>
UNAM
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
ECUACIONES DIFERENCIALES
GRUPO 15 SEMESTRE 2024-1
PRIMER EXAMEN PARCIAL Temas 1 & 2

2023-09-28

> *restart*

PREGUNTA 1 (30 puntos) Obtener la solución general de la siguiente ecuación diferencial ordinaria no lineal, utilizando el método de coeficientes homogéneos (***sin usar dsolve***)

> $Ecua := 4 \cdot x^2 - x \cdot y + y^2 + (x^2 - x \cdot y + 4 \cdot y^2) \cdot y' = 0$

$$Ecua := 4\,x^2 - x\,y(x) + y(x)^2 + \left(x^2 - x\,y(x) + 4\,y(x)^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) = 0 \qquad (1)$$

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RESPUESTA 1)

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[[\,\_homogeneous,\ class\ A\,],\ \_rational,\ \_dAlembert] \qquad (2)$$

> $EcuaDos := simplify(isolate(eval(subs(y(x) = u(x) \cdot x, Ecua)), diff(u(x), x)))$

$$EcuaDos := \frac{\mathrm{d}}{\mathrm{d}x}\,u(x) = -\frac{4\left(u(x)^3 + 1\right)}{x\left(4\,u(x)^2 - u(x) + 1\right)} \qquad (3)$$

> $P := \dfrac{4\left(u^3 + 1\right)}{\left(4\,u^2 - u + 1\right)}$

$$P := \frac{4\left(u^3 + 1\right)}{4\,u^2 - u + 1} \qquad (4)$$

> $R := x$

$$R := x \qquad (5)$$

> $SolUno := int\left(\dfrac{1}{P}, u\right) + int\left(\dfrac{1}{R}, x\right) = \_C1$

$$SolUno := \frac{1}{2}\,\ln(u + 1) + \frac{1}{4}\,\ln(u^2 - u + 1) + \ln(x) = \_C1 \qquad (6)$$

> $SolDos := isolate\left(simplify\left(subs\left(u = \dfrac{y(x)}{x},\ SolUno\right)\right),\ \_C1\right)$

$$SolDos := \_C1 = \frac{1}{2}\,\ln\left(\frac{y(x) + x}{x}\right) + \frac{1}{4}\,\ln\left(\frac{y(x)^2 - x\,y(x) + x^2}{x^2}\right) + \ln(x) \qquad (7)$$

> $SolGral := simplify(\exp(rhs(SolDos))) = \_C1$

$$SolGral := \sqrt{\frac{y(x) + x}{x}}\left(\frac{y(x)^2 - x\,y(x) + x^2}{x^2}\right)^{1/4} x = \_C1 \qquad (8)$$

> $SolFinal := lhs(SolGral)^4 = \_C1$

$$SolFinal := (y(x) + x)^2\left(y(x)^2 - x\,y(x) + x^2\right) = \_C1 \qquad (9)$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x),\ diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = -\frac{y(x)^2 - x\,y(x) + 4\,x^2}{x^2 - x\,y(x) + 4\,y(x)^2} \tag{10}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{-y(x)^2 + x\,y(x) - 4\,x^2}{x^2 - x\,y(x) + 4\,y(x)^2} \tag{11}$$

> $Comprobar := simplify(rhs(DerEcua) - rhs(DerSolFinal)) = 0$

$$Comprobar := 0 = 0 \tag{12}$$

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FIN RESPUESTA 1)

> $restart$

PREGUNTA 2 (20 puntos) Obtener la solución general de la siguiente ecuación diferencial ordinaria de coeficientes variables no homogenea (***sin usar dsolve***)

> $Ecua := \left(\frac{d}{dx}\,y(x) \right) + 2 \cdot x \cdot y(x) = 2 \cdot x \cdot \exp(-x^2)$

$$Ecua := \frac{d}{dx}\,y(x) + 2\,x\,y(x) = 2\,x\,e^{-x^2} \tag{13}$$

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RESPUESTA 2)

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d}{dx}\,y(x) + 2\,x\,y(x) = 0 \tag{14}$$

> $p := 2 \cdot x;\ q := rhs(Ecua)$

$$p := 2\,x$$

$$q := 2\,x\,e^{-x^2} \tag{15}$$

> $SolHom := y(x) = \_C1 \cdot \exp(-int(p, x))$

$$SolHom := y(x) = \_C1\,e^{-x^2} \tag{16}$$

> $SolGral := y(x) = \_C1 \cdot \exp(-int(p, x)) + \exp(-int(p, x)) \cdot int(\exp(int(p, x)) \cdot q, x)$

$$SolGral := y(x) = \_C1\,e^{-x^2} + e^{-x^2}\,x^2 \tag{17}$$

> $Comprobacion := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobacion := 0 = 0 \tag{18}$$

FIN RESPUESTA 2)

> $restart$

PREGUNTA 3 (20 puntos) Obtener la solución particular del siguiente problema de ecuaciones diferenciales ordinarias lineales homogeneas con condiciones iniciales (***sin usar dsolve***)

> $Ecua := y'' - 6 \cdot y' + 8 \cdot y = 0$

$$Ecua := \frac{d^2}{dx^2}\,y(x) - 6\left(\frac{d}{dx}\,y(x) \right) + 8\,y(x) = 0 \tag{19}$$

> $CondIni := y(0) = 4,\ D(y)(0) = -5$

$$CondIni := y(0) = 4,\ D(y)(0) = -5 \tag{20}$$

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RESPUESTA PREGUNTA 3)

> $EcuaCarac := m^2 - 6\,m + 8 = 0$

(21)

$$EcuaCarac := m^2 - 6\,m + 8 = 0 \tag{21}$$

> $Raiz := solve(EcuaCarac)$
$$Raiz := 4, 2 \tag{22}$$

> $SolGralHom := y(x) = \_C1 \cdot \exp(Raiz[1] \cdot x) + \_C2 \cdot \exp(Raiz[2] \cdot x)$
$$SolGralHom := y(x) = \_C1\,e^{4x} + \_C2\,e^{2x} \tag{23}$$

> $EcuaUno := eval(subs(x=0, rhs(SolGralHom)=4))$
$$EcuaUno := \_C1 + \_C2 = 4 \tag{24}$$

> $EcuaDos := eval(subs(x=0, rhs(diff(SolGralHom, x))=-5))$
$$EcuaDos := 4\,\_C1 + 2\,\_C2 = -5 \tag{25}$$

> $Para := solve([EcuaUno, EcuaDos])$
$$Para := \left\{ \_C1 = -\frac{13}{2}, \ \_C2 = \frac{21}{2} \right\} \tag{26}$$

> $SolPart := subs(Para, SolGralHom)$
$$SolPart := y(x) = -\frac{13}{2}\,e^{4x} + \frac{21}{2}\,e^{2x} \tag{27}$$

> $Comprobacion := eval(subs(y(x) = rhs(SolPart), Ecua))$
$$Comprobacion := 0 = 0 \tag{28}$$

> $CondIni$
$$y(0) = 4, \ D(y)(0) = -5 \tag{29}$$

> $CompUno := simplify(subs(x=0, SolPart))$
$$CompUno := y(0) = 4 \tag{30}$$

> $CompDos := D(y)(0) = simplify(subs(x=0, rhs(diff(SolPart, x))))$
$$CompDos := D(y)(0) = -5 \tag{31}$$

FIN RESPUESTA 3)

> $restart$

PREGUNTA 4 (30 puntos) Obtener la solución particular del siguiente problema de ecuaciones diferenciales ordinarias no homogeneas con condiciones iniciales (***sin usar dsolve***)

> $Ecua := \dfrac{d^2}{dx^2}\,y(x) - y(x) = 8 \cdot x \cdot e^{x}$

$$Ecua := \frac{d^2}{dx^2}\,y(x) - y(x) = 8\,x\,e^{x} \tag{32}$$

> $CondIni := y(0) = -1, D(y)(0) = 1$
$$CondIni := y(0) = -1, \ D(y)(0) = 1 \tag{33}$$

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RESPUESTA 4)

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) - y(x) = 0 \tag{34}$$

> $Q := rhs(Ecua)$

$$Q := 8\,x\,e^{x} \tag{35}$$

> $EcuaCarac := m^2 - 1 = 0$

$$EcuaCarac := m^2 - 1 = 0 \tag{36}$$

> $Raiz := solve(EcuaCarac)$
$$Raiz := 1, -1 \qquad (37)$$

> $yy[1] := \exp(Raiz[1] \cdot x); yy[2] := \exp(Raiz[2] \cdot x)$
$$yy_1 := e^x$$
$$yy_2 := e^{-x} \qquad (38)$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$
$$SolHom := y(x) = \_C1\, e^x + \_C2\, e^{-x} \qquad (39)$$

> $SolNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$
$$SolNoHom := y(x) = A\, e^x + B\, e^{-x} \qquad (40)$$

> $with(linalg):$
> $WW := wronskian([yy[1], yy[2]], x)$
$$WW := \begin{bmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{bmatrix} \qquad (41)$$

> $BB := array([0, Q])$
$$BB := \begin{bmatrix} 0 & 8\, x\, e^x \end{bmatrix} \qquad (42)$$

> $Para := simplify(linsolve(WW, BB))$
$$Para := \begin{bmatrix} 4\, x & -4\, x\, e^{2x} \end{bmatrix} \qquad (43)$$

> $Aprima := Para[1]; Bprima := Para[2]$
$$Aprima := 4\, x$$
$$Bprima := -4\, x\, e^{2x} \qquad (44)$$

> $A := int(Aprima, x) + \_C10; B := int(Bprima, x) + \_C20$
$$A := 2\, x^2 + \_C10$$
$$B := -(2\, x - 1)\, e^{2x} + \_C20 \qquad (45)$$

> $SolGral := simplify(subs(\_C10 = \_C1 + 1, \_C20 = \_C2, simplify(SolNoHom)))$
$$SolGral := y(x) = 2\, e^x x^2 + \_C1\, e^x - 2\, x\, e^x + \_C2\, e^{-x} + 2\, e^x \qquad (46)$$

> $CondIni$
$$y(0) = -1, D(y)(0) = 1 \qquad (47)$$

> $ParaUno := simplify(subs(x = 0, rhs(SolGral) = -1))$
$$ParaUno := \_C1 + \_C2 + 2 = -1 \qquad (48)$$

> $ParaDos := simplify(subs(x = 0, rhs(diff(SolGral, x)) = 1))$
$$ParaDos := \_C1 - \_C2 = 1 \qquad (49)$$

> $Parametros := solve([ParaUno, ParaDos])$
$$Parametros := \{\_C1 = -1, \_C2 = -2\} \qquad (50)$$

> $SolPart := subs(Parametros, SolGral)$
$$SolPart := y(x) = 2\, e^x x^2 + e^x - 2\, x\, e^x - 2\, e^{-x} \qquad (51)$$

> $Comprobacion := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$
$$Comprobacion := 0 = 0 \qquad (52)$$

> $CondIni$
$$y(0) = -1, D(y)(0) = 1 \qquad (53)$$

> $CompUno := simplify(subs(x = 0, SolPart))$
$$CompUno := y(0) = -1 \tag{54}$$

> $CompDos := D(y)(0) = simplify(subs(x = 0, rhs(diff(SolPart, x))))$
$$CompDos := D(y)(0) = 1 \tag{55}$$

FIN RESPUESTA 4

> $restart$

FIN DEL EXAMEN

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