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> restart
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SOLUCIÓN

UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL
SEMESTRE 2022-2

9 JUNIO 2022

PARTE I

Ecuacion 1

$Ecua := x^2 + y(x) - x \cdot diff(y(x), x) = 0$

$$x^2 + y(x) - x\left(\frac{d}{dx} y(x)\right) = 0 \quad (1)$$

$Cond := y(1) = 2$

$$y(1) = 2 \quad (2)$$

```
with(DEtools) :
odeadvisor(Ecua)
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$$[\_linear] \quad (3)$$

$M := x^2 + y$

$$x^2 + y \quad (4)$$

$N := -x$

$$-x \quad (5)$$

$DerMy := diff(M, y)$

$$1 \quad (6)$$

$DerNx := diff(N, x)$

$$-1 \quad (7)$$

No es exacta, hay que buscar el factor integrante

$FI := isolate\left(int\left(\frac{1}{\mu}, \mu\right) = int\left(\frac{(DerMy - DerNx)}{N}, x\right), \mu\right)$

$$\mu = \frac{1}{x^2} \quad (8)$$

$MM := expand(M \cdot rhs(FI))$

$$1 + \frac{y}{x^2} \quad (9)$$

$NN := N \cdot rhs(FI)$

$$-\frac{1}{x} \quad (10)$$

$Comprobar := diff(MM, y) = diff(NN, x)$

$$\frac{1}{x^2} = \frac{1}{x^2} \quad (11)$$

$EcuaDos := MM + NN \cdot diff(y(x), x) = 0$

$$1 + \frac{y}{x^2} - \frac{\frac{d}{dx} y(x)}{x} = 0 \quad (12)$$

$EcuaDosDos := 1 + \frac{y(x)}{x^2} - \frac{\frac{d}{dx} y(x)}{x} = 0$

$$1 + \frac{y(x)}{x^2} - \frac{\frac{d}{dx} y(x)}{x} = 0 \quad (13)$$

$odeadvisor(EcuaDosDos)$

$$[\_linear] \quad (14)$$

Ya es una ecuación diferencial exacta

$IntMMx := int(MM, x)$

$$x - \frac{y}{x} \quad (15)$$

$SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$x - \frac{y}{x} = \_C1 \quad (16)$$

$SolGralDos := x - \frac{y(x)}{x} = \_C1$

$$x - \frac{y(x)}{x} = \_C1 \quad (17)$$

$Para := subs(x = 1, y = 2, SolGral)$

$$-1 = \_C1 \quad (18)$$

$SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$x - \frac{y}{x} = -1 \quad (19)$$

$SolPartDos := x - \frac{y(x)}{x} = -1$

$$x - \frac{y(x)}{x} = -1 \quad (20)$$

$DerSolPart := expand(isolate(diff(SolPartDos, x), diff(y(x), x)))$

$$\frac{d}{dx} y(x) = x + \frac{y(x)}{x} \quad (21)$$

$DerEcua := expand(isolate(Ecua, diff(y(x), x)))$

$$\frac{d}{dx} y(x) = x + \frac{y(x)}{x} \quad (22)$$

$CompUno := rhs(DerSolPart) - rhs(DerEcua) = 0$

$$0 = 0 \quad (23)$$

$DerSolGral := expand(isolate(diff(SolGralDos, x), diff(y(x), x)))$

$$\frac{d}{dx} y(x) = x + \frac{y(x)}{x} \tag{24}$$

$CompDos := rhs(DerSolGral) - rhs(DerEcua) = 0$

$$0 = 0 \tag{25}$$

restart

Ecuación 2

$Ecua := y'' + 4 \cdot y' + 4 \cdot y = 4 \cdot \exp(-2x)$

$$\frac{d^2}{dx^2} y(x) + 4 \left(\frac{d}{dx} y(x) \right) + 4 y(x) = 4 e^{-2x} \tag{26}$$

$EcuaHom := lhs(Ecua) = 0$

$$\frac{d^2}{dx^2} y(x) + 4 \left(\frac{d}{dx} y(x) \right) + 4 y(x) = 0 \tag{27}$$

$Q := rhs(Ecua)$

$$4 e^{-2x} \tag{28}$$

$EcuaCarac := m^2 + 4 \cdot m + 4 = 0$

$$m^2 + 4 m + 4 = 0 \tag{29}$$

$Raiz := solve(EcuaCarac)$

$$-2, -2 \tag{30}$$

$yy[1] := \exp(Raiz[1] \cdot x); yy[2] := x \cdot \exp(Raiz[1] \cdot x)$

$$e^{-2x}$$

$$x e^{-2x} \tag{31}$$

$SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$y(x) = \_C1 e^{-2x} + \_C2 x e^{-2x} \tag{32}$$

$SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$

$$y(x) = A e^{-2x} + B x e^{-2x} \tag{33}$$

$with(linalg) :$

$WW := array([[yy[1], yy[2]], [diff(yy[1], x), diff(yy[2], x)]])$

$$\begin{bmatrix} e^{-2x} & x e^{-2x} \\ -2 e^{-2x} & e^{-2x} - 2 x e^{-2x} \end{bmatrix} \tag{34}$$

$BB := array([0, Q])$

$$\begin{bmatrix} 0 & 4 e^{-2x} \end{bmatrix} \tag{35}$$

$Para := linsolve(WW, BB) : Aprima := Para[1]; Bprima := Para[2]$

$$-4 x$$

$$4 \tag{36}$$

$A := int(Aprima, x) + \_C1$

$$-2 x^2 + \_C1 \tag{37}$$

$B := int(Bprima, x) + C2$

$$4\,x + \_C2 \tag{38}$$

SolGral := *simplify*(*SolNoHom*)

$$y(x) = e^{-2x}\left(\_C2\,x + 2\,x^2 + \_C1\right) \tag{39}$$

Comprobacion := *eval*(*subs*(*y*(*x*) = *rhs*(*SolGral*), *lhs*(*Ecua*) − *rhs*(*Ecua*) = 0))

$$0 = 0 \tag{40}$$

PARTE II

restart

Ecuación 3

Ecua := *diff*(*y*(*t*), *t*$2) + *diff*(*y*(*t*), *t*) + *y*(*t*) = exp(*t* − Pi)·Heaviside(*t* − Pi)

$$\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + y(t) = e^{t-\pi}\,\text{Heaviside}(t-\pi) \tag{41}$$

Cond := *y*(0) = 1, D(*y*)(0) = 1

$$y(0) = 1,\ D(y)(0) = 1 \tag{42}$$

with(*inttrans*) :

EcuaLap := *subs*(*Cond*, *laplace*(*Ecua*, *t*, *s*))

$$s^2\,laplace(y(t), t, s) - 2 - s + s\,laplace(y(t), t, s) + laplace(y(t), t, s) = \frac{e^{-s\pi}}{s-1} \tag{43}$$

SolLap := *isolate*(*EcuaLap*, *laplace*(*y*(*t*), *t*, *s*))

$$laplace(y(t), t, s) = \frac{\frac{e^{-s\pi}}{s-1} + s + 2}{s^2 + s + 1} \tag{44}$$

SolPart := *invlaplace*(*SolLap*, *s*, *t*)

$$y(t) = -\frac{1}{3}\,e^{-\frac{1}{2}t+\frac{1}{2}\pi}\,\text{Heaviside}(t-\pi)\left(\sqrt{3}\,\sin\left(\frac{1}{2}\sqrt{3}\,(t-\pi)\right) + \cos\left(\frac{1}{2}\sqrt{3}\,(t-\pi)\right)\right) \tag{45}$$
$$+ e^{-\frac{1}{2}t}\left(\cos\left(\frac{1}{2}\sqrt{3}\,t\right) + \sqrt{3}\,\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\right) + \frac{1}{3}\,(1 - \text{Heaviside}(-t+\pi))\,e^{t-\pi}$$

Ecua

$$\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + y(t) = e^{t-\pi}\,\text{Heaviside}(t-\pi) \tag{46}$$

Comp := *simplify*(*eval*(*subs*(*y*(*t*) = *rhs*(*SolPart*), *lhs*(*Ecua*) − *rhs*(*Ecua*) = 0)))

$$0 = 0 \tag{47}$$

SolPart

$$y(t) = -\frac{1}{3}\,e^{-\frac{1}{2}t+\frac{1}{2}\pi}\,\text{Heaviside}(t-\pi)\left(\sqrt{3}\,\sin\left(\frac{1}{2}\sqrt{3}\,(t-\pi)\right) + \cos\left(\frac{1}{2}\sqrt{3}\,(t-\pi)\right)\right) \tag{48}$$
$$+ e^{-\frac{1}{2}t}\left(\cos\left(\frac{1}{2}\sqrt{3}\,t\right) + \sqrt{3}\,\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\right) + \frac{1}{3}\,(1 - \text{Heaviside}(-t+\pi))\,e^{t-\pi}$$

restart

Ecuación 4

Sistema := *diff*(*x*[1](*t*), *t*) = *x*[1](*t*) − *x*[2](*t*), *diff*(*x*[2](*t*), *t*) = −*x*[1](*t*) + *x*[2](*t*) :

Sistema[1]; *Sistema*[2]

$$\frac{d}{dt}x_1(t) = x_1(t) - x_2(t)$$

$$\frac{d}{dt}x_2(t) = -x_1(t) + x_2(t) \quad (49)$$

Cond := $x[1](0) = 2, x[2](0) = -2$

$$x_1(0) = 2, x_2(0) = -2 \quad (50)$$

AA := *array*([[1, -1], [-1, 1]])

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (51)$$

with(*linalg*) :
MatExp := *exponential*(*AA*, *t*)

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{2}e^{2t} & -\frac{1}{2}e^{2t} + \frac{1}{2} \\ -\frac{1}{2}e^{2t} + \frac{1}{2} & \frac{1}{2} + \frac{1}{2}e^{2t} \end{bmatrix} \quad (52)$$

Xcero := *array*([2, -2])

$$\begin{bmatrix} 2 & -2 \end{bmatrix} \quad (53)$$

SolPart := *evalm*(*MatExp* &* *Xcero*) : $x[1](t) = SolPart[1]; x[2](t) = SolPart[2]$

$$x_1(t) = 2e^{2t}$$

$$x_2(t) = -2e^{2t} \quad (54)$$

Sistema[1]; *Sistema*[2]

$$\frac{d}{dt}x_1(t) = x_1(t) - x_2(t)$$

$$\frac{d}{dt}x_2(t) = -x_1(t) + x_2(t) \quad (55)$$

CompUno := *eval*(*subs*($x[1](t) = SolPart[1], x[2](t) = SolPart[2], lhs(Sistema[1])$
$- rhs(Sistema[1]) = 0$))

$$0 = 0 \quad (56)$$

CompDos := *eval*(*subs*($x[1](t) = SolPart[1], x[2](t) = SolPart[2], lhs(Sistema[2])$
$- rhs(Sistema[2]) = 0$))

$$0 = 0 \quad (57)$$

SolPart := *evalm*(*MatExp* &* *Xcero*) : $x[1](t) = SolPart[1]; x[2](t) = SolPart[2]$

$$x_1(t) = 2e^{2t}$$

$$x_2(t) = -2e^{2t} \quad (58)$$

restart
Ecuación 5

Ecua := $x^2 \cdot diff(u(x, y), x) + y^2 \cdot diff(u(x, y), y) = 0$

$$x^2\left(\frac{\partial}{\partial x}u(x,y)\right)+y^2\left(\frac{\partial}{\partial y}u(x,y)\right)=0 \quad (59)$$

constante de separación menor que cero

$EcuaSeparable := eval(subs(u(x,y)=M(x)\cdot N(y), Ecua))$

$$x^2\left(\frac{d}{dx}M(x)\right)N(y)+y^2M(x)\left(\frac{d}{dy}N(y)\right)=0 \quad (60)$$

$$EcuaSeparada := \frac{\left(lhs(EcuaSeparable)-y^2M(x)\cdot\frac{d}{dy}N(y)\right)}{M(x)\cdot N(y)} = \frac{\left(rhs(EcuaSeparable)-y^2M(x)\left(\frac{d}{dy}N(y)\right)\right)}{M(x)\cdot N(y)}$$

$$\frac{x^2\left(\frac{d}{dx}M(x)\right)}{M(x)} = -\frac{y^2\left(\frac{d}{dy}N(y)\right)}{N(y)} \quad (61)$$

$EcuaX := lhs(EcuaSeparada) = -\beta^2$

$$\frac{x^2\left(\frac{d}{dx}M(x)\right)}{M(x)} = -\beta^2 \quad (62)$$

$EcuaY := rhs(EcuaSeparada) = -\beta^2$

$$-\frac{y^2\left(\frac{d}{dy}N(y)\right)}{N(y)} = -\beta^2 \quad (63)$$

$SolX := dsolve(EcuaX)$

$$M(x) = \_C1\, e^{\frac{\beta^2}{x}} \quad (64)$$

$SolY := dsolve(EcuaY)$

$$N(y) = \_C1\, e^{-\frac{\beta^2}{y}} \quad (65)$$

$SolGral := u(x,y) = rhs(SolX)\cdot subs(\_C1=1, rhs(SolY))$

$$u(x,y) = \_C1\, e^{\frac{\beta^2}{x}} e^{-\frac{\beta^2}{y}} \quad (66)$$

$Comprobación := eval(subs(u(x,y)=rhs(SolGral), Ecua))$

$$0=0 \quad (67)$$

$SolGral$

$$u(x,y) = \_C1\, e^{\frac{\beta^2}{x}} e^{-\frac{\beta^2}{y}} \quad (68)$$

FIN DEL EXAMEN