>

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL
SEMESTRE 2023-2

22 JUNIO 2023

> *restart*

1) (20/100)
OBTENER LA SOLUCIÓN PARTICULAR DE LA SIGUIENTE ECUACIÓN DIFERENCIAL DE PRIMER ORDEN **(SIN UTILIZAR dsolve) (TEMA 1)**

> $Ecua := y(x)\cdot\sin(x) + \left(\frac{1}{x} - \frac{y(x)}{x}\right)\cdot diff(y(x), x) = 0;\ Cond := y(\mathrm{Pi}) = 2$

$$Ecua := y(x)\,\sin(x) + \left(\frac{1}{x} - \frac{y(x)}{x}\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) = 0$$

$$Cond := y(\pi) = 2 \tag{1}$$

>

RESPUESTA

> *Ecua*

$$y(x)\,\sin(x) + \left(\frac{1}{x} - \frac{y(x)}{x}\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) = 0 \tag{2}$$

> *Cond*

$$y(\pi) = 2 \tag{3}$$

> *with(DEtools)* :

> *odeadvisor(Ecua)*

$$[\_separable] \tag{4}$$

> $M := y\cdot\sin(x)$

$$M := y\,\sin(x) \tag{5}$$

> $N := factor\left(\frac{1}{x} - \frac{y}{x}\right)$

$$N := -\frac{-1+y}{x} \tag{6}$$

> $P := \sin(x);\ Q := y;\ R := -\frac{1}{x};\ S := y - 1$

$$P := \sin(x)$$
$$Q := y$$
$$R := -\frac{1}{x}$$
$$S := -1 + y \tag{7}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := -\sin(x) + \cos(x)\,x + y - \ln(y) = \_C1 \tag{8}$$

> $Para := simplify(subs(x = \mathrm{Pi}, y = 2, SolGral))$

$$Para := 2 - \pi - \ln(2) = \_C1 \tag{9}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

(10)

$$SolPart := -\sin(x) + \cos(x)\, x + y - \ln(y) = 2 - \pi - \ln(2) \quad (10)$$

> $SolPartFinal := -\sin(x) + \cos(x)\, x + y(x) - \ln(y(x)) = 10 - \pi - \ln(2) - \ln(5)$

$$SolPartFinal := -\sin(x) + \cos(x)\, x + y(x) - \ln(y(x)) = 10 - \pi - \ln(2) - \ln(5) \quad (11)$$

> $DerSolPart := simplify(isolate(diff(SolPartFinal, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\, y(x) = \frac{\sin(x)\, x\, y(x)}{y(x) - 1} \quad (12)$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{\sin(x)\, x\, y(x)}{y(x) - 1} \quad (13)$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolPart) = 0)$

$$Comprobar := 0 = 0 \quad (14)$$

> $restart$

2) (20/100)
OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL DE PRIMER ORDEN **(SIN UTILIZAR dsolve) (TEMA 2)**

> $Ecua := y'' - 2\cdot y' + y = 2\cdot \mathrm{sqrt}(x)\cdot \exp(x)$

$$Ecua := \frac{d^2}{dx^2}\, y(x) - 2\left(\frac{d}{dx}\, y(x)\right) + y(x) = 2\sqrt{x}\, e^x \quad (15)$$

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RESPUESTA

> $Ecua$

$$\frac{d^2}{dx^2}\, y(x) - 2\left(\frac{d}{dx}\, y(x)\right) + y(x) = 2\sqrt{x}\, e^x \quad (16)$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) - 2\left(\frac{d}{dx}\, y(x)\right) + y(x) = 0 \quad (17)$$

> $Q := rhs(Ecua)$

$$Q := 2\sqrt{x}\, e^x \quad (18)$$

> $EcuaCarac := m^2 - 2\cdot m + 1 = 0$

$$EcuaCarac := m^2 - 2\, m + 1 = 0 \quad (19)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, 1 \quad (20)$$

> $yy[1] := \exp(Raiz[1]\cdot x);\ yy[2] := x\cdot \exp(Raiz[1]\cdot x)$

$$yy_1 := e^x$$

$$yy_2 := x\, e^x \quad (21)$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(x) = \_C1\, e^x + \_C2\, x\, e^x \quad (22)$$

> $SolNoHom := y(x) = A\cdot yy[1] + B\cdot yy[2]$

$$SolNoHom := y(x) = A\, e^x + B\, x\, e^x \quad (23)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & x\,e^x \\ e^x & e^x + x\,e^x \end{bmatrix} \qquad (24)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 2\sqrt{x}\,e^x \end{bmatrix} \qquad (25)$$

> $Para := linsolve(WW, BB) : Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -2\,x^{3/2}$$

$$Bprima := 2\sqrt{x} \qquad (26)$$

> $A := int(Aprima, x) + \_C1$

$$A := -\frac{4}{5}\,x^{5/2} + \_C1 \qquad (27)$$

> $B := int(Bprima, x) + \_C2$

$$B := \frac{4}{3}\,x^{3/2} + \_C2 \qquad (28)$$

> $SolGral := expand(SolNoHom)$

$$SolGral := y(x) = \frac{8}{15}\,e^x\,x^{5/2} + \_C1\,e^x + \_C2\,x\,e^x \qquad (29)$$

> $Comprobacion := eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0))$

$$Comprobacion := 0 = 0 \qquad (30)$$

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> $restart$

3) (20/100)
OBTENER LA SOLUCIÓN PARTICULAR DEL SIGUIENTE PROBLEMA DE UNA ECUACIÓN DIFERENCIAL HOMOGENEA CON CONDICIONES INICIALES MEDIANTE EL MÉTODO DE TRANSFORMADA DE LAPLACE **(SIN UTILIZAR dsolve)**
(TEMA 3)

> $Ecua := diff(y(t), t\$2) + 2\cdot diff(y(t), t) + 2\cdot y(t) = \mathrm{Dirac}(t - \mathrm{Pi}); Cond := y(0) = -1, \mathrm{D}(y)(0) = 2$

$$Ecua := \frac{d^2}{dt^2}\,y(t) + 2\left(\frac{d}{dt}\,y(t)\right) + 2\,y(t) = \mathrm{Dirac}(t - \pi)$$

$$Cond := y(0) = -1,\ \mathrm{D}(y)(0) = 2 \qquad (31)$$

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RESPUESTA

> $Ecua$

$$\frac{d^2}{dt^2}\,y(t) + 2\left(\frac{d}{dt}\,y(t)\right) + 2\,y(t) = \mathrm{Dirac}(t - \pi) \qquad (32)$$

> $Cond$

$$y(0) = -1,\ \mathrm{D}(y)(0) = 2 \qquad (33)$$

> $with(inttrans):$

> $EcuaLap := subs(Cond, laplace(Ecua, t, s))$

$$EcuaLap := s^2\, laplace(y(t), t, s) + s + 2\, s\, laplace(y(t), t, s) + 2\, laplace(y(t), t, s) = e^{-s\pi} \quad (34)$$

> $SolLap := isolate(EcuaLap, laplace(y(t), t, s))$

$$SolLap := laplace(y(t), t, s) = \frac{e^{-s\pi} - s}{s^2 + 2\, s + 2} \quad (35)$$

> $SolPart := invlaplace(SolLap, s, t)$

$$SolPart := y(t) = e^{-t}\left(-\cos(t) + \sin(t)\right) - \mathrm{Heaviside}(t - \pi)\, e^{-t + \pi} \sin(t) \quad (36)$$

> $Ecua$

$$\frac{d^2}{dt^2}\, y(t) + 2\left(\frac{d}{dt}\, y(t)\right) + 2\, y(t) = \mathrm{Dirac}(t - \pi) \quad (37)$$

> $Comprobacion := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobacion := 0 = 0 \quad (38)$$

>

> $restart$

4) (20/100)
OBTENER LA SOLUCIÓN PARTICULAR DEL SISTEMA DE ECUACIONES DIFERENCIALES CON CONDICIONES INICIALES, MEDIANTE EL MÉTODO DE LA MATRIZ EXPONENCIAL **(SIN UTILIZAR dsolve) (TEMA 3)**

> $Sistema := diff(x[1](t), t) = -x[1](t) + x[2](t), diff(x[2](t), t) = x[1](t) - x[2](t) :$
> $Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\, x_1(t) = -x_1(t) + x_2(t)$$

$$\frac{d}{dt}\, x_2(t) = x_1(t) - x_2(t) \quad (39)$$

> $Cond := x[1](0) = 2, x[2](0) = 1$

$$Cond := x_1(0) = 2, x_2(0) = 1 \quad (40)$$

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RESPUESTA

> $AA := array([[-1, 1], [1, -1]])$

$$AA := \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (41)$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{1}{2}\, e^{-2t} + \frac{1}{2} & \frac{1}{2} - \frac{1}{2}\, e^{-2t} \\ \frac{1}{2} - \frac{1}{2}\, e^{-2t} & \frac{1}{2}\, e^{-2t} + \frac{1}{2} \end{bmatrix} \quad (42)$$

> $Xcero := array([2, 1])$

$$Xcero := \begin{bmatrix} 2 & 1 \end{bmatrix} \quad (43)$$

> $SolPart := evalm(MatExp \,\&*\, Xcero) : x[1](t) = SolPart[1]; x[2](t) = SolPart[2]$

$$x_1(t) = \frac{1}{2}\,e^{-2t} + \frac{3}{2}$$
$$x_2(t) = \frac{3}{2} - \frac{1}{2}\,e^{-2t} \quad \textbf{(44)}$$

> *Sistema*[1]; *Sistema*[2]

$$\frac{d}{dt}\,x_1(t) = -x_1(t) + x_2(t)$$
$$\frac{d}{dt}\,x_2(t) = x_1(t) - x_2(t) \quad \textbf{(45)}$$

> *CompUno* := *eval*(*subs*(*x*[1](*t*) = *SolPart*[1], *x*[2](*t*) = *SolPart*[2], *lhs*(*Sistema*[1]) − *rhs*(*Sistema*[1]) = 0))

$$CompUno := 0 = 0 \quad \textbf{(46)}$$

> *CompDos* := *eval*(*subs*(*x*[1](*t*) = *SolPart*[1], *x*[2](*t*) = *SolPart*[2], *lhs*(*Sistema*[2]) − *rhs*(*Sistema*[2]) = 0))

$$CompDos := 0 = 0 \quad \textbf{(47)}$$

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> *restart*

5) (20/100)
RESOLVER LA ECUACIÓN EN DERIVADAS PARCIALES PARA UNA CONSTANTE DE SEPARACIÓN MENOR QUE CERO
(SIN UTILIZAR dsolve) (TEMA 4)

> $Ecua := y^3 \cdot diff(u(x,y),x) + x^3 \cdot diff(u(x,y),y) = 0$

$$Ecua := y^3 \left(\frac{\partial}{\partial x}\,u(x,y)\right) + x^3 \left(\frac{\partial}{\partial y}\,u(x,y)\right) = 0 \quad \textbf{(48)}$$

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RESPUESTA

> *EcuaSeparable* := *eval*(*subs*(*u*(*x*, *y*) = *M*(*x*)·*N*(*y*), *Ecua*))

$$EcuaSeparable := y^3 \left(\frac{d}{dx}\,M(x)\right) N(y) + x^3\,M(x) \left(\frac{d}{dy}\,N(y)\right) = 0 \quad \textbf{(49)}$$

> $$EcuaSeparada := \frac{\left(lhs(EcuaSeparable) - x^3\,M(x)\left(\frac{d}{dy}\,N(y)\right)\right)}{x^3 \cdot M(x) \cdot y^3 \cdot N(y)} = \frac{\left(rhs(EcuaSeparable) - x^3\,M(x)\left(\frac{d}{dy}\,N(y)\right)\right)}{x^3 \cdot M(x) \cdot y^3 \cdot N(y)}$$

$$EcuaSeparada := \frac{\frac{d}{dx}\,M(x)}{x^3\,M(x)} = -\frac{\frac{d}{dy}\,N(y)}{y^3\,N(y)} \quad \textbf{(50)}$$

> $EcuaX := lhs(EcuaSeparada) = -\beta^2$

$$EcuaX := \frac{\frac{d}{dx}\,M(x)}{x^3\,M(x)} = -\beta^2 \quad \textbf{(51)}$$

> $EcuaY := rhs(EcuaSeparada) = -\beta^2$

$$EcuaY := -\frac{\frac{d}{dy} N(y)}{y^3 N(y)} = -\beta^2 \quad \textbf{(52)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := M(x) = \_C1\, e^{-\frac{1}{4} \beta^2 x^4} \quad \textbf{(53)}$$

> $SolY := dsolve(EcuaY)$

$$SolY := N(y) = \_C1\, e^{\frac{1}{4} \beta^2 y^4} \quad \textbf{(54)}$$

> $SolGral := u(x, y) = rhs(SolX) \cdot subs(\_C1 = 1, rhs(SolY))$

$$SolGral := u(x, y) = \_C1\, e^{-\frac{1}{4} \beta^2 x^4}\, e^{\frac{1}{4} \beta^2 y^4} \quad \textbf{(55)}$$

> $Comprobacion := eval(subs(u(x, y) = rhs(SolGral), Ecua))$

$$Comprobacion := 0 = 0 \quad \textbf{(56)}$$

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FIN DEL EXAMEN

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