> restart

SERIE 4

>

>

> restart

1)

> $SolEnDerPar := z(x,y) = f(y^2 - x^2) + \left(\frac{\exp(x^2)}{\exp(y^2)} + 2\right)$

$$SolEnDerPar := z(x,y) = f(-x^2 + y^2) + \frac{e^{x^2}}{e^{y^2}} + 2 \quad \textbf{(1)}$$

> $SolHom := z(x,y) = f(y^2 - x^2);\ SolNoHom := z(x,y) = \left(\frac{\exp(x^2)}{\exp(y^2)} + 2\right)$

$$SolHom := z(x,y) = f(-x^2 + y^2)$$

$$SolNoHom := z(x,y) = \frac{e^{x^2}}{e^{y^2}} + 2 \quad \textbf{(2)}$$

> $DerSolHomX := diff(SolHom, x)$

$$DerSolHomX := \frac{\partial}{\partial x} z(x,y) = -2\, \mathrm{D}(f)(-x^2 + y^2)\, x \quad \textbf{(3)}$$

> $DerSolHomY := diff(SolHom, y)$

$$DerSolHomY := \frac{\partial}{\partial y} z(x,y) = 2\, \mathrm{D}(f)(-x^2 + y^2)\, y \quad \textbf{(4)}$$

> $DerUno := isolate(DerSolHomX, \mathrm{D}(f)(-x^2 + y^2))$

$$DerUno := \mathrm{D}(f)(-x^2 + y^2) = -\frac{1}{2}\, \frac{\frac{\partial}{\partial x} z(x,y)}{x} \quad \textbf{(5)}$$

> $DerDos := isolate(DerSolHomY, \mathrm{D}(f)(-x^2 + y^2))$

$$DerDos := \mathrm{D}(f)(-x^2 + y^2) = \frac{1}{2}\, \frac{\frac{\partial}{\partial y} z(x,y)}{y} \quad \textbf{(6)}$$

> $EcuaHomEnDerPar := 2 \cdot rhs(DerDos) - 2 \cdot rhs(DerUno) = 0$

$$EcuaHomEnDerPar := \frac{\frac{\partial}{\partial y} z(x,y)}{y} + \frac{\frac{\partial}{\partial x} z(x,y)}{x} = 0 \quad \textbf{(7)}$$

> $EcuaHomDos := x \cdot \frac{\partial}{\partial y} z(x,y) + y \cdot \frac{\partial}{\partial x} z(x,y) = 0$

$$EcuaHomDos := x \left(\frac{\partial}{\partial y} z(x,y)\right) + y \left(\frac{\partial}{\partial x} z(x,y)\right) = 0 \quad \textbf{(8)}$$

> $Comprobar := pdsolve(EcuaHomEnDerPar)$

$$Comprobar := z(x,y) = \_F1(-x^2 + y^2) \quad \textbf{(9)}$$

> $ComprobarDos := pdsolve(EcuaHomDos)$

$$ComprobarDos := z(x,y) = \_F1(-x^2 + y^2) \quad \textbf{(10)}$$

> $FunNoHom := subs(z(x, y) = rhs(SolNoHom), lhs(EcuaHomDos))$

$$FunNoHom := x\left(\frac{\partial}{\partial y}\left(\frac{e^{x^2}}{e^{y^2}}+2\right)\right)+y\left(\frac{\partial}{\partial x}\left(\frac{e^{x^2}}{e^{y^2}}+2\right)\right) \quad \textbf{(11)}$$

> $EcuaFinalEnDerPar := x\cdot\frac{\partial}{\partial y}z(x, y) + y\cdot\frac{\partial}{\partial x}z(x, y) = \left(x\cdot diff\left(\left(\frac{e^{x^2}}{e^{y^2}}+2\right), y\right)\right) + \left(y \cdot diff\left(\left(\frac{e^{x^2}}{e^{y^2}}+2\right), x\right)\right)$

$$EcuaFinalEnDerPar := x\left(\frac{\partial}{\partial y}z(x, y)\right)+y\left(\frac{\partial}{\partial x}z(x, y)\right)=0 \quad \textbf{(12)}$$

> $pdsolve(EcuaFinalEnDerPar)$

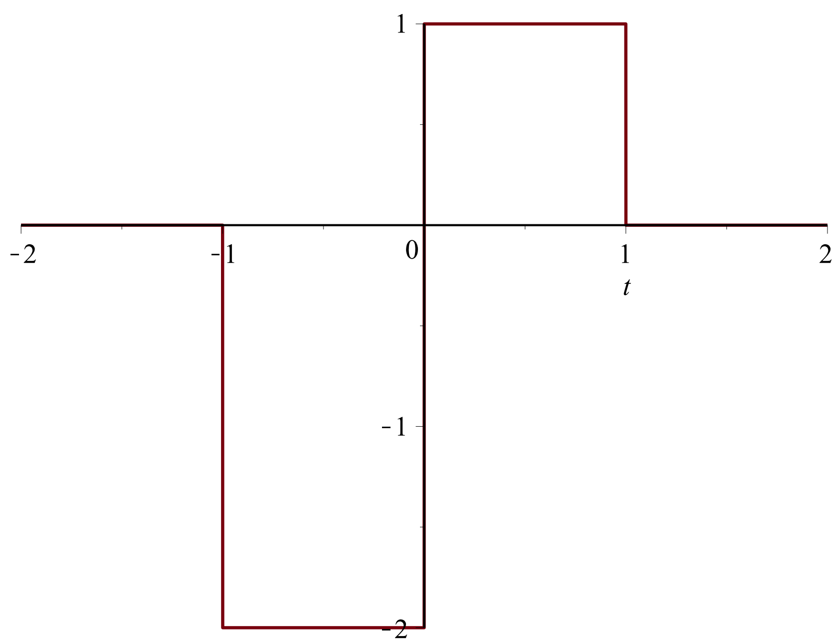
$$z(x, y) = \_F1(-x^2+y^2) \quad \textbf{(13)}$$

> $restart$

2

> $f := -2\cdot \text{Heaviside}(t+1) + 3\cdot \text{Heaviside}(t) - \text{Heaviside}(t-1); plot(f, t=-2..2, scaling = CONSTRAINED)$

$$f := -2\ \text{Heaviside}(t+1) + 3\ \text{Heaviside}(t) - \text{Heaviside}(t-1)$$



> $L := 2$

$$L := 2 \tag{14}$$

> $a[0] := \frac{1}{L} \cdot int(f, t = -L..L)$

$$a_0 := -\frac{1}{2} \tag{15}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$a_n := -\frac{\sin\left(\frac{1}{2} n \pi\right)}{n \pi} \tag{16}$$

> $b[n] := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

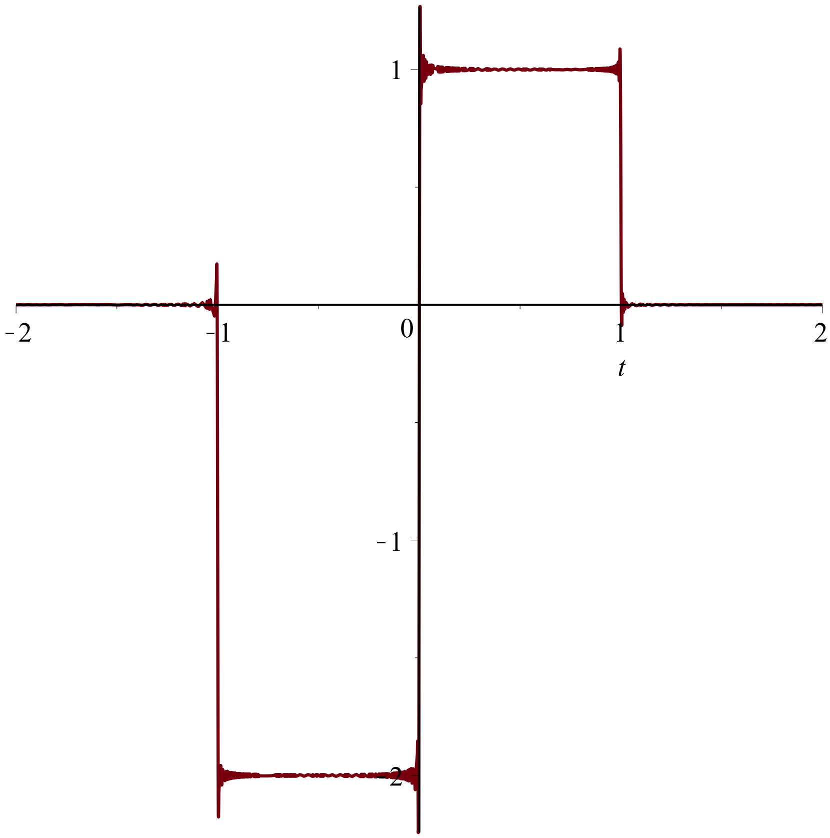
$$b_n := -\frac{3 \cos\left(\frac{1}{2} n \pi\right)}{n \pi} + \frac{3}{n \pi} \tag{17}$$

> $STF := \frac{a[0]}{2} + Sum\left(\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right)\right), n = 1..infinity\right)$

$$STF := -\frac{1}{4} + \sum_{n=1}^{\infty} \left(-\frac{\sin\left(\frac{1}{2} n \pi\right) \cos\left(\frac{1}{2} n \pi t\right)}{n \pi} + \left(-\frac{3 \cos\left(\frac{1}{2} n \pi\right)}{n \pi} + \frac{3}{n \pi} \right) \sin\left(\frac{1}{2} n \pi t\right) \right) \tag{18}$$

> $STF500 := \frac{a[0]}{2} + sum\left(\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right)\right), n = 1..500\right)$:

> $plot(STF500, t = -L..\mathrm{L})$



> restart

3)

> $Ecua := diff(u(x, t), x\$2) + diff(u(x, t), t, x) = 4 \cdot t \cdot diff(u(x, t), x)$

$$Ecua := \frac{\partial^2}{\partial x^2} u(x, t) + \frac{\partial^2}{\partial x\, \partial t} u(x, t) = 4\, t \left(\frac{\partial}{\partial x} u(x, t)\right) \quad \textbf{(19)}$$

> $EcuaDos := eval(subs(u(x, t) = F(x) \cdot G(t), Ecua))$

$$EcuaDos := \left(\frac{d^2}{dx^2} F(x)\right) G(t) + \left(\frac{d}{dx} F(x)\right) \left(\frac{d}{dt} G(t)\right) = 4\, t \left(\frac{d}{dx} F(x)\right) G(t) \quad \textbf{(20)}$$

> $EcuaTres := lhs(EcuaDos) - \left(\frac{d}{dx} F(x)\right) \left(\frac{d}{dt} G(t)\right) = rhs(EcuaDos) - \left(\frac{d}{dx} F(x)\right) \left(\frac{d}{dt} G(t)\right)$

$$EcuaTres := \left(\frac{d^2}{dx^2} F(x)\right) G(t) = 4\, t \left(\frac{d}{dx} F(x)\right) G(t) - \left(\frac{d}{dx} F(x)\right) \left(\frac{d}{dt} G(t)\right) \quad \textbf{(21)}$$

> $EcuaCuatro := \frac{lhs(EcuaTres)}{\left(\frac{d}{dx} F(x)\right) G(t)} = simplify\left(\frac{rhs(EcuaTres)}{\left(\frac{d}{dx} F(x)\right) G(t)}\right)$

$$EcuaCuatro := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \frac{4\, G(t)\, t - \left(\frac{d}{dt} G(t)\right)}{G(t)} \qquad \textbf{(22)}$$

> $EcuaX := lhs(EcuaCuatro) = \beta^2$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \beta^2 \qquad \textbf{(23)}$$

> $EcuaT := rhs(EcuaCuatro) = \beta^2$

$$EcuaT := \frac{4\, G(t)\, t - \left(\frac{d}{dt} G(t)\right)}{G(t)} = \beta^2 \qquad \textbf{(24)}$$

> $EcuaXdos := lhs(EcuaX) \cdot \frac{d}{dx} F(x) = rhs(EcuaX) \cdot \frac{d}{dx} F(x)$

$$EcuaXdos := \frac{d^2}{dx^2} F(x) = \beta^2 \left(\frac{d}{dx} F(x)\right) \qquad \textbf{(25)}$$

> $EcuaXtres := lhs(EcuaXdos) - rhs(EcuaXdos) = 0$

$$EcuaXtres := \frac{d^2}{dx^2} F(x) - \beta^2 \left(\frac{d}{dx} F(x)\right) = 0 \qquad \textbf{(26)}$$

> $EcuaTdos := lhs(EcuaT) \cdot G(t) = rhs(EcuaT) \cdot G(t)$

$$EcuaTdos := 4\, G(t)\, t - \left(\frac{d}{dt} G(t)\right) = \beta^2\, G(t) \qquad \textbf{(27)}$$

> $EcuaTtres := -lhs(EcuaTdos) + rhs(EcuaTdos) = 0$

$$EcuaTtres := -4\, G(t)\, t + \frac{d}{dt} G(t) + \beta^2\, G(t) = 0 \qquad \textbf{(28)}$$

> $SolX := dsolve(EcuaXtres)$

$$SolX := F(x) = \_C1 + \_C2\, e^{\beta^2 x} \qquad \textbf{(29)}$$

> $SolT := dsolve(EcuaTtres)$

$$SolT := G(t) = \_C1\, e^{-t\,(\beta^2 - 2\,t)} \qquad \textbf{(30)}$$

> $SolGral := u(x, t) = rhs(SolX) \cdot subs(\_C1 = 1, rhs(SolT))$

$$SolGral := u(x, t) = \left(\_C1 + \_C2\, e^{\beta^2 x}\right) e^{-t\,(\beta^2 - 2\,t)} \qquad \textbf{(31)}$$

> $Ecua$

$$\frac{\partial^2}{\partial x^2} u(x, t) + \frac{\partial^2}{\partial x\, \partial t} u(x, t) = 4\, t \left(\frac{\partial}{\partial x} u(x, t)\right) \qquad \textbf{(32)}$$

> $Comprobar := simplify(eval(subs(u(x, t) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \qquad \textbf{(33)}$$

> $restart$

4)

> $Ecua := diff(u(x,t),x) = \frac{5}{x} \cdot diff(u(x,t),t)$

$$Ecua := \frac{\partial}{\partial x} u(x,t) = \frac{5 \left(\frac{\partial}{\partial t} u(x,t)\right)}{x} \quad \textbf{(34)}$$

> alpha := 3

$$\alpha := 3 \quad \textbf{(35)}$$

> $EcuaDos := eval(subs(u(x,t) = F(x) \cdot G(t), Ecua))$

$$EcuaDos := \left(\frac{d}{dx} F(x)\right) G(t) = \frac{5\, F(x) \left(\frac{d}{dt} G(t)\right)}{x} \quad \textbf{(36)}$$

> $EcuaTres := \frac{lhs(EcuaDos)}{\left(\frac{5 \cdot F(x)}{x}\right) \cdot G(t)} = \frac{rhs(EcuaDos)}{\left(\frac{5 \cdot F(x)}{x}\right) \cdot G(t)}$

$$EcuaTres := \frac{1}{5} \frac{\left(\frac{d}{dx} F(x)\right) x}{F(x)} = \frac{\frac{d}{dt} G(t)}{G(t)} \quad \textbf{(37)}$$

> $EcuaX := lhs(EcuaTres) =$ alpha

$$EcuaX := \frac{1}{5} \frac{\left(\frac{d}{dx} F(x)\right) x}{F(x)} = 3 \quad \textbf{(38)}$$

> $EcuaT := rhs(EcuaTres) =$ alpha

$$EcuaT := \frac{\frac{d}{dt} G(t)}{G(t)} = 3 \quad \textbf{(39)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1\, x^{15} \quad \textbf{(40)}$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = \_C1\, e^{3t} \quad \textbf{(41)}$$

> $SolGral := u(x,t) = rhs(SolX) \cdot subs(\_C1 = 1, rhs(SolT))$

$$SolGral := u(x,t) = \_C1\, x^{15}\, e^{3t} \quad \textbf{(42)}$$

> $Ecua$

$$\frac{\partial}{\partial x} u(x,t) = \frac{5 \left(\frac{\partial}{\partial t} u(x,t)\right)}{x} \quad \textbf{(43)}$$

> $Comprobar := eval(subs(u(x,t) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0))$

$$Comprobar := 0 = 0 \quad \textbf{(44)}$$

> $restart$

5)

> $Ecua := diff(u(x,t),t) = 2 \cdot diff(u(x,t), x\$2)$

$$Ecua := \frac{\partial}{\partial t} u(x,t) = 2 \left(\frac{\partial^2}{\partial x^2} u(x,t)\right) \quad \textbf{(45)}$$

> alpha := β^2

$$\alpha := \beta^2 \quad (46)$$

> $EcuaDos := eval(subs(u(x,t) = F(x)\cdot G(t), Ecua))$

$$EcuaDos := F(x)\left(\frac{d}{dt}G(t)\right) = 2\left(\frac{d^2}{dx^2}F(x)\right)G(t) \quad (47)$$

> $EcuaTres := \frac{lhs(EcuaDos)}{F(x)\cdot G(t)} = \frac{rhs(EcuaDos)}{F(x)\cdot G(t)}$

$$EcuaTres := \frac{\frac{d}{dt}G(t)}{G(t)} = \frac{2\left(\frac{d^2}{dx^2}F(x)\right)}{F(x)} \quad (48)$$

> $EcuaX := rhs(EcuaTres) = \text{alpha}$

$$EcuaX := \frac{2\left(\frac{d^2}{dx^2}F(x)\right)}{F(x)} = \beta^2 \quad (49)$$

> $EcuaT := lhs(EcuaTres) = \text{alpha}$

$$EcuaT := \frac{\frac{d}{dt}G(t)}{G(t)} = \beta^2 \quad (50)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1\, e^{\frac{1}{2}\sqrt{2}\,\beta x} + \_C2\, e^{-\frac{1}{2}\sqrt{2}\,\beta x} \quad (51)$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = \_C1\, e^{\beta^2 t} \quad (52)$$

> $SolGral := u(x,t) = rhs(SolX)\cdot subs(\_C1 = 1, rhs(SolT))$

$$SolGral := u(x,t) = \left(\_C1\, e^{\frac{1}{2}\sqrt{2}\,\beta x} + \_C2\, e^{-\frac{1}{2}\sqrt{2}\,\beta x}\right) e^{\beta^2 t} \quad (53)$$

> $Ecua$

$$\frac{\partial}{\partial t}u(x,t) = 2\left(\frac{\partial^2}{\partial x^2}u(x,t)\right) \quad (54)$$

> $Comprobar := simplify(eval(subs(u(x,t) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (55)$$

> $restart$

6)

> $SolGral := u(x,y) = \_F1(y) + \_F2\left(-\frac{2\cdot x}{5} + y^2\right)$

$$SolGral := u(x,y) = \_F1(y) + \_F2\left(-\frac{2}{5}x + y^2\right) \quad (56)$$

> $DerSolY := diff(SolGral, y)$

$$DerSolY := \frac{\partial}{\partial y}u(x,y) = \frac{d}{dy}\_F1(y) + 2\,D(\_F2)\left(-\frac{2}{5}x + y^2\right)y \quad (57)$$

> $DerSolYY := diff(SolGral, y\$2)$

$$DerSolYY := \frac{\partial^2}{\partial y^2}u(x,y) = \frac{d^2}{dy^2}\_F1(y) + 4\,D^{(2)}(\_F2)\left(-\frac{2}{5}x + y^2\right)y^2 + 2\,D(\_F2)\left(-\frac{2}{5}x \quad (58)$$

$+ y^2\Big)$

> DerSolX := diff(SolGral, x)

$$DerSolX := \frac{\partial}{\partial x} u(x, y) = -\frac{2}{5} \mathrm{D}(\_F2)\left(-\frac{2}{5} x + y^2\right) \quad (59)$$

> DerSolXX := diff(SolGral, x$2)

$$DerSolXX := \frac{\partial^2}{\partial x^2} u(x, y) = \frac{4}{25} \mathrm{D}^{(2)}(\_F2)\left(-\frac{2}{5} x + y^2\right) \quad (60)$$

> DerSolXY := diff(SolGral, x, y)

$$DerSolXY := \frac{\partial^2}{\partial y\, \partial x} u(x, y) = -\frac{4}{5} \mathrm{D}^{(2)}(\_F2)\left(-\frac{2}{5} x + y^2\right) y \quad (61)$$

> $DerSolXdos := rhs(DerSolX) \cdot \left(-\frac{5}{2}\right) = lhs(DerSolX) \cdot \left(-\frac{5}{2}\right)$

$$DerSolXdos := \mathrm{D}(\_F2)\left(-\frac{2}{5} x + y^2\right) = -\frac{5}{2} \frac{\partial}{\partial x} u(x, y) \quad (62)$$

> $DerSolXXdos := rhs(DerSolXX) \cdot \left(\frac{25}{4}\right) = lhs(DerSolXX) \cdot \left(\frac{25}{4}\right)$

$$DerSolXXdos := \mathrm{D}^{(2)}(\_F2)\left(-\frac{2}{5} x + y^2\right) = \frac{25}{4} \frac{\partial^2}{\partial x^2} u(x, y) \quad (63)$$

> $Ecua := lhs(DerSolYY) = 4 \cdot y^2 \cdot \frac{25}{4} \frac{\partial^2}{\partial x^2} u(x, y) + 2 \cdot \left(-\frac{5}{2} \frac{\partial}{\partial x} u(x, y)\right)$

$$Ecua := \frac{\partial^2}{\partial y^2} u(x, y) = 25\, y^2 \left(\frac{\partial^2}{\partial x^2} u(x, y)\right) - 5 \left(\frac{\partial}{\partial x} u(x, y)\right) \quad (64)$$

> restart

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