>

SOLUCIÓN DE LA SERIE 2023-2-2

> *restart*

1)

> $Ecuacion := y'' - 2\cdot y' + y = x^{-1}\cdot \exp(x)$

$$Ecuacion := \frac{d^2}{dx^2} y(x) - 2\left(\frac{d}{dx} y(x)\right) + y(x) = \frac{e^x}{x} \quad (1)$$

Respuesta

> $EcuaHom := lhs(Ecuacion) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) - 2\left(\frac{d}{dx} y(x)\right) + y(x) = 0 \quad (2)$$

> $Q := rhs(Ecuacion)$

$$Q := \frac{e^x}{x} \quad (3)$$

> $EcuaCarac := m^2 - 2\cdot m + 1 = 0$

$$EcuaCarac := m^2 - 2\,m + 1 = 0 \quad (4)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, 1 \quad (5)$$

Caso II

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := x\cdot\exp(Raiz[1]\cdot x)$

$$yy_1 := e^x$$

$$yy_2 := x\,e^x \quad (6)$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(x) = \_C1\,e^x + \_C2\,x\,e^x \quad (7)$$

> $SolNoHom := y(x) = A\cdot e^x + B\cdot x\,e^x$

$$SolNoHom := y(x) = A\,e^x + B\,x\,e^x \quad (8)$$

> $with(linalg)$:

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & x\,e^x \\ e^x & e^x + x\,e^x \end{bmatrix} \quad (9)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{e^x}{x} \end{bmatrix} \quad (10)$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} -1 & \frac{1}{x} \end{bmatrix} \quad (11)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -1$$

(12)

$$Bprima := \frac{1}{x} \tag{12}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -x + \_C1$$
$$B := \ln(x) + \_C2 \tag{13}$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = -x\,e^{x} + \_C1\,e^{x} + x\,e^{x}\ln(x) + \_C2\,x\,e^{x} \tag{14}$$

> $restart$

2)

> $Ecuacion := 2\cdot x\cdot y'' - 2((x+1)\cdot y' - y) = 2\cdot x^{2}$

$$Ecuacion := 2\,x\left(\frac{d^{2}}{dx^{2}}\,y(x)\right) - 2\,(x+1)\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 2\,x^{2} \tag{15}$$

> $yy[1] := \exp(x); yy[2] := x + 1$

$$yy_{1} := e^{x}$$
$$yy_{2} := x + 1 \tag{16}$$

> $EcuaHom := lhs(Ecuacion) = 0$

$$EcuaHom := 2\,x\left(\frac{d^{2}}{dx^{2}}\,y(x)\right) - 2\,(x+1)\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0 \tag{17}$$

Resuesta

> $EcuaHomDos := expand\left(\frac{lhs(EcuaHom)}{2\cdot x}\right) = 0$

$$EcuaHomDos := \frac{d^{2}}{dx^{2}}\,y(x) - \left(\frac{d}{dx}\,y(x)\right) - \frac{\frac{d}{dx}\,y(x)}{x} + \frac{y(x)}{x} = 0 \tag{18}$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(x) = \_C1\,e^{x} + \_C2\,(x+1) \tag{19}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHomDos)))$

$$Comprobar := 0 = 0 \tag{20}$$

> $EcuaNoHomDos := expand\left(\frac{lhs(Ecuacion)}{2\cdot x}\right) = \frac{rhs(Ecuacion)}{2\cdot x}$

$$EcuaNoHomDos := \frac{d^{2}}{dx^{2}}\,y(x) - \left(\frac{d}{dx}\,y(x)\right) - \frac{\frac{d}{dx}\,y(x)}{x} + \frac{y(x)}{x} = x \tag{21}$$

> $Q := rhs(EcuaNoHomDos)$

$$Q := x \tag{22}$$

> $SolNoHom := y(x) = A\,e^{x} + B\cdot(x+1)$

$$SolNoHom := y(x) = A\,e^{x} + B\,(x+1) \tag{23}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

(24)

$$WW := \begin{bmatrix} e^{x} & x+1 \\ e^{x} & 1 \end{bmatrix} \quad (24)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x \end{bmatrix} \quad (25)$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} \frac{x+1}{e^{x}} & -1 \end{bmatrix} \quad (26)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := \frac{x+1}{e^{x}}$$

$$Bprima := -1 \quad (27)$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\frac{x+2}{e^{x}} + \_C1$$

$$B := -x + \_C2 \quad (28)$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = -2x - 2 + \_C1\, e^{x} + \_C2\, x - x^{2} + \_C2 \quad (29)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecuacion)))$

$$Comprobar := 2x^{2} = 2x^{2} \quad (30)$$

> $restart$

3)

> $Ecuacion := y'' + y = \sec(x)^{2}$

$$Ecuacion := \frac{d^{2}}{dx^{2}} y(x) + y(x) = \sec(x)^{2} \quad (31)$$

> $EcuaHom := lhs(Ecuacion) = 0$

$$EcuaHom := \frac{d^{2}}{dx^{2}} y(x) + y(x) = 0 \quad (32)$$

> $Q := rhs(Ecuacion)$

$$Q := \sec(x)^{2} \quad (33)$$

> $EcuaCarac := m^{2} + 1 = 0$

$$EcuaCarac := m^{2} + 1 = 0 \quad (34)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := I, -I \quad (35)$$

Caso III

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x); yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_{1} := \cos(x)$$

$$yy_{2} := \sin(x) \quad (36)$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1 \cos(x) + \_C2 \sin(x) \quad (37)$$

> $SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$

$$SolNoHom := y(x) = A\cos(x) + B\sin(x) \quad (38)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad (39)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \sec(x)^2 \end{bmatrix} \quad (40)$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{\sin(x)}{\cos(x)^2} & \frac{1}{\cos(x)} \end{bmatrix} \quad (41)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{\sin(x)}{\cos(x)^2}$$

$$Bprima := \frac{1}{\cos(x)} \quad (42)$$

> $A := int(Aprima, x) + \_C1;$

$$A := -\frac{1}{\cos(x)} + \_C1 \quad (43)$$

> $B := int(Bprima, x) + \_C2$

$$B := \ln(\sec(x) + \tan(x)) + \_C2 \quad (44)$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = -1 + \_C1\cos(x) + \sin(x)\ln(\sec(x) + \tan(x)) + \_C2\sin(x) \quad (45)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecuacion)))$

$$Comprobar := \frac{1}{\cos(x)^2} = \frac{1}{\cos(x)^2} \quad (46)$$

> $restart$

4)

> $Ecuacion := y'' + y = 2 \cdot \sec(x)^3$

$$Ecuacion := \frac{d^2}{dx^2}y(x) + y(x) = 2\sec(x)^3 \quad (47)$$

> $EcuaHom := lhs(Ecuacion) = 0$

$$EcuaHom := \frac{d^2}{dx^2}y(x) + y(x) = 0 \quad (48)$$

> $Q := rhs(Ecuacion)$

$$Q := 2\sec(x)^3 \quad (49)$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \quad (50)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := I, -I \quad (51)$$

Caso III

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x); yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(x)$$

$$yy_2 := \sin(x) \quad (52)$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1 \cos(x) + \_C2 \sin(x) \quad (53)$$

> $SolNoHom := y(x) = A \cdot \cos(x) + B \cdot \sin(x)$

$$SolNoHom := y(x) = A \cos(x) + B \sin(x) \quad (54)$$

> $with(linalg)$:

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad (55)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 2\sec(x)^3 \end{bmatrix} \quad (56)$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\dfrac{2\sin(x)}{\cos(x)^3} & \dfrac{2}{\cos(x)^2} \end{bmatrix} \quad (57)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{2\sin(x)}{\cos(x)^3}$$

$$Bprima := \frac{2}{\cos(x)^2} \quad (58)$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\frac{1}{\cos(x)^2} + \_C1$$

$$B := \frac{2\sin(x)}{\cos(x)} + \_C2 \quad (59)$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = -\frac{1}{\cos(x)} + \_C1\cos(x) + \frac{2\sin(x)^2}{\cos(x)} + \_C2\sin(x) \quad (60)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecuacion)))$

$$Comprobar := \frac{2}{\cos(x)^3} = \frac{2}{\cos(x)^3} \quad (61)$$

> $restart$

5)

> $Ecuacion := y'' + 4 \cdot y = \cot(2 \cdot x)$

$$Ecuacion := \frac{d^2}{dx^2} y(x) + 4\,y(x) = \cot(2\,x) \quad (62)$$

> $EcuaHom := lhs(Ecuacion) = 0$

(63)

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 4\,y(x) = 0 \tag{63}$$

> $Q := rhs(Ecuacion)$

$$Q := \cot(2\,x) \tag{64}$$

> $EcuaCarac := m^2 + 4 = 0$

$$EcuaCarac := m^2 + 4 = 0 \tag{65}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2\,I,\ -2\,I \tag{66}$$

Caso III

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot x); yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot x)$

$$yy_1 := \cos(2\,x)$$

$$yy_2 := \sin(2\,x) \tag{67}$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(x) = \_C1\cos(2\,x) + \_C2\sin(2\,x) \tag{68}$$

> $SolNoHom := y(x) = A\cdot yy[1] + B\cdot yy[2]$

$$SolNoHom := y(x) = A\cos(2\,x) + B\sin(2\,x) \tag{69}$$

> $with(linalg)$:

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(2\,x) & \sin(2\,x) \\ -2\sin(2\,x) & 2\cos(2\,x) \end{bmatrix} \tag{70}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \cot(2\,x) \end{bmatrix} \tag{71}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{1}{2}\cos(2\,x) & \frac{1}{2}\,\frac{\cos(2\,x)^2}{\sin(2\,x)} \end{bmatrix} \tag{72}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{1}{2}\cos(2\,x)$$

$$Bprima := \frac{1}{2}\,\frac{\cos(2\,x)^2}{\sin(2\,x)} \tag{73}$$

> $A := int(Aprima, x) + \_C1; B := int(Bprima, x) + \_C2$

$$A := -\frac{1}{4}\sin(2\,x) + \_C1$$

$$B := \frac{1}{4}\cos(2\,x) + \frac{1}{4}\ln(\csc(2\,x) - \cot(2\,x)) + \_C2 \tag{74}$$

> $SolFinal := simplify(SolNoHom)$

$$SolFinal := y(x) = \frac{1}{4}\sin(2\,x)\ln\left(-\frac{-1+\cos(2\,x)}{\sin(2\,x)}\right) + \_C2\sin(2\,x) + \_C1\cos(2\,x) \tag{75}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecuacion)))$

$$Comprobar := \frac{\cos(2\,x)}{\sin(2\,x)} = \cot(2\,x) \tag{76}$$

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> restart
FIN DE LA SERIE 2
>
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