```
> restart
```

PRIMER EXAMEN FINAL ECUACIONES DIFERENCIALES GRUPO 15 SEMESTRE 2024-1

```
>
```

1)

```
> Ecua := (r(theta)^2·sec(theta) − tan(theta))·diff(r(theta), theta) + r(theta)·sec(theta)
      ·(sec(theta) + r(theta)^2·tan(theta)) = 0
```

$$Ecua := \left(r(\theta)^2 \sec(\theta) - \tan(\theta)\right)\left(\frac{d}{d\theta} r(\theta)\right) + r(\theta)\sec(\theta)\left(\sec(\theta) + r(\theta)^2\tan(\theta)\right) = 0 \quad (1)$$

```
> M := r·sec(theta)·(sec(theta) + r^2·tan(theta))
```

$$M := r\sec(\theta)\left(\sec(\theta) + r^2\tan(\theta)\right) \quad (2)$$

```
> N := (r^2·sec(θ) − tan(θ))
```

$$N := r^2\sec(\theta) - \tan(\theta) \quad (3)$$

```
>
```

```
> diff(M, r)
```

$$\sec(\theta)\left(\sec(\theta) + r^2\tan(\theta)\right) + 2r^2\sec(\theta)\tan(\theta) \quad (4)$$

```
> diff(N, theta)
```

$$r^2\sec(\theta)\tan(\theta) - 1 - \tan(\theta)^2 \quad (5)$$

```
> F := simplify((diff(N, theta) − diff(M, r))/M)
```

$$F := -\frac{2}{r} \quad (6)$$

```
> EcuaDos := isolate(int(1/mu, mu) = int(F, r), mu)
```

$$EcuaDos := \mu = \frac{1}{r^2} \quad (7)$$

```
> MM := expand(rhs(EcuaDos)·M)
```

$$MM := \frac{\sec(\theta)^2}{r} + r\sec(\theta)\tan(\theta) \quad (8)$$

```
> NN := expand(rhs(EcuaDos)·N)
```

$$NN := \sec(\theta) - \frac{\tan(\theta)}{r^2} \quad (9)$$

```
> simplify(diff(MM, r) − diff(NN, theta) = 0)
```

$$0 = 0 \quad (10)$$

```
> IntMM := int(MM, theta)
```

$$IntMM := \frac{\tan(\theta)}{r} + r\sec(\theta) \quad (11)$$

```
> SolGral := IntMM + int((NN − diff(IntMM, r)), r) = _C1
```

$$SolGral := \frac{\tan(\theta)}{r} + r\sec(\theta) = \_C1 \quad (12)$$

> $SolFinal := \frac{\tan(\theta)}{r(\text{theta})} + r(\text{theta}) \cdot \sec(\theta) = \_C1$

$$SolFinal := \frac{\tan(\theta)}{r(\theta)} + r(\theta)\,\sec(\theta) = \_C1 \quad (13)$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, \text{theta}), diff(r(\text{theta}), \text{theta})))$

$$DerSolFinal := \frac{d}{d\theta} r(\theta) = -\frac{r(\theta)\left(\sec(\theta) + r(\theta)^2 \tan(\theta)\right)}{r(\theta)^2 - \sin(\theta)} \quad (14)$$

> $DerEcua := simplify(isolate(Ecua, diff(r(\text{theta}), \text{theta})))$

$$DerEcua := \frac{d}{d\theta} r(\theta) = -\frac{r(\theta)\left(\sec(\theta) + r(\theta)^2 \tan(\theta)\right)}{r(\theta)^2 - \sin(\theta)} \quad (15)$$

> $Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua) = 0)$

$$Comprobar := 0 = 0 \quad (16)$$

> $restart$

2)

> $Ecua := 2 \cdot y'' + 3 \cdot y' - 5 \cdot y = 7 \cdot \exp(x)$

$$Ecua := 2\frac{d^2}{dx^2} y(x) + 3\frac{d}{dx} y(x) - 5\,y(x) = 7\,e^x \quad (17)$$

> $EcuaStandard := \frac{lhs(Ecua)}{2} = \frac{rhs(Ecua)}{2}$

$$EcuaStandard := \frac{d^2}{dx^2} y(x) + \frac{3\frac{d}{dx} y(x)}{2} - \frac{5\,y(x)}{2} = \frac{7\,e^x}{2} \quad (18)$$

> $EcuaHom := lhs(EcuaStandard) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + \frac{3\frac{d}{dx} y(x)}{2} - \frac{5\,y(x)}{2} = 0 \quad (19)$$

> $Q := rhs(EcuaStandard)$

$$Q := \frac{7\,e^x}{2} \quad (20)$$

> $EcuaCarac := m^2 + \frac{3 \cdot m}{2} - \frac{5}{2} = 0$

$$EcuaCarac := m^2 + \frac{3}{2} m - \frac{5}{2} = 0 \quad (21)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -\frac{5}{2} \quad (22)$$

> $yy[1] := \exp(Raiz[1] \cdot x)$

$$yy_1 := e^x \quad (23)$$

> $yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_2 := e^{-\frac{5x}{2}} \tag{24}$$

```
> with(linalg) :
```

```
> WW := wronskian([yy[1], yy[2]], x)
```

$$WW := \begin{bmatrix} e^{x} & e^{-\frac{5x}{2}} \\ e^{x} & -\frac{5\,e^{-\frac{5x}{2}}}{2} \end{bmatrix} \tag{25}$$

```
> BB := array([0, Q])
```

$$BB := \begin{bmatrix} 0 & \frac{7\,e^{x}}{2} \end{bmatrix} \tag{26}$$

```
> Para := linsolve(WW, BB)
```

$$Para := \begin{bmatrix} 1 & -\frac{e^{x}}{e^{-\frac{5x}{2}}} \end{bmatrix} \tag{27}$$

```
> Aprima := Para[1]
```

$$Aprima := 1 \tag{28}$$

```
> Bprima := simplify(Para[2])
```

$$Bprima := -e^{\frac{7x}{2}} \tag{29}$$

```
> A := int(Aprima, x) + _C1
```

$$A := x + \_C1 \tag{30}$$

```
> B := int(Bprima, x) + _C2
```

$$B := -\frac{2\,e^{\frac{7x}{2}}}{7} + \_C2 \tag{31}$$

```
> SolGral := y(x) = expand(A·yy[1] + B·yy[2])
```

$$SolGral := y(x) = e^{x}x + e^{x}\_C1 - \frac{2\,e^{-\frac{5x}{2}}\,e^{\frac{7x}{2}}}{7} + e^{-\frac{5x}{2}}\,\_C2 \tag{32}$$

```
> SolPart := expand(simplify(e^x x - 2 e^(-5/2 x) e^(7/2 x) / 7))
```

$$SolPart := e^{x}x - \frac{2\,e^{x}}{7} \tag{33}$$

```
> SolNoHom := y(x) = e^x _C1 + e^(-5/2 x) _C2 + x·exp(x)
```

$$SolNoHom := y(x) = e^{x}\,\_C1 + e^{-\frac{5x}{2}}\,\_C2 + e^{x}x \tag{34}$$

```
> restart
```

3)

> $Ecua := diff(y(t), t\$2) + 3 \cdot diff(y(t), t) + 4 \cdot y(t) = 4 \cdot \exp(4 \cdot t)$

$$Ecua := \frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 4 y(t) = 4 e^{4t} \quad (35)$$

> $CondIni := y(0) = 0, D(y)(0) = 2$

$$CondIni := y(0) = 0, D(y)(0) = 2 \quad (36)$$

> $with(inttrans)$:

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) - 2 + 3 s \mathcal{L}(y(t), t, s) + 4 \mathcal{L}(y(t), t, s) = \frac{4}{s - 4} \quad (37)$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{\frac{4}{s - 4} + 2}{s^2 + 3 s + 4} \quad (38)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \frac{e^{4t}}{8} + \frac{\left(3 \sqrt{7} \sin\left(\frac{\sqrt{7}\, t}{2}\right) - \cos\left(\frac{\sqrt{7}\, t}{2}\right)\right) e^{-\frac{3t}{2}}}{8} \quad (39)$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (40)$$

> $restart$

4)

> $with(inttrans)$:

> $G := simplify\left(diff\left(\frac{(s + 2)}{(s + 2)^2 + 9}, s\right)\right)$

$$G := \frac{-s^2 - 4 s + 5}{\left(s^2 + 4 s + 13\right)^2} \quad (41)$$

> $g := invlaplace(G, s, t)$

$$g := -t e^{-2t} \cos(3 t) \quad (42)$$

> $restart$

5)

> $EcuaDP := x \cdot diff(u(x, y), x) + y \cdot diff(u(x, y), y\$2) = u(x, y)$

$$EcuaDP := x \left(\frac{\partial}{\partial x} u(x, y)\right) + y \left(\frac{\partial^2}{\partial y^2} u(x, y)\right) = u(x, y) \quad (43)$$

> $EcuaSeparable := eval(subs(u(x, y) = F(x) \cdot G(y), EcuaDP))$

$$EcuaSeparable := x \left(\frac{d}{dx} F(x)\right) G(y) + y F(x) \left(\frac{d^2}{dy^2} G(y)\right) = F(x) G(y) \quad (44)$$

PRIMERA SOLUCIÓN

> $EcuaSeparada := \dfrac{\left(lhs(EcuaSeparable) - x\left(\dfrac{d}{dx}F(x)\right)G(y)\right)}{F(x)\cdot G(y)}$

$$= simplify\left(\frac{\left(rhs(EcuaSeparable) - x\left(\frac{d}{dx}F(x)\right)G(y)\right)}{F(x)\cdot G(y)}\right)$$

$$EcuaSeparada := \frac{y\left(\frac{d^2}{dy^2}G(y)\right)}{G(y)} = \frac{-x\left(\frac{d}{dx}F(x)\right) + F(x)}{F(x)} \quad (45)$$

> $EcuaY := lhs(EcuaSeparada) = 0$

$$EcuaY := \frac{y\left(\frac{d^2}{dy^2}G(y)\right)}{G(y)} = 0 \quad (46)$$

> $EcuaX := rhs(EcuaSeparada) = 0$

$$EcuaX := \frac{-x\left(\frac{d}{dx}F(x)\right) + F(x)}{F(x)} = 0 \quad (47)$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = c_1 y + c_2 \quad (48)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 x \quad (49)$$

> $SolFinal := u(x,y) = subs(c_1 = 1, rhs(SolX))\cdot rhs(SolY)$

$$SolFinal := u(x,y) = x\left(c_1 y + c_2\right) \quad (50)$$

SEGUNDA SOLUCIÓN

> $EcuaSeparable$

$$x\left(\frac{d}{dx}F(x)\right)G(y) + yF(x)\left(\frac{d^2}{dy^2}G(y)\right) = F(x)\,G(y) \quad (51)$$

> $EcuaSeparadaDos :=$

$$simplify\left(\frac{\left(lhs(EcuaSeparable) - x\left(\frac{d}{dx}F(x)\right)G(y) - F(x)\cdot G(y)\right)}{F(x)\cdot G(y)}\right)$$

$$= simplify\left(\frac{\left(rhs(EcuaSeparable) - x\left(\frac{d}{dx}F(x)\right)G(y)\right) - F(x)\cdot G(y)}{F(x)\cdot G(y)}\right)$$

$$EcuaSeparadaDos := \frac{\left(\frac{d^2}{dy^2}G(y)\right)y - G(y)}{G(y)} = -\frac{x\left(\frac{d}{dx}F(x)\right)}{F(x)} \quad (52)$$

> $EcuaYY := lhs(EcuaSeparadaDos) = 0$

(53)

$$EcuaYY := \frac{\left(\frac{d^2}{dy^2}\, G(y)\right) y - G(y)}{G(y)} = 0 \quad (53)$$

> $EcuaXX := rhs(EcuaSeparadaDos) = 0$

$$EcuaXX := -\frac{x\left(\frac{d}{dx}\, F(x)\right)}{F(x)} = 0 \quad (54)$$

> $SolXX := dsolve(EcuaXX)$

$$SolXX := F(x) = c_1 \quad (55)$$

> $SolYY := dsolve(EcuaYY)$

$$SolYY := G(y) = c_1 \sqrt{y}\ \mathrm{BesselI}\left(1, 2\sqrt{y}\right) + c_2 \sqrt{y}\ \mathrm{BesselK}\left(1, 2\sqrt{y}\right) \quad (56)$$

> $SolFinalDos := u(x,y) = subs\left(c_1 = 1, rhs(SolXX)\right) \cdot rhs(SolYY)$

$$SolFinalDos := u(x,y) = c_1 \sqrt{y}\ \mathrm{BesselI}\left(1, 2\sqrt{y}\right) + c_2 \sqrt{y}\ \mathrm{BesselK}\left(1, 2\sqrt{y}\right) \quad (57)$$

> $restart$

>

FIN EXAMEN

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