

Ecuaciones Diferenciales  
 grupo 10 semestre 2022-2  
 Segundo Examen Parcial: Temas 3 & 4  
 SOLUCIÓN

2022-05-24

PREGUNTA 1 (20 puntos) Mediante la Transformada de Laplace obtenga la solución de la ecuación diferencial, sujeta a las condiciones iniciales dadas (*sin usar dsolve*)

>  $Ecua := 3 \text{ (diff}(y(t), t\$2)) + y(t) = \sin(t) \text{ Heaviside}(t - 2 \pi)$

$$Ecua := 3 \left( \frac{d^2}{dt^2} y(t) \right) + y(t) = \sin(t) \text{ Heaviside}(t - 2 \pi) \quad (1)$$

>  $Cond := y(0) = 1, D(y)(0) = 0$

$$Cond := y(0) = 1, D(y)(0) = 0 \quad (2)$$

>  $with(inttrans) :$

>  $EcuaTransLap := subs(Cond, laplace(Ecua, t, s))$

$$EcuaTransLap := 3 s^2 \text{ laplace}(y(t), t, s) - 3 s + \text{laplace}(y(t), t, s) = \frac{e^{-2 s \pi}}{s^2 + 1} \quad (3)$$

>  $SolTransLap := isolate(EcuaTransLap, \text{laplace}(y(t), t, s))$

$$SolTransLap := \text{laplace}(y(t), t, s) = \frac{\frac{e^{-2 s \pi}}{s^2 + 1} + 3 s}{3 s^2 + 1} \quad (4)$$

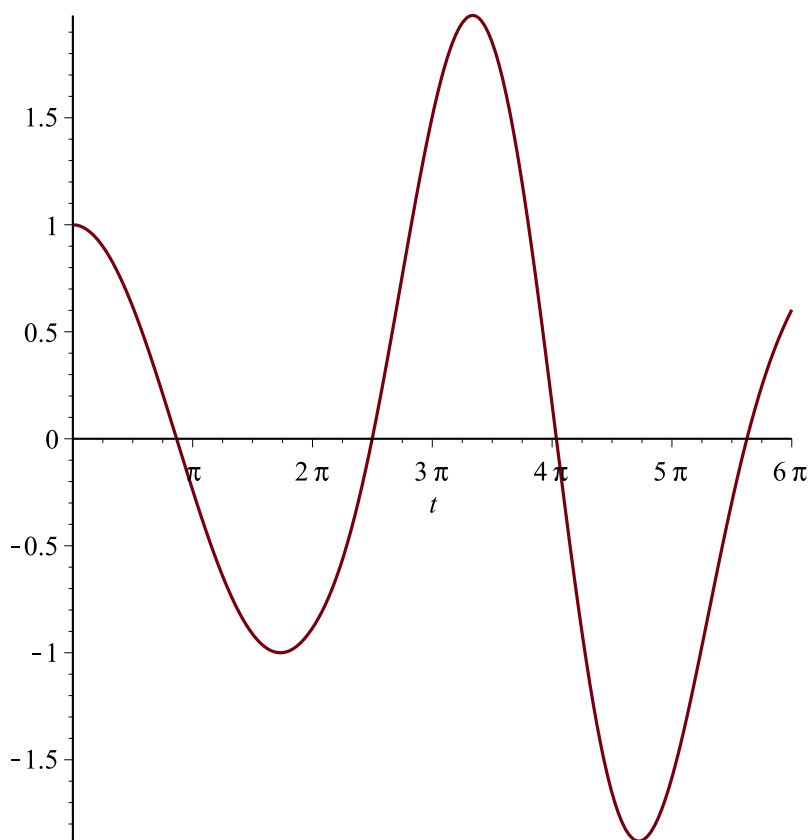
>  $SolPart := invlaplace(SolTransLap, s, t)$

$$SolPart := y(t) = \cos\left(\frac{1}{3} \sqrt{3} t\right) + \frac{1}{2} \left( -\sin(t) + \sqrt{3} \sin\left(\frac{1}{3} \sqrt{3} (t - 2 \pi)\right) \right) \text{ Heaviside}(t - 2 \pi) \quad (5)$$

>  $Comprob := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprob := 0 = 0 \quad (6)$$

>  $plot(rhs(SolPart), t = 0 .. 6 \cdot \text{Pi})$



> restart

PREGUNTA 2 (20 puntos) Obtener la solución particular del sistema de ecuaciones diferenciales con las condiciones iniciales dadas (*sin usar dsolve*)

>  $SistEcua := diff(y[1](t), t) = y[2](t), diff(y[2](t), t) = -4 \cdot y[1](t) + 2 \cdot \cos(t) : SistEcua[1];$   
 $SistEcua[2];$

$$\frac{d}{dt} y_1(t) = y_2(t)$$

$$\frac{d}{dt} y_2(t) = -4 y_1(t) + 2 \cos(t) \quad (7)$$

>  $Cond := y[1](0) = 0, y[2](0) = 0$

$$Cond := y_1(0) = 0, y_2(0) = 0 \quad (8)$$

>  $AA := array([[0, 1], [-4, 0]])$

$$AA := \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \quad (9)$$

>  $BB := array([0, 2 \cdot \cos(t)])$

$$BB := \begin{bmatrix} 0 & 2 \cos(t) \end{bmatrix} \quad (10)$$

> with(linalg) :

> *MatExp* := exponential(*AA*, *t*)

$$\mathit{MatExp} := \begin{bmatrix} \cos(2t) & \frac{1}{2} \sin(2t) \\ -2 \sin(2t) & \cos(2t) \end{bmatrix} \quad (11)$$

> *Xcero* := array([0, 0])

$$\mathit{Xcero} := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (12)$$

> *SolGral* := evalm(*MatExp* &\* *Xcero*)

$$\mathit{SolGral} := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (13)$$

> *MatExpTau* := map(rcurry(eval, t='t - tau'), *MatExp*)

$$\mathit{MatExpTau} := \begin{bmatrix} \cos(2t - 2\tau) & \frac{1}{2} \sin(2t - 2\tau) \\ -2 \sin(2t - 2\tau) & \cos(2t - 2\tau) \end{bmatrix} \quad (14)$$

> *BBtau* := map(rcurry(eval, t='tau'), *BB*)

$$\mathit{BBtau} := \begin{bmatrix} 0 & 2 \cos(\tau) \end{bmatrix} \quad (15)$$

> *AAtau* := evalm(*MatExpTau* &\* *BBtau*)

$$\mathit{AAtau} := \begin{bmatrix} \sin(2t - 2\tau) \cos(\tau) & 2 \cos(2t - 2\tau) \cos(\tau) \end{bmatrix} \quad (16)$$

> *SolPart* := map(int, *AAtau*, tau = 0 .. t) : *y*[1](*t*) = *SolPart*[1]; *y*[2](*t*) = *SolPart*[2]

$$y_1(t) = -\frac{2}{3} \cos(2t) + \frac{2}{3} \cos(t)$$
$$y_2(t) = \frac{4}{3} \sin(2t) - \frac{2}{3} \sin(t) \quad (17)$$

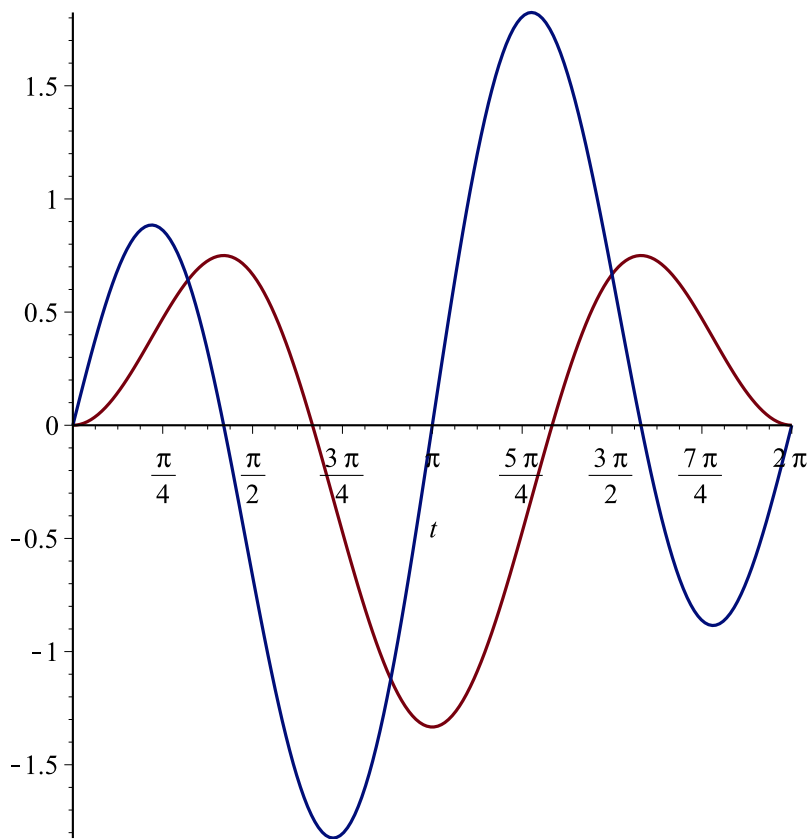
> *CompUno* := eval(subs(*y*[1](*t*) = *SolPart*[1], *y*[2](*t*) = *SolPart*[2], lhs(*SistEcua*[1])  
- rhs(*SistEcua*[1]) = 0))

$$\mathit{CompUno} := 0 = 0 \quad (18)$$

> *CompDos* := eval(subs(*y*[1](*t*) = *SolPart*[1], *y*[2](*t*) = *SolPart*[2], lhs(*SistEcua*[2])  
- rhs(*SistEcua*[2]) = 0))

$$\mathit{CompDos} := 0 = 0 \quad (19)$$

> plot([*SolPart*[1], *SolPart*[2]], t = 0 .. 2·Pi)



> restart

PREGUNTA 3 (30 puntos) Detremine una solución completa de la ecuación diferencial utilizando el método de separación de variables para una constante de separación nula (*sin usar pdsolve*)

> Ecua := diff(y(x, t), t\$2) + diff(y(x, t), x, t) = 2 \* x^3 \* diff(y(x, t), t)

$$Ecua := \frac{\partial^2}{\partial t^2} y(x, t) + \frac{\partial^2}{\partial x \partial t} y(x, t) = 2 x^3 \left( \frac{\partial}{\partial t} y(x, t) \right) \quad (20)$$

> EcuaSeparable := eval(subs(y(x, t) = F(x) \* G(t), Ecua))

$$EcuaSeparable := F(x) \left( \frac{d^2}{dt^2} G(t) \right) + \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dt} G(t) \right) = 2 x^3 F(x) \left( \frac{d}{dt} G(t) \right) \quad (21)$$

> EcuaSeparada := 
$$\frac{\left( lhs(EcuaSeparable) - \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dt} G(t) \right) \right)}{F(x) \cdot diff(G(t), t)}$$

$$= simplify \left( \frac{\left( rhs(EcuaSeparable) - \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dt} G(t) \right) \right)}{F(x) \cdot diff(G(t), t)} \right)$$

(22)

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{\frac{d}{dt} G(t)} = \frac{2 F(x) x^3 - \left( \frac{d}{dx} F(x) \right)}{F(x)} \quad (22)$$

>  $EcuaX := rhs(EcuaSeparada) = 0$

$$EcuaX := \frac{2 F(x) x^3 - \left( \frac{d}{dx} F(x) \right)}{F(x)} = 0 \quad (23)$$

>  $EcuaT := lhs(EcuaSeparada) = 0$

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{\frac{d}{dt} G(t)} = 0 \quad (24)$$

>  $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1 e^{\frac{1}{2} x^4} \quad (25)$$

>  $SolT := dsolve(EcuaT)$

$$SolT := G(t) = \_C1 t + \_C2 \quad (26)$$

>  $SolGralCero := y(x, t) = rhs(SolT) \cdot subs(\_C1 = 1, rhs(SolX))$

$$SolGralCero := y(x, t) = (\_C1 t + \_C2) e^{\frac{1}{2} x^4} \quad (27)$$

>  $Comprobacion := simplify(eval(subs(y(x, t) = rhs(SolGralCero), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobacion := 0 = 0 \quad (28)$$

> restart

PREGUNTA 4 (30 puntos) Determinar la solución de la ecuación diferencial considerando una constante de separación positiva (**sin usar pdsolve**)

>  $Ecua := diff(z(x, y), x^2, y) = diff(z(x, y), x)$

$$Ecua := \frac{\partial^3}{\partial y \partial x^2} z(x, y) = \frac{\partial}{\partial x} z(x, y) \quad (29)$$

>  $EcuaSeparable := eval(subs(z(x, y) = F(x) \cdot G(y), Ecua))$

$$EcuaSeparable := \left( \frac{d^2}{dx^2} F(x) \right) \left( \frac{d}{dy} G(y) \right) = \left( \frac{d}{dx} F(x) \right) G(y) \quad (30)$$

>  $EcuaSeparada := \frac{lhs(EcuaSeparable)}{\left( \frac{d}{dx} F(x) \right) \cdot \left( \frac{d}{dy} G(y) \right)} = \frac{rhs(EcuaSeparable)}{\left( \frac{d}{dx} F(x) \right) \cdot \left( \frac{d}{dy} G(y) \right)}$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \frac{G(y)}{\frac{d}{dy} G(y)} \quad (31)$$

>  $EcuaX := lhs(EcuaSeparada) = \beta^2$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \beta^2 \quad (32)$$

>  $EcuaY := rhs(EcuaSeparada) = \beta^2$

$$EcuaY := \frac{G(y)}{\frac{d}{dy} G(y)} = \beta^2 \quad (33)$$

>  $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1 + \_C2 e^{\beta^2 x} \quad (34)$$

>  $SolY := dsolve(EcuaY)$

$$SolY := G(y) = \_C1 e^{\frac{y}{\beta^2}} \quad (35)$$

>  $SolGral := z(x, y) = subs(\_C1 = 1, rhs(SolY)) \cdot rhs(SolX)$

$$SolGral := z(x, y) = e^{\frac{y}{\beta^2}} (\_C1 + \_C2 e^{\beta^2 x}) \quad (36)$$

>  $Comp := eval(subs(z(x, y) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0))$

$$Comp := 0 = 0 \quad (37)$$

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FIN DE LA SOLUCIÓN

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