\right] \quad (22)$$

> $with(PDEtools):$

> $SolDos := simplify(subs(\_c_1 = 1, build(SolUno)))$

$$SolDos := u(x,y) = \_C4\,(e^{x}\,\_C1 + e^{-x}\,\_C2) \quad (23)$$

> $EcuaSepUno := eval(subs(u(x,y) = F(x)\cdot G(y), Ecua))$

$$EcuaSepUno := \left(\frac{d^2}{dx^2}F(x)\right)G(y) + F(x)\left(\frac{d^2}{dy^2}G(y)\right) = F(x)\,G(y) \quad (24)$$

> $EcuaSepDos := simplify\left(\frac{1}{F(x)\cdot G(y)}\left(lhs(EcuaSepUno) - F(x)\left(\frac{d^2}{dy^2}G(y)\right) = rhs(EcuaSepUno) - F(x)\left(\frac{d^2}{dy^2}G(y)\right)\right)\right)$

$$EcuaSepDos := \frac{\frac{d^2}{dx^2}F(x)}{F(x)} = \frac{G(y) - \left(\frac{d^2}{dy^2}G(y)\right)}{G(y)} \quad (25)$$

> $EcuaX := lhs(EcuaSepDos) = \mathrm{alpha};\ EcuaY := rhs(EcuaSepDos) = \mathrm{alpha}$

$$EcuaX := \frac{\frac{d^2}{dx^2}F(x)}{F(x)} = \alpha$$

$$EcuaY := \frac{G(y) - \left(\frac{d^2}{dy^2}G(y)\right)}{G(y)} = \alpha \quad (26)$$

> $SolX := dsolve(subs(\mathrm{alpha} = 1, EcuaX))$

$$SolX := F(x) = \_C1\,e^{-x} + \_C2\,e^{x} \quad (27)$$

> $SolY := dsolve(subs(\mathrm{alpha} = 1, EcuaY))$

$$SolY := G(y) = \_C1\,y + \_C2 \quad (28)$$

> $SolucionGeneral := u(x,y) = rhs(SolX)\cdot subs(\_C1 = \_C3, \_C2 = \_C4, rhs(SolY))$

$$SolucionGeneral := u(x,y) = (\_C1\,e^{-x} + \_C2\,e^{x})\,(\_C3\,y + \_C4) \quad (29)$$

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