UNAM
FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEMESTRE 2024-1
GRUPO 15
SOLUCIÓN
SERIE TEMA 1

> restart

1) Obtener la solución particular del siguiente problema de ecuaciones diferenciales de primer orden con condiciones iniciales

> $Ecua := (1 + e^x)\, y(x) \left(\frac{d}{dx}\, y(x)\right) = e^x$

$$Ecua := (1 + e^x)\, y(x) \left(\frac{d}{dx}\, y(x)\right) = e^x \qquad (1)$$

> $CondIni := y(0) = 1$

$$CondIni := y(0) = 1 \qquad (2)$$

RESPUESTA 1)

> $with(DEtools)$:

> $odeadvisor(Ecua)$

$$[\_separable] \qquad (3)$$

> $M := -rhs(Ecua)$

$$M := -e^x \qquad (4)$$

> $N := (1 + e^x)\, y$

$$N := (1 + e^x)\, y \qquad (5)$$

> $P := e^x; Q := -1; R := (1 + e^x); S := y$

$$P := e^x$$
$$Q := -1$$
$$R := 1 + e^x$$
$$S := y \qquad (6)$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := \ln(1 + e^x) - \frac{1}{2}\, y^2 = \_C1 \qquad (7)$$

> $Para := simplify(subs(x = 0, y = 1, SolGral))$

$$Para := \ln(2) - \frac{1}{2} = \_C1 \qquad (8)$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$SolPart := \ln(1 + e^x) - \frac{1}{2}\, y^2 = \ln(2) - \frac{1}{2} \qquad (9)$$

> $SolFinal := \ln(1 + e^x) - \frac{1}{2}\, y(x)^2 = \ln(2) - \frac{1}{2}$

$$SolFinal := \ln(1 + e^x) - \frac{1}{2}\, y(x)^2 = \ln(2) - \frac{1}{2} \qquad (10)$$

> $DerSolFinal := isolate(diff(SolFinal, x), diff(y(x), x))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{e^x}{(1 + e^x)\, y(x)} \quad (11)$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{e^x}{(1 + e^x)\, y(x)} \quad (12)$$

> $Comprobar := rhs(DerSolFinal) - rhs(DerEcua) = 0$

$$Comprobar := 0 = 0 \quad (13)$$

>

FIN RESPUESTA 1)

> $restart$

2) Obtener la solución general de la siguiente ecuación diferencial de primer orden

> $Ecuacion := x\cdot(2\cdot x^2 + y^2) + y\cdot(x^2 + 2\cdot y^2)\cdot y' = 0$

$$Ecuacion := x\,(2\,x^2 + y(x)^2) + y(x)\,(x^2 + 2\,y(x)^2)\left(\frac{d}{dx}\, y(x)\right) = 0 \quad (14)$$

RESPUESTA 2)

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

[[_homogeneous, class A], _exact, _rational, _dAlembert] (15)

Se resolverá por exacta

> $M := x\,(2\,x^2 + y^2)$

$$M := x\,(2\,x^2 + y^2) \quad (16)$$

> $N := y\,(x^2 + 2\,y^2)$

$$N := y\,(x^2 + 2\,y^2) \quad (17)$$

> $Comprobar := diff(M, y) = diff(N, x)$

$$Comprobar := 2\,x\,y = 2\,x\,y \quad (18)$$

> $IntMx := int(M, x)$

$$IntMx := \frac{1}{2}\,x^4 + \frac{1}{2}\,x^2\,y^2 \quad (19)$$

> $SolucionGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolucionGral := \frac{1}{2}\,x^4 + \frac{1}{2}\,x^2\,y^2 + \frac{1}{2}\,y^4 = \_C1 \quad (20)$$

> $SolGralFinal := \frac{1}{2}\,x^4 + \frac{1}{2}\,x^2\,y(x)^2 + \frac{1}{2}\,y(x)^4 = \_C1$

$$SolGralFinal := \frac{1}{2}\,x^4 + \frac{1}{2}\,x^2\,y(x)^2 + \frac{1}{2}\,y(x)^4 = \_C1 \quad (21)$$

> $DerSolGral := isolate(diff(SolGralFinal, x), diff(y(x), x))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{-2\,x^3 - x\,y(x)^2}{x^2\,y(x) + 2\,y(x)^3} \quad (22)$$

> $DerEcuacion := isolate(Ecuacion, diff(y(x), x))$

$$DerEcuacion := \frac{d}{dx}\, y(x) = -\frac{x\,(2\,x^2 + y(x)^2)}{y(x)\,(x^2 + 2\,y(x)^2)} \quad (23)$$

> $Comprobar := simplify(rhs(DerSolGral) - rhs(DerEcuacion)) = 0$

$$Comprobar := 0 = 0 \quad (24)$$

>

FIN RESPUESTA 2)

> restart

3) Resuelva la siguiente ecuacion diferencial de primer orden por dos métodos distintos y pruebe que las soluciones generales obtenidas son iguales

> $Ecuacion := x\,(2\,x^2 + y(x)^2) + y(x)\,(x^2 + 2\,y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = 0$

$$Ecuacion := x\,(2\,x^2 + y(x)^2) + y(x)\,(x^2 + 2\,y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (25)$$

RESPUESTA 3)

> $with(DEtools)$:

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous, class\ A], \_exact, \_rational, \_dAlembert] \quad (26)$$

>

Por el método de coeficientes homogeneos

> $EcuaDos := simplify(isolate(eval(subs(y(x) = x \cdot u(x), Ecuacion)), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx}\,u(x) = -\frac{2\,(u(x)^4 + u(x)^2 + 1)}{x\,u(x)\,(2\,u(x)^2 + 1)} \quad (27)$$

> $odeadvisor(EcuaDos)$

$$[\_separable] \quad (28)$$

> $M := 2\,(u^4 + u^2 + 1)$

$$M := 2\,u^4 + 2\,u^2 + 2 \quad (29)$$

> $N := x\,u\,(2\,u^2 + 1)$

$$N := x\,u\,(2\,u^2 + 1) \quad (30)$$

> $P := 2; Q := u^4 + u^2 + 1; R := x; S := expand(u\,(2\,u^2 + 1))$

$$P := 2$$

$$Q := u^4 + u^2 + 1$$

$$R := x$$

$$S := 2\,u^3 + u \quad (31)$$

> $SolGralUno := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = \_C1$

$$SolGralUno := 2\ln(x) + \frac{1}{2}\ln(u^4 + u^2 + 1) = \_C1 \quad (32)$$

> $SolGralDos := simplify(\exp(lhs(SolGralUno))) = \_C1$

$$SolGralDos := x^2\sqrt{(u^2 + u + 1)\,(u^2 - u + 1)} = \_C1 \quad (33)$$

> $SolGralTres := lhs(SolGralDos)^2 = \_C1$

$$SolGralTres := x^4\,(u^2 + u + 1)\,(u^2 - u + 1) = \_C1 \quad (34)$$

> $SolGralFinal := expand\left(subs\left(u = \frac{y(x)}{x}, SolGralTres\right)\right)$

$$SolGralFinal := y(x)^4 + x^2\,y(x)^2 + x^4 = \_C1 \quad (35)$$

>

Cómo la ecuación diferencial es la misma que la pregunta 2) que fue resuelta como exacta vamos a compararla con este resultado

> $SolGralPrevia := \frac{1}{2}\,x^4 + \frac{1}{2}\,x^2\,y(x)^2 + \frac{1}{2}\,y(x)^4 = \_C1$

$$SolGralPrevia := \frac{1}{2}\,x^4 + \frac{1}{2}\,x^2\,y(x)^2 + \frac{1}{2}\,y(x)^4 = \_C1 \tag{36}$$

> $SolGralPreviaFinal := lhs(SolGralPrevia)\cdot 2 = \_C1$

$$SolGralPreviaFinal := y(x)^4 + x^2\,y(x)^2 + x^4 = \_C1 \tag{37}$$

>

FIN RESPUESTA 3)

> *restart*

4) Obtener la solución general de la siguiente ecuación diferencial de primer orden

> $Ecuacion := 2\,x^2\,y(x) + 2\,y(x) + 5 + \left(2\,x^3 + 2\,x\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) = 0$

$$Ecuacion := 2\,x^2\,y(x) + 2\,y(x) + 5 + \left(2\,x^3 + 2\,x\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) = 0 \tag{38}$$

>

RESPUESTA 4)

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[\_linear] \tag{39}$$

> $IntFact := intfactor(Ecuacion)$

$$IntFact := \frac{1}{x^2 + 1} \tag{40}$$

> $M := 2\,x^2\,y + 2\,y + 5$

$$M := 2\,x^2\,y + 2\,y + 5 \tag{41}$$

> $N := \left(2\,x^3 + 2\,x\right)$

$$N := 2\,x^3 + 2\,x \tag{42}$$

> $MM := IntFact\cdot M$

$$MM := \frac{2\,x^2\,y + 2\,y + 5}{x^2 + 1} \tag{43}$$

> $MMx := 2\cdot y + \frac{5}{x^2 + 1}$

$$MMx := 2\,y + \frac{5}{x^2 + 1} \tag{44}$$

> $NN := simplify(IntFact\cdot N)$

$$NN := 2\,x \tag{45}$$

> $Comprobar := simplify(diff(MM, y) = diff(NN, x))$

$$Comprobar := 2 = 2 \tag{46}$$

Por lo tanto es exacta

> $IntNNy := int(NN, y)$

$$IntNNy := 2\,x\,y \tag{47}$$

> $SolGral := IntNNy + int((MMx - diff(IntNNy, x)), x) = \_C1$

$$SolGral := 2\,x\,y + 5\arctan(x) = \_C1 \quad (48)$$

> $SolGralFinal := 2\,x\,y(x) + 5\arctan(x) = \_C1$

$$SolGralFinal := 2\,x\,y(x) + 5\arctan(x) = \_C1 \quad (49)$$

> $DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\,y(x) = -\frac{1}{2}\,\frac{2\,x^2\,y(x) + 2\,y(x) + 5}{x\,(x^2+1)} \quad (50)$$

> $DerEcua := isolate(Ecuacion, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{-2\,x^2\,y(x) - 2\,y(x) - 5}{2\,x^3 + 2\,x} \quad (51)$$

> $Comprobar := simplify(rhs(DerSolGral) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \quad (52)$$

>

FIN RESPUESTA 4)

> $restart$

5) Obtener la solución de la ecuación de primer orden lineal

> $Ecuacion := x\,(x^3+1)\left(\frac{d}{dx}\,y(x)\right) + (2\,x^3 - 1)\,y(x) = 0$

$$Ecuacion := x\,(x^3+1)\left(\frac{d}{dx}\,y(x)\right) + (2\,x^3 - 1)\,y(x) = 0 \quad (53)$$

>

RESPUESTA 5)

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[\_separable] \quad (54)$$

> $M := (2\,x^3 - 1)\,y$

$$M := y\,(2\,x^3 - 1) \quad (55)$$

> $N := expand(x\,(x^3+1))$

$$N := x^4 + x \quad (56)$$

> $P := (2\,x^3 - 1);\ Q := y;\ R := x^4 + x;\ S := 1$

$$P := 2\,x^3 - 1$$
$$Q := y$$
$$R := x^4 + x$$
$$S := 1 \quad (57)$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := \ln(x^2 - x + 1) + \ln(x + 1) - \ln(x) + \ln(y) = \_C1 \quad (58)$$

> $SolGralDos := expand(\exp(lhs(SolGral))) = \_C1$

$$SolGralDos := y\,x^2 + \frac{y}{x} = \_C1 \quad (59)$$

> $SolGralTres := y(x)x^2 + \frac{y(x)}{x} = \_C1$

$$SolGralTres := y(x)\,x^2 + \frac{y(x)}{x} = \_C1 \quad \textbf{(60)}$$

> $DerSolGral := simplify(isolate(diff(lhs(SolGralTres), x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\,y(x) = -\frac{y(x)\,(2\,x^3-1)}{x\,(x^3+1)} \quad \textbf{(61)}$$

> $DerEcua := isolate(Ecuacion, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = -\frac{y(x)\,(2\,x^3-1)}{x\,(x^3+1)} \quad \textbf{(62)}$$

> $Comprobacion := simplify(rhs(DerSolGral) - rhs(DerEcua)) = 0$

$$Comprobacion := 0 = 0 \quad \textbf{(63)}$$

>

OTRA RESPUESTA COMO LINEAL

> $Ecuacion$

$$x\,(x^3+1)\left(\frac{d}{dx}\,y(x)\right) + (2\,x^3-1)\,y(x) = 0 \quad \textbf{(64)}$$

> $EcuacionCero := diff(y(x), x) + \frac{(2\,x^3-1)\,y(x)}{x\,(x^3+1)} = 0$

$$EcuacionCero := \frac{d}{dx}\,y(x) + \frac{y(x)\,(2\,x^3-1)}{x\,(x^3+1)} = 0 \quad \textbf{(65)}$$

> $p := \frac{(2\,x^3-1)}{x\,(x^3+1)}$

$$p := \frac{2\,x^3-1}{x\,(x^3+1)} \quad \textbf{(66)}$$

> $SolGral := expand(isolate(y(x) = expand(\_C1 \cdot \exp(-int(p, x))), \_C1))$

$$SolGral := \_C1 = y(x)\,x^2 + \frac{y(x)}{x} \quad \textbf{(67)}$$

> $SolGralTres$

$$y(x)\,x^2 + \frac{y(x)}{x} = \_C1 \quad \textbf{(68)}$$

>

FIN RESPUESTA 5)

> $restart$

6) Obtener la solución particular de ecuación de primer orden lineal con condiciones iniciales

> $Ecuacion := \frac{d}{dx}\,y(x) + y(x)\cos(x) = \sin(x)\cos(x)$

$$Ecuacion := \frac{d}{dx}\,y(x) + y(x)\cos(x) = \sin(x)\cos(x) \quad \textbf{(69)}$$

> $CondIni := y(0) = 1$

$$CondIni := y(0) = 1 \quad \textbf{(70)}$$

>

RESPUESTA 6)

> $p := \cos(x)$

(71)

$$p := \cos(x) \quad (71)$$

> $q := \sin(x) \cdot \cos(x)$

$$q := \sin(x)\,\cos(x) \quad (72)$$

> $SolGral := y(x) = expand(\_C1 \cdot \exp(int(-p, x)) + \exp(int(-p, x)) \cdot int(\exp(int(p, x)) \cdot q, x))$

$$SolGral := y(x) = \frac{\_C1}{e^{\sin(x)}} + \sin(x) - 1 \quad (73)$$

> $Para := simplify(isolate(subs(x = 0, rhs(SolGral) = 1), \_C1))$

$$Para := \_C1 = 2 \quad (74)$$

> $SolPart := subs(\_C1 = rhs(Para), SolGral)$

$$SolPart := y(x) = \frac{2}{e^{\sin(x)}} + \sin(x) - 1 \quad (75)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolPart), Ecuacion)))$

$$Comprobar := \sin(x)\,\cos(x) = \sin(x)\,\cos(x) \quad (76)$$

>

FIN RESPUESTA 6)

> *restart*

FIN SERIE 1)

>

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