>

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN PARCIAL (TEMAS 3 Y 4)
SEMESTRE 2023-2

2023 JUNIO 08

> *restart*

1) (20/100 puntos) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE, OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN CON LAS CONDICIONES INICIALES, DADAS **(sin usar dsolve)**

$$\frac{d^2}{dt^2}\,y(t)+4\,y(t)=64\,(t-2)\,\mathrm{Heaviside}(t-2)\,\sin(2\,t-4)$$

$$y(0)=2$$

$$\mathrm{D}(y)(0)=0 \qquad (1)$$

> *restart*

RESPUESTA 1

> $Ecuacion := \frac{d^2}{dt^2}\,y(t)+4\,y(t)=64\,(t-2)\,\mathrm{Heaviside}(t-2)\,\sin(2\,t-4);\ Condiciones := y(0)=2,\ \mathrm{D}(y)(0)=0$

$$Ecuacion := \frac{d^2}{dt^2}\,y(t)+4\,y(t)=64\,(t-2)\,\mathrm{Heaviside}(t-2)\,\sin(2\,t-4)$$

$$Condiciones := y(0)=2,\ \mathrm{D}(y)(0)=0 \qquad (2)$$

> *with(inttrans) :*

> *TransLapEcuacion := simplify(subs(Condiciones, laplace(Ecuacion, t, s)))*

$$TransLapEcuacion := s^2\,laplace(y(t),t,s)-2\,s+4\,laplace(y(t),t,s)=\frac{256\,e^{-2\,s}\,s}{(s^2+4)^2} \qquad (3)$$

> *TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))*

$$TransLapSolucion := laplace(y(t),t,s)=\frac{2\,s\,(128\,e^{-2\,s}+s^4+8\,s^2+16)}{(s^2+4)^3} \qquad (4)$$

> *SolucionParticular := invlaplace(TransLapSolucion, s, t)*

$$SolucionParticular := y(t)=2\cos(2\,t)+4\,(t-2)\,(\sin(2\,t-4)-2\cos(2\,t-4)\,(t-2))\,\mathrm{Heaviside}(t-2) \qquad (5)$$

>

FIN RESPUESTA 1)

> *restart*

2) (30/100 puntos) UTILIZANDO MATRIZ EXPONENCIAL OBTENER LA SOLUCIÓN PARTICULAR DEL SISTEMA DE ECUACIONES DIFERENCIALES CON LAS CONDICIONES INICIALES DADAS **(sin usar dsolve)**

$$\frac{d}{dt}\,x(t) = -x(t) - 2\,y(t) + e^{t}$$
$$\frac{d}{dt}\,y(t) = -2\,x(t) - y(t) \quad \textbf{(6)}$$

$$x(0) = 4,\, y(0) = 0 \quad \textbf{(7)}$$

> *restart*

RESPUESTA 2)

> *Sistema* := *diff*(*x*(*t*), *t*) =−*x*(*t*) − 2·*y*(*t*) + exp(*t*), *diff*(*y*(*t*), *t*) =−2·*x*(*t*) − *y*(*t*) : *Sistema*[1]; *Sistema*[2]

$$\frac{d}{dt}\,x(t) = -x(t) - 2\,y(t) + e^{t}$$
$$\frac{d}{dt}\,y(t) = -2\,x(t) - y(t) \quad \textbf{(8)}$$

> *CondIni* := *x*(0) =4, *y*(0) =0

$$CondIni := x(0) = 4,\, y(0) = 0 \quad \textbf{(9)}$$

>

RESPUESTA 2)

> *AA* := *array*([[-1,-2], [-2,-1]])

$$AA := \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} \quad \textbf{(10)}$$

> *Xcero* := *array*([4, 0])

$$Xcero := \begin{bmatrix} 4 & 0 \end{bmatrix} \quad \textbf{(11)}$$

> *BB* := *array*([exp(*t*), 0])

$$BB := \begin{bmatrix} e^{t} & 0 \end{bmatrix} \quad \textbf{(12)}$$

> *with*(*linalg*) :

> *MatExp* := *exponential*(*AA*, *t*)

$$MatExp := \begin{bmatrix} \frac{1}{2}\,e^{-3\,t} + \frac{1}{2}\,e^{t} & -\frac{1}{2}\,e^{t} + \frac{1}{2}\,e^{-3\,t} \\ -\frac{1}{2}\,e^{t} + \frac{1}{2}\,e^{-3\,t} & \frac{1}{2}\,e^{-3\,t} + \frac{1}{2}\,e^{t} \end{bmatrix} \quad \textbf{(13)}$$

> *SolHom* := *evalm*(*MatExp* &* *Xcero*)

$$SolHom := \begin{bmatrix} 2\,e^{-3\,t} + 2\,e^{t} & -2\,e^{t} + 2\,e^{-3\,t} \end{bmatrix} \quad \textbf{(14)}$$

> *MatExpTau* := *map*(*rcurry*(*eval*, *t* ='*t* − tau'), *MatExp*)

$$MatExpTau := \begin{bmatrix} \frac{1}{2}\,e^{-3\,t+3\,\tau} + \frac{1}{2}\,e^{t-\tau} & -\frac{1}{2}\,e^{t-\tau} + \frac{1}{2}\,e^{-3\,t+3\,\tau} \\ -\frac{1}{2}\,e^{t-\tau} + \frac{1}{2}\,e^{-3\,t+3\,\tau} & \frac{1}{2}\,e^{-3\,t+3\,\tau} + \frac{1}{2}\,e^{t-\tau} \end{bmatrix} \quad \textbf{(15)}$$

> *BBtau* := *map*(*rcurry*(*eval*, *t* ='tau'), *BB*)

$$BBtau := \begin{bmatrix} e^{\tau} & 0 \end{bmatrix} \quad (16)$$

> *ProdTau* := *simplify*(*evalm*(*MatExpTau* &* *BBtau*))

$$ProdTau := \begin{bmatrix} \frac{1}{2}\left(e^{-3t+3\tau} + e^{t-\tau}\right) e^{\tau} & \frac{1}{2}\left(-e^{t-\tau} + e^{-3t+3\tau}\right) e^{\tau} \end{bmatrix} \quad (17)$$

> *SolNoHom* := *map*(*int*, *ProdTau*, tau = 0 ..*t*)

$$SolNoHom := \begin{bmatrix} \frac{1}{8}\left(4\, t\, e^{4t} + e^{4t} - 1\right) e^{-3t} & -\frac{1}{8}\left(4\, t\, e^{4t} - e^{4t} + 1\right) e^{-3t} \end{bmatrix} \quad (18)$$

> *ComprobarUno* := *map*(*rcurry*(*eval*, *t* ='0'), *SolNoHom*)

$$ComprobarUno := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (19)$$

> *SolFinal* := *evalm*(*SolHom* + *SolNoHom*) : *x*(*t*) = *SolFinal*[1]; *y*(*t*) = *SolFinal*[2]

$$x(t) = 2\, e^{-3t} + 2\, e^{t} + \frac{1}{8}\left(4\, t\, e^{4t} + e^{4t} - 1\right) e^{-3t}$$

$$y(t) = -2\, e^{t} + 2\, e^{-3t} - \frac{1}{8}\left(4\, t\, e^{4t} - e^{4t} + 1\right) e^{-3t} \quad (20)$$

> *CondicionInicial* := *x*(0) = *simplify*(*eval*(*subs*(*t* = 0, *SolFinal*[1]))), *y*(0) = *simplify*(*eval*(*subs*(*t* = 0, *SolFinal*[2])))

$$CondicionInicial := x(0) = 4,\ y(0) = 0 \quad (21)$$

> *ComprobarDos* := *simplify*(*eval*(*subs*(*x*(*t*) = *SolFinal*[1], *y*(*t*) = *SolFinal*[2], *lhs*(*Sistema*[1]) − *rhs*(*Sistema*[1]) = 0)))

$$ComprobarDos := 0 = 0 \quad (22)$$

> *ComprobarTres* := *simplify*(*eval*(*subs*(*x*(*t*) = *SolFinal*[1], *y*(*t*) = *SolFinal*[2], *lhs*(*Sistema*[2]) − *rhs*(*Sistema*[2]) = 0)))

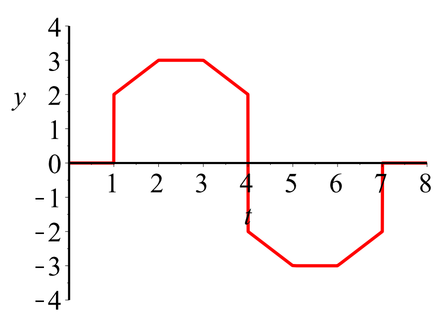
$$ComprobarTres := 0 = 0 \quad (23)$$

>

FIN RESPUESTA 2)

> *restart*

3) (20/100 puntos) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:

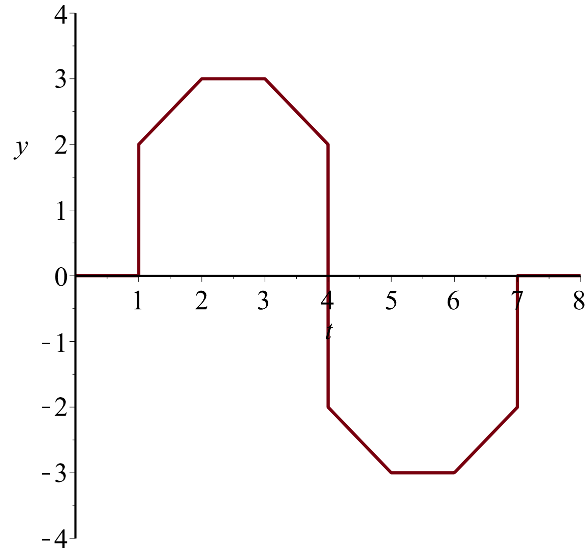


OBTENER Y GRAFICAR SU SERIE TRIGONOMÉTRICA DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

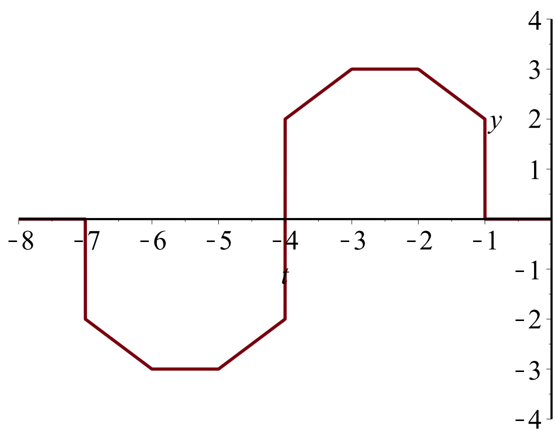
> *restart*

RESPUESTA 3)

> $f := 2\cdot \text{Heaviside}(t-1) + (t-1)\cdot\text{Heaviside}(t-1) - (t-2)\cdot\text{Heaviside}(t-2) - (t-3)\cdot\text{Heaviside}(t-3) + (t-4)\cdot\text{Heaviside}(t-4) - 4\cdot\text{Heaviside}(t-4) - (t-4)\cdot\text{Heaviside}(t-4) + (t-5)\cdot\text{Heaviside}(t-5) + (t-6)\cdot\text{Heaviside}(t-6) - (t-7)\cdot\text{Heaviside}(t-7) + 2\cdot\text{Heaviside}(t-7) : plot(f, t=0..8, y=-4..4)$



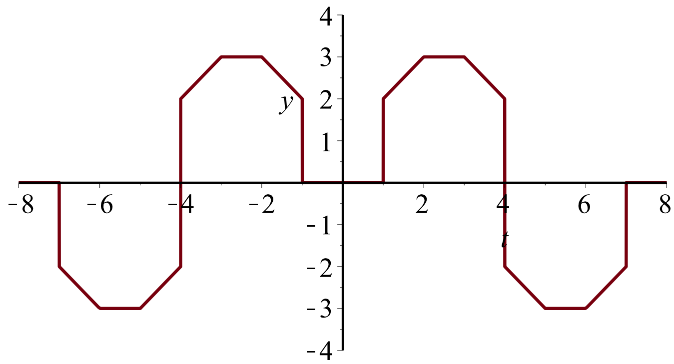
> $g := -2\cdot \text{Heaviside}(t+7) - (t+7)\cdot\text{Heaviside}(t+7) + (t+6)\cdot\text{Heaviside}(t+6) + (t+5)\cdot\text{Heaviside}(t+5) - (t+4)\cdot\text{Heaviside}(t+4) + 4\cdot\text{Heaviside}(t+4) + (t+4)\cdot\text{Heaviside}(t+4) - (t+3)\cdot\text{Heaviside}(t+3) - (t+2)\cdot\text{Heaviside}(t+2) + (t+1)\cdot\text{Heaviside}(t+1) - 2\cdot\text{Heaviside}(t+1) : plot(g, t=-8..0, y=-4..4)$



> $h := f + g; plot(h, t=-8..8, y=-4..4)$

$h := 2\,\text{Heaviside}(t-1) + (t-1)\,\text{Heaviside}(t-1) - (t-2)\,\text{Heaviside}(t-2) - (t-3)\,\text{Heaviside}(t-3) - 4\,\text{Heaviside}(t-4) + (t-5)\,\text{Heaviside}(t-5) + (t-6)\,\text{Heaviside}(t-6) - (t-7)\,\text{Heaviside}(t-7) + 2\,\text{Heaviside}(t-7) - 2\,\text{Heaviside}(t$

$+7) - (t+7)\,\text{Heaviside}(t+7) + (t+6)\,\text{Heaviside}(t+6) + (t+5)\,\text{Heaviside}(t+5)$
$+4\,\text{Heaviside}(t+4) - (t+3)\,\text{Heaviside}(t+3) - (t+2)\,\text{Heaviside}(t+2) + (t$
$+1)\,\text{Heaviside}(t+1) - 2\,\text{Heaviside}(t+1)$



> $L := 8$

$$L := 8 \quad \textbf{(24)}$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(h, t=-L..L);\ C := \frac{a_0}{2}$

$$a_0 := 0$$
$$C := 0 \quad \textbf{(25)}$$

> $a_n := simplify\left(\left(\frac{1}{L}\right) \cdot int\left(h \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t=-L..L\right)\right)$

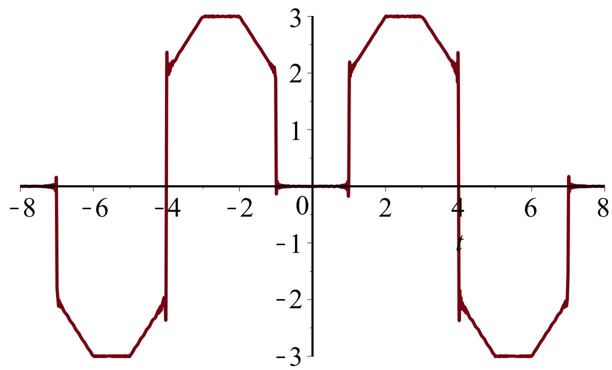
$$a_n := -\frac{1}{n^2 \pi^2}\left(4\left(n\pi \sin\left(\frac{1}{8} n\pi\right) + n\pi \sin\left(\frac{7}{8} n\pi\right) - 2\sin\left(\frac{1}{2} n\pi\right) n\pi + 4\cos\left(\frac{1}{8} n\pi\right) - 4\cos\left(\frac{1}{4} n\pi\right) - 4\cos\left(\frac{3}{8} n\pi\right) + 4\cos\left(\frac{5}{8} n\pi\right) + 4\cos\left(\frac{3}{4} n\pi\right) - 4\cos\left(\frac{7}{8} n\pi\right)\right)\right) \quad \textbf{(26)}$$

> $b_n := simplify\left(\left(\frac{1}{L}\right) \cdot int\left(h \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t=-L..L\right)\right)$

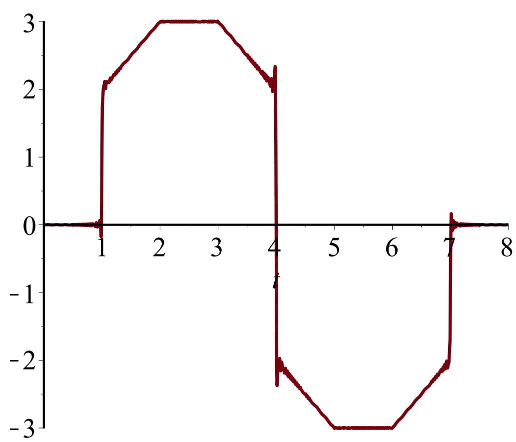
$$b_n := 0 \quad \textbf{(27)}$$

> $STF_{500} := sum\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n=1..500\right):$

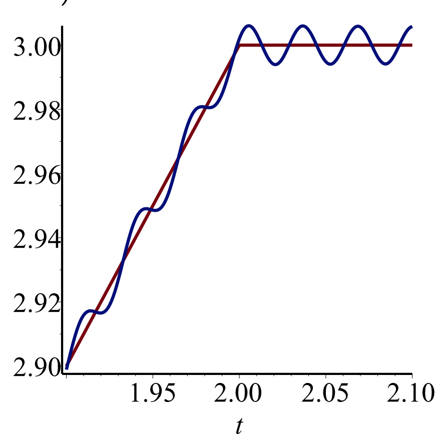
> $plot(STF_{500}, t=-8..8)$



> $plot(STF_{500}, t = 0 ..8)$



> $plot([f, STF_{500}], t = 1.9 ..2.1)$



OTRA FORMA DE RESOLVERLO

> $LL := 4$

$$LL := 4 \tag{28}$$

> $aa[0] := \frac{1}{LL} \cdot int(f, t = 0..2 \cdot LL)$

$$aa_0 := 0 \tag{29}$$

> $aa[n] := simplify\left(subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \left(\frac{1}{LL}\right) \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{LL} \cdot t\right), t = 0..2\,LL\right)\right)\right)$

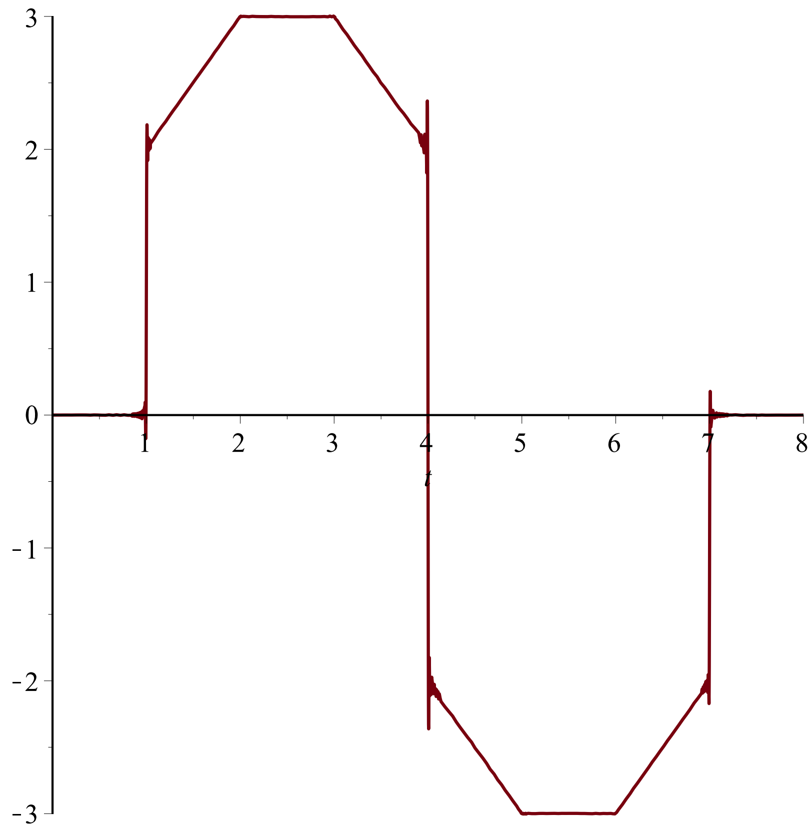
$$aa_n := -\frac{1}{n^2 \pi^2}\left(2\left(n\pi \sin\left(\frac{1}{4} n\pi\right) + n\pi \sin\left(\frac{7}{4} n\pi\right) - 2\cos\left(\frac{7}{4} n\pi\right) + 2\cos\left(\frac{3}{2} n\pi\right) + 2\cos\left(\frac{5}{4} n\pi\right) - 2\cos\left(\frac{1}{2} n\pi\right) + 2\cos\left(\frac{1}{4} n\pi\right) - 2\cos\left(\frac{3}{4} n\pi\right)\right)\right) \tag{30}$$

> $bb[n] := simplify\left(subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \left(\frac{1}{LL}\right) \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{LL} \cdot t\right), t = 0..2\,LL\right)\right)\right)$

$$bb_n := -\frac{1}{n^2 \pi^2}\left(2\left(2\cos(n\pi)\, n\pi - \cos\left(\frac{7}{4} n\pi\right) n\pi - \cos\left(\frac{1}{4} n\pi\right) n\pi + 2\sin\left(\frac{1}{4} n\pi\right) - 2\sin\left(\frac{3}{4} n\pi\right) - 2\sin\left(\frac{1}{2} n\pi\right) - 2\sin\left(\frac{7}{4} n\pi\right) + 2\sin\left(\frac{3}{2} n\pi\right) + 2\sin\left(\frac{5}{4} n\pi\right)\right)\right) \tag{31}$$

> $STF600 := sum\left(aa[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{LL} \cdot t\right) + bb[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{LL} \cdot t\right), n = 1..600\right):$

> $plot(STF600, t = 0..2 \cdot LL)$



>

>

FIN RESPUESTA 3)

> *restart*

4) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA **CONSTANTE DE SEPARACIÓN NEGATIVA**:

$$\frac{\partial^2}{\partial x^2} y(x,t) + t^2 \left(\frac{\partial}{\partial t} y(x,t)\right) = \frac{\partial}{\partial x} y(x,t) \qquad \textbf{(32)}$$

> *restart*

RESPUESTA 4)

> $Ecuacion := \frac{\partial^2}{\partial x^2} y(x,t) + t^2 \left(\frac{\partial}{\partial t} y(x,t)\right) = \frac{\partial}{\partial x} y(x,t)$

$$Ecuacion := \frac{\partial^2}{\partial x^2} y(x,t) + t^2 \left(\frac{\partial}{\partial t} y(x,t)\right) = \frac{\partial}{\partial x} y(x,t) \qquad \textbf{(33)}$$

> $EcuacionDos := eval(subs(y(x,t) = F(x) \cdot G(t), Ecuacion))$

(34)

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x)\right) G(t) + t^2 F(x) \left(\frac{d}{dt} G(t)\right) = \left(\frac{d}{dx} F(x)\right) G(t) \quad \textbf{(34)}$$

> EcuacionTres := lhs(EcuacionDos) − t² F(x) (d/dt G(t)) − (d/dx F(x)) G(t) = rhs(EcuacionDos) − t² F(x) (d/dt G(t)) − (d/dx F(x)) G(t)

$$EcuacionTres := \left(\frac{d^2}{dx^2} F(x)\right) G(t) - \left(\frac{d}{dx} F(x)\right) G(t) = -t^2 F(x) \left(\frac{d}{dt} G(t)\right) \quad \textbf{(35)}$$

> EcuacionSeparada := simplify(lhs(EcuacionTres) / (F(x)·G(t)) = rhs(EcuacionTres) / (F(x)·G(t)))

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} \quad \textbf{(36)}$$

> EcuacionX := lhs(EcuacionSeparada) =−beta··2; EcuacionT := rhs(EcuacionSeparada) = −beta··2

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\beta^2$$

$$EcuacionT := -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} = -\beta^2 \quad \textbf{(37)}$$

> SolucionX := dsolve(EcuacionX); SolucionT := dsolve(EcuacionT)

$$SolucionX := F(x) = \_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-4\beta^2}\right)x}$$

$$SolucionT := G(t) = \_C1\, e^{-\frac{\beta^2}{t}} \quad \textbf{(38)}$$

>

> SolucionNegativa := y(x, t) = rhs(SolucionX) ·subs(_C1 = 1, rhs(SolucionT))

$$SolucionNegativa := y(x,t) = \left(\_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-4\beta^2}\right)x}\right) e^{-\frac{\beta^2}{t}} \quad \textbf{(39)}$$

>

FIN RESPUESTA 4)

> restart

FIN DEL SOLUCIÓN

>