>

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
PRIMER EXAMEN PARCIAL
SEMESTRE 2023-2

25 ABRIL 2023

> *restart*

1) (20/100)
OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL DE PRIMER ORDEN **(SIN UTILIZAR dsolve)**

> $\frac{d}{dt}x(t)+x(t)\cdot\cos(t)=\sin(t)\cdot\cos(t)$

$$\frac{d}{dt}x(t)+x(t)\cos(t)=\sin(t)\cos(t) \quad \textbf{(1)}$$

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INICIA RESPUESTA 1)

> $Ecuacion := \frac{d}{dt}x(t)+x(t)\cos(t)=\sin(t)\cos(t)$

$$Ecuacion := \frac{d}{dt}x(t)+x(t)\cos(t)=\sin(t)\cos(t) \quad \textbf{(2)}$$

> $p := \cos(t); q := rhs(Ecuacion)$

$$p := \cos(t)$$
$$q := \sin(t)\cos(t) \quad \textbf{(3)}$$

> $IntP := int(p, t)$

$$IntP := \sin(t) \quad \textbf{(4)}$$

> $IntPneg := int(-p, t)$

$$IntPneg := -\sin(t) \quad \textbf{(5)}$$

> $SolucionGeneral := x(t) = expand\left(C_1\cdot\exp(IntPneg)+\exp(IntPneg)\cdot int(\exp(IntP)\cdot q, t)\right)$

$$SolucionGeneral := x(t)=\frac{C_1}{e^{\sin(t)}}+\sin(t)-1 \quad \textbf{(6)}$$

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comprobacion

> $SolGral := dsolve(Ecuacion)$

$$SolGral := x(t)=\sin(t)-1+e^{-\sin(t)}\_C1 \quad \textbf{(7)}$$

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FIN RESPUESTA 1)

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2) (30/100)
OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL DE PRIMER ORDEN **(SIN UTILIZAR dsolve)**

> $2\,x\,y(x)-\left(3\,x^2-y(x)^2\right)\left(\frac{d}{dx}y(x)\right)=0$

$$2\,x\,y(x)-\left(3\,x^2-y(x)^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right)=0 \quad \textbf{(8)}$$

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INICIA RESPUESTA 2)

> $Ecua := 2\,x\,y(x)-\left(3\,x^2-y(x)^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right)=0$

$$Ecua := 2\,x\,y(x)-\left(3\,x^2-y(x)^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right)=0 \quad \textbf{(9)}$$

> *with(DEtools) :*

> *odeadvisor(Ecua)*

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_dAlembert] \quad \textbf{(10)}$$

> *EcuaDos := simplify(isolate(eval(subs(y(x) = u(x)·x, Ecua)), diff(u(x), x)))*

$$EcuaDos := \frac{\mathrm{d}}{\mathrm{d}x}\,u(x) = -\frac{u(x)\left(u(x)^2-1\right)}{x\left(u(x)^2-3\right)} \quad \textbf{(11)}$$

> *odeadvisor(EcuaDos)*

$$[\_separable] \quad \textbf{(12)}$$

> $P := -\frac{u\left(u^2-1\right)}{\left(u^2-3\right)}$

$$P := -\frac{u\left(u^2-1\right)}{u^2-3} \quad \textbf{(13)}$$

> $R := x$

$$R := x \quad \textbf{(14)}$$

> $SolUno := int\left(\frac{1}{P}, u\right) = int\left(\frac{1}{R}, x\right) + \_C1$

$$SolUno := -3\ln(u)+\ln(u-1)+\ln(u+1)=\ln(x)+\_C1 \quad \textbf{(15)}$$

> $SolDos := isolate\left(simplify\left(subs\left(u=\frac{y(x)}{x}, SolUno\right)\right), \_C1\right)$

$$SolDos := \_C1 = -3\ln\left(\frac{y(x)}{x}\right)+\ln\left(\frac{y(x)-x}{x}\right)+\ln\left(\frac{y(x)+x}{x}\right)-\ln(x) \quad \textbf{(16)}$$

> *SolGral := simplify(exp(rhs(SolDos))) = _C1*

$$SolGral := \frac{y(x)^2-x^2}{y(x)^3} = \_C1 \quad \textbf{(17)}$$

> *DerSolGral := isolate(diff(SolGral, x), diff(y(x), x))*

$$DerSolGral := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = -\frac{2\,x\,y(x)}{-3\,x^2+y(x)^2} \quad \textbf{(18)}$$

> *DerEcua := isolate(Ecua, diff(y(x), x))*

$$DerEcua := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = -\frac{2\,x\,y(x)}{-3\,x^2+y(x)^2} \quad \textbf{(19)}$$

> *Comprobacion := rhs(DerEcua) − rhs(DerSolGral) = 0*

$$Comprobacion := 0 = 0 \quad \textbf{(20)}$$

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FIN RESPUESTA 2)

> *restart*

3) (20/100)
OBTENER LA SOLUCIÓN PARTICULAR DEL SIGUIENTE PROBLEMA DE UNA ECUACIÓN DIFERENCIAL HOMOGENEA CON CONDICIONES INICIALES **(SIN UTILIZAR dsolve)**

> $\frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x)\right) + 3 y(x) = 0$

$$\frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x)\right) + 3 y(x) = 0 \quad \textbf{(21)}$$

> $y(0) = 6, D(y)(0) = 10$

$$y(0) = 6, D(y)(0) = 10 \quad \textbf{(22)}$$

>

INICIA RESPUESTA 2)

> $Ecua := \frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x)\right) + 3 y(x) = 0$

$$Ecua := \frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x)\right) + 3 y(x) = 0 \quad \textbf{(23)}$$

> $CondIni := y(0) = 6, D(y)(0) = 10$

$$CondIni := y(0) = 6, D(y)(0) = 10 \quad \textbf{(24)}$$

> $EcuaCarac := m^2 - 4 m + 3 = 0$

$$EcuaCarac := m^2 - 4 m + 3 = 0 \quad \textbf{(25)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 3, 1 \quad \textbf{(26)}$$

> $SolGralHom := y(x) = \_C1 \cdot \exp(Raiz[1] \cdot x) + \_C2 \cdot \exp(Raiz[2] \cdot x)$

$$SolGralHom := y(x) = \_C1\, e^{3x} + \_C2\, e^{x} \quad \textbf{(27)}$$

> $EcuaUno := eval(subs(x = 0, rhs(SolGralHom) = 6))$

$$EcuaUno := \_C1 + \_C2 = 6 \quad \textbf{(28)}$$

> $EcuaDos := eval(subs(x = 0, rhs(diff(SolGralHom, x)) = 10))$

$$EcuaDos := 3\ \_C1 + \_C2 = 10 \quad \textbf{(29)}$$

> $Para := solve([EcuaUno, EcuaDos])$

$$Para := \{\_C1 = 2, \_C2 = 4\} \quad \textbf{(30)}$$

> $SolPart := subs(Para, SolGralHom)$

$$SolPart := y(x) = 2\, e^{3x} + 4\, e^{x} \quad \textbf{(31)}$$

> $Comprobacion := eval(subs(y(x) = rhs(SolPart), Ecua))$

$$Comprobacion := 0 = 0 \quad \textbf{(32)}$$

> $CondIni$

$$y(0) = 6, D(y)(0) = 10 \quad \textbf{(33)}$$

> $CompUno := simplify(subs(x = 0, SolPart))$

$$CompUno := y(0) = 6 \quad \textbf{(34)}$$

> $CompDos := D(y)(0) = simplify(subs(x=0, rhs(diff(SolPart, x))))$

$$CompDos := D(y)(0) = 10 \quad (35)$$

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FIN RESPUESTA 3)

> $restart$

4) (30/100)
DADA LA SIGUIENTE **SOLUCIÓN GENERAL** DE UNA ECUACIÓN DIFERENCIAL DESCONOCIDA

> $y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) + 5\cos(2x) + \sin(2x)$

$$y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) + 5\cos(2x) + \sin(2x) \quad (36)$$

a) OBTENGA LA **SOLUCIÓN PARTICULAR** DADAS LAS CONDICIONES DE FRONTERA SIGUIENTES **(15 puntos)**

> $Condicion := y(0) = 4, y\left(\frac{Pi}{4}\right) = 4 : Condicion_1; Condicion_2$

$$y(0) = 4$$

$$y\left(\frac{1}{4}\pi\right) = 4 \quad (37)$$

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INICIA RESPUESTAS 4)

> $SolucionGeneral := y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) + 5\cos(2x) + \sin(2x)$

$$SolucionGeneral := y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) + 5\cos(2x) + \sin(2x) \quad (38)$$

> $Condicion := y(0) = 4, y\left(\frac{Pi}{4}\right) = 4 : Condicion_1; Condicion_2$

$$y(0) = 4$$

$$y\left(\frac{1}{4}\pi\right) = 4 \quad (39)$$

> $Sistema := eval(subs(x=0, rhs(SolucionGeneral) = rhs(Condicion_1))), eval\left(subs\left(x = \frac{Pi}{4}, rhs(SolucionGeneral) = rhs(Condicion_2)\right)\right) : Sistema_1; Sistema_2$

$$C_1 + 5 = 4$$

$$1 + C_2 e^{-\frac{1}{2}\pi} = 4 \quad (40)$$

> $Parametro := simplify(solve(\{Sistema\}, \{C_1, C_2\})) : Parametro_1; Parametro_2$

$$C_1 = -1$$

$$C_2 = 3 e^{\frac{1}{2}\pi} \quad (41)$$

> $SolucionParticular := subs(C_1 = rhs(Parametro_1), C_2 = rhs(Parametro_2), SolucionGeneral)$

$$SolucionParticular := y(x) = -e^{-2x}\cos(2x) + 3 e^{\frac{1}{2}\pi} e^{-2x}\sin(2x) + 5\cos(2x) + \sin(2x) \quad (42)$$

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b) OBTENGA SU **ECUACIÓN DIFERENCIAL ORDINARIA LINEAL** CORRESPONDIENTE Y **CLASIFÍQUELA** (por tipo de coeficientes y tipo de homogeneidad). **(15 puntos)**

> *SolucionGeneral*

$$y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) + 5\cos(2x) + \sin(2x) \quad \textbf{(43)}$$

> $SolucionHomogenea := y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x)$

$$SolucionHomogenea := y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) \quad \textbf{(44)}$$

> $SolucionParticular := y(x) = 5\cos(2x) + \sin(2x)$

$$SolucionParticular := y(x) = 5\cos(2x) + \sin(2x) \quad \textbf{(45)}$$

> $EcuacionCaracteristica := expand((m - (-2 + 2\cdot I))\cdot(m - (-2 - 2\cdot I))) = 0$

$$EcuacionCaracteristica := m^2 + 4m + 8 = 0 \quad \textbf{(46)}$$

> $EcuacionHomogenea := y'' + 4\cdot y' + 8\cdot y = 0$

$$EcuacionHomogenea := \frac{d^2}{dx^2} y(x) + 4\left(\frac{d}{dx} y(x)\right) + 8\, y(x) = 0 \quad \textbf{(47)}$$

> $Q := eval(subs(y(x) = rhs(SolucionParticular), lhs(EcuacionHomogenea)))$

$$Q := 28\cos(2x) - 36\sin(2x) \quad \textbf{(48)}$$

> $EcuacionFinal := lhs(EcuacionHomogenea) = Q$

$$EcuacionFinal := \frac{d^2}{dx^2} y(x) + 4\left(\frac{d}{dx} y(x)\right) + 8\, y(x) = 28\cos(2x) - 36\sin(2x) \quad \textbf{(49)}$$

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comprobacion

> $SolGral := simplify(dsolve(EcuacionFinal))$

$$SolGral := y(x) = e^{-2x} \sin(2x)\ \_C2 + e^{-2x} \cos(2x)\ \_C1 + 5\cos(2x) + \sin(2x) \quad \textbf{(50)}$$

> *SolucionGeneral*

$$y(x) = C_1 e^{-2x} \cos(2x) + C_2 e^{-2x} \sin(2x) + 5\cos(2x) + \sin(2x) \quad \textbf{(51)}$$

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FIN RESPUESTAS 4)

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> *restart*

FIN DEL EXAMEN

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