

ECUACIONES DIFERENCIALES  
 GRUPO 15 SEMESTRE 2024-1  
 SEGUNDO EXAMEN FINAL COLEGIADO  
 Solución

Diciembre 15 de 2023

- ```

> restart
  
```
- 1)
- ```

> Ecua := (y/x - 5) + y' = 0
  
```
- $$Ecua := \frac{y(x)}{x} - 5 + \frac{d}{dx} y(x) = 0 \quad (1)$$
- ```

> with(DEtools):
> odeadvisor(Ecua)
  
```
- $$[_linear] \quad (2)$$
- ```

> FI := intfactor(Ecua)
  
```
- $$FI := x \quad (3)$$
- ```

> M := y/x - 5
  
```
- $$M := \frac{y}{x} - 5 \quad (4)$$
- ```

> N := 1
  
```
- $$N := 1 \quad (5)$$
- ```

> MM := simplify(M*FI)
  
```
- $$MM := y - 5x \quad (6)$$
- ```

> NN := simplify(N*FI)
  
```
- $$NN := x \quad (7)$$
- ```

> Comprobar := diff(MM, y) - diff(NN, x) = 0
  
```
- $$Comprobar := 0 = 0 \quad (8)$$
- ```

> IntMMx := int(MM, x)
  
```
- $$IntMMx := yx - \frac{5}{2}x^2 \quad (9)$$
- ```

> Solucion := IntMMx + int((NN - diff(IntMMx, y)), y) = _C1
  
```
- $$Solucion := yx - \frac{5}{2}x^2 = \_C1 \quad (10)$$
- ```

> SolFinal := y(x) * x - 5/2 * x^2 = _C1
  
```
- $$SolFinal := y(x)x - \frac{5x^2}{2} = \_C1 \quad (11)$$
- ```

> DerSolFinal := isolate(diff(SolFinal, x), diff(y(x), x))
  
```
- $$(12)$$

$$DerSolFinal := \frac{d}{dx} y(x) = \frac{-y(x) + 5x}{x} \quad (12)$$

> DerEcua := isolate(Ecua, diff(y(x), x))

$$DerEcua := \frac{d}{dx} y(x) = -\frac{y(x)}{x} + 5 \quad (13)$$

> Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua) = 0)

$$Comprobar := 0 = 0 \quad (14)$$

> SolFinalDos := expand(isolate(SolFinal, y(x)))

$$SolFinalDos := y(x) = \frac{C1}{x} + \frac{5x}{2} \quad (15)$$

> SolComprobar := dsolve(Ecua)

$$SolComprobar := y(x) = \frac{c_1}{x} + \frac{5x}{2} \quad (16)$$

> restart

2)

> Ecua := y'' - 5·y' - 6·y = cos(x)

$$Ecua := \frac{d^2}{dx^2} y(x) - 5 \frac{d}{dx} y(x) - 6 y(x) = \cos(x) \quad (17)$$

> EcuaHom := lhs(Ecua) = 0

$$EcuaHom := \frac{d^2}{dx^2} y(x) - 5 \frac{d}{dx} y(x) - 6 y(x) = 0 \quad (18)$$

> Q := rhs(Ecua)

$$Q := \cos(x) \quad (19)$$

> EcuaCarac := m<sup>2</sup> - 5·m - 6 = 0

$$EcuaCarac := m^2 - 5m - 6 = 0 \quad (20)$$

> Raiz := solve(EcuaCarac)

$$Raiz := 6, -1 \quad (21)$$

> yy[1] := exp(Raiz[1]·x); yy[2] := exp(Raiz[2]·x)

$$yy_1 := e^{6x}$$

$$yy_2 := e^{-x}$$

(22)

> with(linalg) :

> WW := wronskian([yy[1], yy[2]], x)

$$WW := \begin{bmatrix} e^{6x} & e^{-x} \\ 6e^{6x} & -e^{-x} \end{bmatrix} \quad (23)$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & \cos(x) \end{bmatrix} \quad (24)$$

> Para := linsolve(WW, BB)

$$\text{Para} := \left[ \frac{\cos(x)}{7 e^{6x}} \quad - \frac{\cos(x)}{7 e^{-x}} \right] \quad (25)$$

> *Aprima* := Para[1]; *Bprima* := Para[2]

$$\text{Aprima} := \frac{\cos(x)}{7 e^{6x}}$$

$$\text{Bprima} := - \frac{\cos(x)}{7 e^{-x}} \quad (26)$$

> *SolGralNoHom* := y(x) = expand((int(*Aprima*, x) + \_C1)·yy[1] + (int(*Bprima*, x) + \_C2)·yy[2])

$$\text{SolGralNoHom} := y(x) = - \frac{7 \cos(x)}{74} - \frac{5 \sin(x)}{74} + (e^x)^6 \_C1 + \frac{\_C2}{e^x} \quad (27)$$

> *Comprobar* := simplify(eval(subs(y(x) = rhs(*SolGralNoHom*), lhs(*Ecua*) - rhs(*Ecua*) = 0)))

$$\text{Comprobar} := 0 = 0 \quad (28)$$

> restart

3)

> *Ecua* := diff(y(t), t) + y(t) = Heaviside(t - 1)·t

$$\text{Ecua} := \frac{d}{dt} y(t) + y(t) = \text{Heaviside}(t - 1) t \quad (29)$$

> *Cond* := y(0) = 0

$$\text{Cond} := y(0) = 0 \quad (30)$$

> with(inttrans) :

> *EcuaTL* := subs(*Cond*, laplace(*Ecua*, t, s))

$$\text{EcuaTL} := s \mathcal{L}(y(t), t, s) + \mathcal{L}(y(t), t, s) = \frac{e^{-s} (s + 1)}{s^2} \quad (31)$$

> *SolTL* := isolate(*EcuaTL*, laplace(y(t), t, s))

$$\text{SolTL} := \mathcal{L}(y(t), t, s) = \frac{e^{-s}}{s^2} \quad (32)$$

> *SolPart* := invlaplace(*SolTL*, s, t)

$$\text{SolPart} := y(t) = \text{Heaviside}(t - 1) (t - 1) \quad (33)$$

> *Comprobar* := simplify(eval(subs(y(t) = rhs(*SolPart*), lhs(*Ecua*) - rhs(*Ecua*) = 0)))

$$\text{Comprobar} := 0 = 0 \quad (34)$$

> restart

4)

> *f* := 2·sin(2·t)·cosh(3·t)

$$f := 2 \sin(2 t) \cosh(3 t) \quad (35)$$

> with(inttrans) :

> *F* := laplace(*f*, t, s)

$$F := \frac{4 (s^2 + 13)}{((s + 3)^2 + 4) ((s - 3)^2 + 4)} \quad (36)$$

>  $FF := \text{simplify}(F)$

$$FF := \frac{4s^2 + 52}{(s^2 + 6s + 13)(s^2 - 6s + 13)} \quad (37)$$

>  $\text{FraccTL} := \frac{4s^2 + 52}{(s^2 + 6s + 13)(s^2 - 6s + 13)} = \frac{A}{((s+3)^2 + 4)} + \frac{B}{((s-3)^2 + 4)}$

$$\text{FraccTL} := \frac{4s^2 + 52}{(s^2 + 6s + 13)(s^2 - 6s + 13)} = \frac{A}{(s+3)^2 + 4} + \frac{B}{(s-3)^2 + 4} \quad (38)$$

>  $\text{DespTL} := \text{lhs}(\text{FraccTL}) \cdot ((s^2 + 6s + 13)(s^2 - 6s + 13)) = \text{simplify}(\text{rhs}(\text{FraccTL}) \cdot ((s^2 + 6s + 13)(s^2 - 6s + 13)))$

$$\text{DespTL} := 4s^2 + 52 = (A + B)s^2 + (-6A + 6B)s + 13A + 13B \quad (39)$$

>  $\text{SistTL} := A + B = 4, -6 \cdot A + 6 \cdot B = 0$

$$\text{SistTL} := A + B = 4, -6A + 6B = 0 \quad (40)$$

>  $\text{Raiz} := \text{solve}([\text{SistTL}[1], \text{SistTL}[2]])$

$$\text{Raiz} := \{A=2, B=2\} \quad (41)$$

>  $\text{FFF} := \frac{2}{(s+3)^2 + 4} + \frac{2}{(s-3)^2 + 4}$

$$\text{FFF} := \frac{2}{(s+3)^2 + 4} + \frac{2}{(s-3)^2 + 4} \quad (42)$$

> *restart*

5)

>  $\text{EDP} := x \cdot \text{diff}(y(x, t), x, t) + y(x, t) = 0$

$$\text{EDP} := x \left( \frac{\partial^2}{\partial t \partial x} y(x, t) \right) + y(x, t) = 0 \quad (43)$$

>  $\text{EcuaSeparable} := \text{eval}(\text{subs}(y(x, t) = F(x) \cdot G(t), \text{EDP}))$

$$\text{EcuaSeparable} := x \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dt} G(t) \right) + F(x) G(t) = 0 \quad (44)$$

>  $\text{EcuaSeparada} := \frac{(\text{lhs}(\text{EcuaSeparable}) - F(x) \cdot G(t))}{F(x) \cdot \text{diff}(G(t), t)}$

$$= \frac{(\text{rhs}(\text{EcuaSeparable}) - F(x) \cdot G(t))}{F(x) \cdot \text{diff}(G(t), t)}$$

$$\text{EcuaSeparada} := \frac{x \left( \frac{d}{dx} F(x) \right)}{F(x)} = - \frac{G(t)}{\frac{d}{dt} G(t)} \quad (45)$$

>  $\text{EcuaXX} := \text{lhs}(\text{EcuaSeparada}) = \beta^2$

$$\text{EcuaXX} := \frac{x \left( \frac{d}{dx} F(x) \right)}{F(x)} = \beta^2 \quad (46)$$

>  $\text{EcuaTT} := \text{rhs}(\text{EcuaSeparada}) = \beta^2$

(47)

$$EcuaTT := -\frac{G(t)}{\frac{d}{dt} G(t)} = \beta^2 \quad (47)$$

> SolXX := dsolve(EcuaXX)

$$SolXX := F(x) = c_1 x^{\beta^2} \quad (48)$$

> SolTT := dsolve(EcuaTT)

$$SolTT := G(t) = c_1 e^{-\frac{t}{\beta^2}} \quad (49)$$

> SolFinal := y(x, t) = rhs(SolXX) · subs(c1 = 1, rhs(SolTT))

$$SolFinal := y(x, t) = c_1 x^{\beta^2} e^{-\frac{t}{\beta^2}} \quad (50)$$

> Comprobar := simplify(eval(subs(y(x, t) = rhs(SolFinal), EDP)))

$$Comprobar := 0 = 0 \quad (51)$$

> DerSegunda := diff(SolFinal, x, t)

$$DerSegunda := \frac{\partial^2}{\partial t \partial x} y(x, t) = -\frac{c_1 x^{\beta^2} e^{-\frac{t}{\beta^2}}}{x} \quad (52)$$

> PorDerSegunda := x · DerSegunda

$$PorDerSegunda := x \left( \frac{\partial^2}{\partial t \partial x} y(x, t) \right) = -c_1 x^{\beta^2} e^{-\frac{t}{\beta^2}} \quad (53)$$

> SolFinal

$$y(x, t) = c_1 x^{\beta^2} e^{-\frac{t}{\beta^2}} \quad (54)$$

> Ecua := rhs(PorDerSegunda) + rhs(SolFinal) = 0

$$Ecua := 0 = 0 \quad (55)$$

> restart

FIN EXAMEN

>

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