```
> restart
```

SERIE 3

```
> restart
```

1)

```
> F := s/(2*s^2 - 4*s + 20)
```

$$F := \frac{s}{2\,s^2 - 4\,s + 20} \tag{1}$$

```
> G := 4*exp(-2*s)/(s - 3)
```

$$G := \frac{4\,e^{-2\,s}}{s - 3} \tag{2}$$

SOLUCION

```
> with(inttrans):
> f := invlaplace(F, s, t)
```

$$f := \frac{1}{6}\,e^{t}\,(3\cos(3\,t) + \sin(3\,t)) \tag{3}$$

```
> g := invlaplace(G, s, t)
```

$$g := 4\,\mathrm{Heaviside}(t - 2)\,e^{3\,t - 6} \tag{4}$$

```
> restart
```

2)

```
> Sist := diff(x(t), t) = -y(t) - 1, diff(y(t), t) = -3*x(t) + 2*y(t): Sist[1]; Sist[2]
```

$$\frac{d}{dt}x(t) = -y(t) - 1$$

$$\frac{d}{dt}y(t) = -3\,x(t) + 2\,y(t) \tag{5}$$

```
> CondIni := x(0) = 0, y(0) = 0
```

$$CondIni := x(0) = 0, y(0) = 0 \tag{6}$$

SOLUCION

```
> AA := array([[0, -1], [-3, 2]])
```

$$AA := \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix} \tag{7}$$

```
> Xcero := array([0, 0])
```

$$Xcero := \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{8}$$

```
> BB := array([-1, 0])
```

$$BB := \begin{bmatrix} -1 & 0 \end{bmatrix} \tag{9}$$

```
> with(linalg):
> MatExp := exponential(AA, t)
```

(10)

$$MatExp := \begin{bmatrix} \frac{3}{4}\,e^{-t} + \frac{1}{4}\,e^{3t} & -\frac{1}{4}\,e^{3t} + \frac{1}{4}\,e^{-t} \\ -\frac{3}{4}\,e^{3t} + \frac{3}{4}\,e^{-t} & \frac{1}{4}\,e^{-t} + \frac{3}{4}\,e^{3t} \end{bmatrix} \quad (10)$$

> $SolHom := evalm(MatExp \&* Xcero)$

$$SolHom := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (11)$$

> $MatExpTau := map(rcurry(eval, t = 't - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} \frac{3}{4}\,e^{-t+\tau} + \frac{1}{4}\,e^{3t-3\tau} & -\frac{1}{4}\,e^{3t-3\tau} + \frac{1}{4}\,e^{-t+\tau} \\ -\frac{3}{4}\,e^{3t-3\tau} + \frac{3}{4}\,e^{-t+\tau} & \frac{1}{4}\,e^{-t+\tau} + \frac{3}{4}\,e^{3t-3\tau} \end{bmatrix} \quad (12)$$

> $BBtau := map(rcurry(eval, t = 'tau'), BB)$

$$BBtau := \begin{bmatrix} -1 & 0 \end{bmatrix} \quad (13)$$

> $ProdTau := evalm(MatExpTau \&* BBtau)$

$$ProdTau := \begin{bmatrix} -\frac{3}{4}\,e^{-t+\tau} - \frac{1}{4}\,e^{3t-3\tau} & \frac{3}{4}\,e^{3t-3\tau} - \frac{3}{4}\,e^{-t+\tau} \end{bmatrix} \quad (14)$$

> $SolNoHom := map(int, ProdTau, tau = 0..t)$

$$SolNoHom := \begin{bmatrix} \frac{3}{4}\,e^{-t} - \frac{1}{12}\,e^{3t} - \frac{2}{3} & \frac{3}{4}\,e^{-t} + \frac{1}{4}\,e^{3t} - 1 \end{bmatrix} \quad (15)$$

> $ComprobarUno := map(rcurry(eval, t = '0'), SolNoHom)$

$$ComprobarUno := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (16)$$

> $SolFinal := evalm(SolHom + SolNoHom) : x(t) = SolFinal[1]; y(t) = SolFinal[2]$

$$x(t) = \frac{3}{4}\,e^{-t} - \frac{1}{12}\,e^{3t} - \frac{2}{3}$$

$$y(t) = \frac{3}{4}\,e^{-t} + \frac{1}{4}\,e^{3t} - 1 \quad (17)$$

> $CondicionInicial := x(0) = simplify(eval(subs(t = 0, SolFinal[1]))), y(0) = simplify(eval(subs(t = 0, SolFinal[2])))$

$$CondicionInicial := x(0) = 0, y(0) = 0 \quad (18)$$

>

> $ComprobarDos := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sist[1]) - rhs(Sist[1]) = 0)))$

$$ComprobarDos := 0 = 0 \quad (19)$$

> $ComprobarTres := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sist[2]) - rhs(Sist[2]) = 0)))$

$$ComprobarTres := 0 = 0 \quad (20)$$

> $restart$

3)

> $Ecua := diff(y(t), t\$3) - 2 \cdot diff(y(t), t\$2) + diff(y(t), t) = 0$

(21)

$$Ecua := \frac{d^3}{dt^3} y(t) - 2\left(\frac{d^2}{dt^2} y(t)\right) + \frac{d}{dt} y(t) = 0 \quad (21)$$

> $CondIni := y(0) = 0, D(y)(0) = 1, D(D(y))(0) = -3$

$$CondIni := y(0) = 0, D(y)(0) = 1, D^{(2)}(y)(0) = -3 \quad (22)$$

SOLUCION

> $with(inttrans)$:

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^3\, laplace(y(t), t, s) + 5 - s - 2\, s^2\, laplace(y(t), t, s) + s\, laplace(y(t), t, s) = 0 \quad (23)$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := laplace(y(t), t, s) = \frac{s - 5}{s^3 - 2\, s^2 + s} \quad (24)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = -5 - e^t\,(4\,t - 5) \quad (25)$$

> $CondIniUno := y(0) = simplify(subs(t = 0, rhs(SolPart)))$

$$CondIniUno := y(0) = 0 \quad (26)$$

> $CondIniDos := D(y)(0) = simplify(subs(t = 0, rhs(diff(SolPart, t))))$

$$CondIniDos := D(y)(0) = 1 \quad (27)$$

> $CondIniTres := D(D(y))(0) = simplify(subs(t = 0, rhs(diff(SolPart, t\$2))))$

$$CondIniTres := D^{(2)}(y)(0) = -3 \quad (28)$$

> $ComproUno := simplify(eval(subs(y(t) = rhs(SolPart), Ecua)))$

$$ComproUno := 0 = 0 \quad (29)$$

> $restart$

4)

> $Ecua := diff(y(t), t\$2) + 2 \cdot diff(y(t), t) + y(t) = Dirac(t - 1)$

$$Ecua := \frac{d^2}{dt^2} y(t) + 2\left(\frac{d}{dt} y(t)\right) + y(t) = Dirac(t - 1) \quad (30)$$

> $CondIni := y(0) = 0, D(y)(0) = 0$

$$CondIni := y(0) = 0, D(y)(0) = 0 \quad (31)$$

SOLUCION

> $with(inttrans)$:

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2\, laplace(y(t), t, s) + 2\, s\, laplace(y(t), t, s) + laplace(y(t), t, s) = e^{-s} \quad (32)$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := laplace(y(t), t, s) = \frac{e^{-s}}{s^2 + 2\, s + 1} \quad (33)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = Heaviside(t - 1)\ (t - 1)\ e^{1 - t} \quad (34)$$

> $ComproUno := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComproUno := 0 = 0 \quad (35)$$

> $restart$

5)

> $Ecua := diff(y(t), t\$2) + 4 \cdot y(t) = \sin(t) \cdot Heaviside(t - 2 \cdot Pi)$

$$Ecua := \frac{d^2}{dt^2} y(t) + 4\,y(t) = \sin(t)\ \text{Heaviside}(t - 2\,\pi) \tag{36}$$

> $CondIni := y(0) = 1, D(y)(0) = 0$

$$CondIni := y(0) = 1, D(y)(0) = 0 \tag{37}$$

SOLUCION

> $with(inttrans)$:

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2\ laplace(y(t), t, s) - s + 4\ laplace(y(t), t, s) = \frac{e^{-2\,s\,\pi}}{s^2 + 1} \tag{38}$$

> $SolTL := simplify(isolate(EcuaTL, laplace(y(t), t, s)))$

$$SolTL := laplace(y(t), t, s) = \frac{s^3 + e^{-2\,s\,\pi} + s}{(s^2 + 1)\ (s^2 + 4)} \tag{39}$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \cos(2\,t) + \frac{1}{6}\ (2\sin(t) - \sin(2\,t))\ \text{Heaviside}(t - 2\,\pi) \tag{40}$$

> $ComproUno := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComproUno := 0 = 0 \tag{41}$$

> $CondIniUno := y(0) = simplify(subs(t = 0, rhs(SolPart)))$

$$CondIniUno := y(0) = 1 \tag{42}$$

> $CondIniDos := D(y)(0) = simplify(subs(t = 0, rhs(diff(SolPart, t))))$

$$CondIniDos := D(y)(0) = 0 \tag{43}$$

> $restart$

6)

> $Sist := 2 \cdot diff(x(t), t) + diff(y(t), t) - 2 \cdot x(t) = 1, diff(x(t), t) + diff(y(t), t) - 3 \cdot x(t) - 3 \cdot y(t) = 2 : Sist[1]; Sist[2]$

$$2\left(\frac{d}{dt} x(t)\right) + \frac{d}{dt} y(t) - 2\,x(t) = 1$$

$$\frac{d}{dt} x(t) + \frac{d}{dt} y(t) - 3\,x(t) - 3\,y(t) = 2 \tag{44}$$

> $Sistema := isolate(lhs(Sist[1]) - lhs(Sist[2]) = rhs(Sist[1]) - rhs(Sist[2]), diff(x(t), t)),$
> $isolate(lhs(2 \cdot Sist[2]) - lhs(Sist[1]) = rhs(2 \cdot Sist[2]) - rhs(Sist[1]), diff(y(t), t)) :$
> $Sistema[1]; Sistema[2];$

$$\frac{d}{dt} x(t) = -1 - x(t) - 3\,y(t)$$

$$\frac{d}{dt} y(t) = 3 + 4\,x(t) + 6\,y(t) \tag{45}$$

> $CondIni := x(0) = 0, y(0) = 0$

$$CondIni := x(0) = 0, y(0) = 0 \tag{46}$$

SOLUCION

> $AA := array([[-1, -3], [4, 6]])$

$$AA := \begin{bmatrix} -1 & -3 \\ 4 & 6 \end{bmatrix} \tag{47}$$

```
> Xcero := array([0, 0])
```

$$Xcero := \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{48}$$

```
> BB := array([-1, 3])
```

$$BB := \begin{bmatrix} -1 & 3 \end{bmatrix} \tag{49}$$

```
> with(linalg) :
> MatExp := exponential(AA, t)
```

$$MatExp := \begin{bmatrix} 4\,e^{2t} - 3\,e^{3t} & -3\,e^{3t} + 3\,e^{2t} \\ 4\,e^{3t} - 4\,e^{2t} & -3\,e^{2t} + 4\,e^{3t} \end{bmatrix} \tag{50}$$

```
> SolHom := evalm(MatExp &* Xcero)
```

$$SolHom := \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{51}$$

```
> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)
```

$$MatExpTau := \begin{bmatrix} 4\,e^{2t-2\tau} - 3\,e^{3t-3\tau} & -3\,e^{3t-3\tau} + 3\,e^{2t-2\tau} \\ 4\,e^{3t-3\tau} - 4\,e^{2t-2\tau} & -3\,e^{2t-2\tau} + 4\,e^{3t-3\tau} \end{bmatrix} \tag{52}$$

```
> BBtau := map(rcurry(eval, t='tau'), BB)
```

$$BBtau := \begin{bmatrix} -1 & 3 \end{bmatrix} \tag{53}$$

```
> ProdTau := evalm(MatExpTau &* BBtau)
```

$$ProdTau := \begin{bmatrix} 5\,e^{2t-2\tau} - 6\,e^{3t-3\tau} & 8\,e^{3t-3\tau} - 5\,e^{2t-2\tau} \end{bmatrix} \tag{54}$$

```
> SolNoHom := map(int, ProdTau, tau = 0..t)
```

$$SolNoHom := \begin{bmatrix} \frac{5}{2}\,e^{2t} - 2\,e^{3t} - \frac{1}{2} & \frac{8}{3}\,e^{3t} - \frac{5}{2}\,e^{2t} - \frac{1}{6} \end{bmatrix} \tag{55}$$

```
> ComprobarUno := map(rcurry(eval, t='0'), SolNoHom)
```

$$ComprobarUno := \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{56}$$

```
> SolFinal := evalm(SolHom + SolNoHom) : x(t) = SolFinal[1]; y(t) = SolFinal[2]
```

$$x(t) = \frac{5}{2}\,e^{2t} - 2\,e^{3t} - \frac{1}{2}$$

$$y(t) = \frac{8}{3}\,e^{3t} - \frac{5}{2}\,e^{2t} - \frac{1}{6} \tag{57}$$

```
> CondicionInicial := x(0) = simplify(eval(subs(t = 0, SolFinal[1]))), y(0)
      = simplify(eval(subs(t = 0, SolFinal[2])))
```

$$CondicionInicial := x(0) = 0, y(0) = 0 \tag{58}$$

```
> ComprobarDos := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sist[1])
      - rhs(Sist[1]) = 0)))
```

$$ComprobarDos := 0 = 0 \tag{59}$$

```
> ComprobarTres := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sist[2])
      - rhs(Sist[2]) = 0)))
```

$$ComprobarTres := 0 = 0 \tag{60}$$

```
> ComprobarCuatro := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2],
      lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))
```

(61)

$$ComprobarCuatro := 0 = 0 \quad \textbf{(61)}$$

```
> ComprobarCinco := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sistema[2])
      - rhs(Sistema[2]) = 0)))
```

$$ComprobarCinco := 0 = 0 \quad \textbf{(62)}$$

```
>
>
```