>

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SERIE 2
DE EJERCICIOS DEL TEMA 2
SEMESTRE 2024-1
SOLUCIÓN

2022-09-21

> *restart*

1) OBTENER LA SOLUCIÓN GENERAL DE LA ECUACIÓN DIFERENCIAL SIGUIENTE **(sin utilizar dsolve)**

> $x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{1}{2}\sqrt{x}\,(2+\ln(x))=0$

$$x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{1}{2}\sqrt{x}\,(2+\ln(x))=0 \qquad \textbf{(1)}$$

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SOLUCIÓN 1

> $Ecuacion := x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{1}{2}\sqrt{x}\,(2+\ln(x))=0$

$$Ecuacion := x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{1}{2}\sqrt{x}\,(2+\ln(x))=0 \qquad \textbf{(2)}$$

> $EcuaDos := expand\left(\frac{lhs(Ecuacion)}{x\ln(x)}\right)=0$

$$EcuaDos := \frac{d}{dx}y(x)-\frac{y(x)}{x\ln(x)}-\frac{y(x)}{x}+\frac{1}{\sqrt{x}\ln(x)}+\frac{1}{2\sqrt{x}}=0 \qquad \textbf{(3)}$$

> $EcuaTres := lhs(EcuaDos)-\left(\frac{1}{\sqrt{x}\ln(x)}+\frac{1}{2\sqrt{x}}\right)=rhs(EcuaDos)-\left(\frac{1}{\sqrt{x}\ln(x)}+\frac{1}{2\sqrt{x}}\right)$

$$EcuaTres := \frac{d}{dx}y(x)-\frac{y(x)}{x\ln(x)}-\frac{y(x)}{x}=-\frac{1}{\sqrt{x}\ln(x)}-\frac{1}{2\sqrt{x}} \qquad \textbf{(4)}$$

> $p := factor\left(-\frac{1}{x\ln(x)}-\frac{1}{x}\right);\ q := factor\left(-\frac{1}{\sqrt{x}\ln(x)}-\frac{1}{2\sqrt{x}}\right)$

$$p := -\frac{1+\ln(x)}{x\ln(x)}$$

$$q := -\frac{1}{2}\,\frac{2+\ln(x)}{\sqrt{x}\ln(x)} \qquad \textbf{(5)}$$

> $IntMasP := simplify(\exp(int(p,x)))$

$$IntMasP := \frac{1}{x\ln(x)} \qquad \textbf{(6)}$$

> $IntMenosP := simplify(\exp(-int(p,x)))$

$$IntMenosP := x\ln(x) \qquad \textbf{(7)}$$

> $SolGral := y(x) = \_C1\cdot IntMenosP + IntMenosP\cdot int(IntMasP\cdot q, x)$

$$SolGral := y(x) = \_C1\, x \ln(x) + \sqrt{x} \quad \textbf{(8)}$$

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> *restart* :

2) DADA LA ECUACIÓN DIFERENCIAL

> $\frac{d^2}{dx^2} y(x) - 4\, y(x) = 0$

$$\frac{d^2}{dx^2} y(x) - 4\, y(x) = 0 \quad \textbf{(9)}$$

a) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL HOMOGENEA **(sin utilizar dsolve)**
b) CON LA SOLUCIÓN GENERAL OBTENIDA EN EL INCISO a) Y DADAS LAS CONDICIONES INICIALES **y(0) = -6 &**
y'(0) = 8 OBTENER LA SOLUCIÓN PARTICULAR **(sin utilizar dsolve)**
c) GRAFIQUE (JUNTAS) LA SOLUCIÓN PARTICULAR OBTENIDA EN EL INCISO b) Y LA PRIMERA DERIVADA DE ÉSTA, CONSIDERNADO UN INTERVALO $0 < x < 1$

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SOLUCIÓN 2

> $Ecua := \frac{d^2}{dx^2} y(x) - 4\, y(x) = 0$

$$Ecua := \frac{d^2}{dx^2} y(x) - 4\, y(x) = 0 \quad \textbf{(10)}$$

> $EcuaCarac := m^2 - 4 = 0$

$$EcuaCarac := m^2 - 4 = 0 \quad \textbf{(11)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2, -2 \quad \textbf{(12)}$$

> $yy[1] := \exp(Raiz[1] \cdot x);\ yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_1 := e^{2x}$$

$$yy_2 := e^{-2x} \quad \textbf{(13)}$$

> $SolGral := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGral := y(x) = \_C1\, e^{2x} + \_C2\, e^{-2x} \quad \textbf{(14)}$$

> $EcuaUno := simplify(subs(x = 0, rhs(SolGral) = -6))$

$$EcuaUno := \_C1 + \_C2 = -6 \quad \textbf{(15)}$$

> $EcuaDos := simplify(subs(x = 0, rhs(diff(SolGral, x)) = 8))$

$$EcuaDos := 2\, \_C1 - 2\, \_C2 = 8 \quad \textbf{(16)}$$

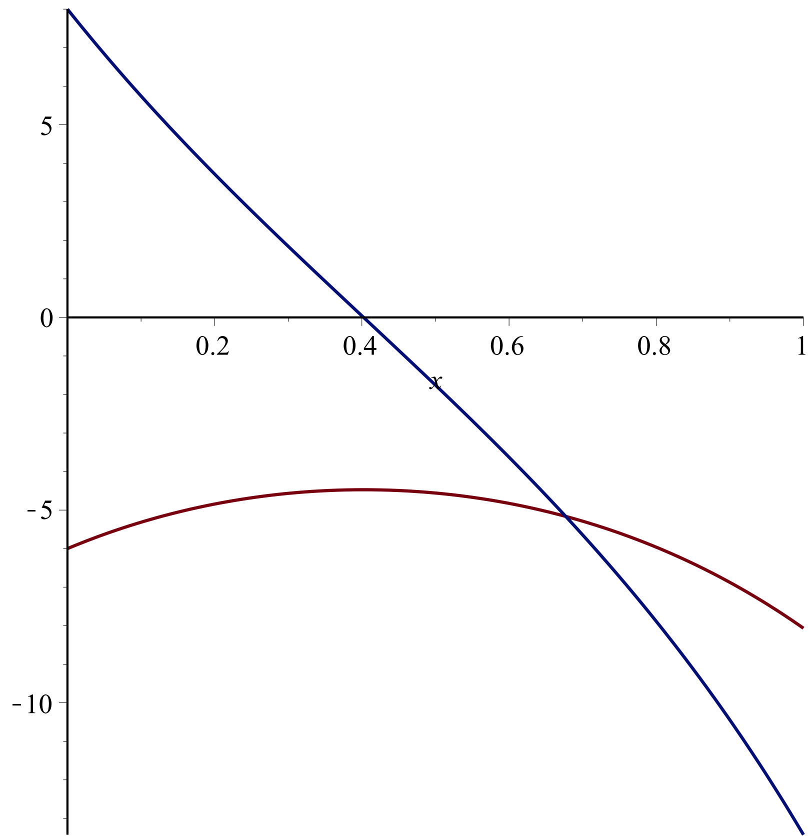
> $Para := solve(\{EcuaUno, EcuaDos\})$

$$Para := \{\_C1 = -1, \_C2 = -5\} \quad \textbf{(17)}$$

> $SolPart := subs(\_C1 = rhs(Para[1]), \_C2 = rhs(Para[2]), SolGral)$

$$SolPart := y(x) = -e^{2x} - 5\,e^{-2x} \quad (18)$$

> $plot([rhs(SolPart), rhs(diff(SolPart, x))], x = 0..1)$



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> $restart:$

3) OBTENGA Y GRAFIQUE { EN EL INTERVALO - 1..1 } LA SOLUCIÓN PARTICULAR DE LOS SIGUIENTES PROBLEMAS **(sin utilizar dsolve):**

a) CON CONDICIONES EN LA FRONTERA

> $\frac{d^3}{dx^3}\,y(x) + \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) + y(x) = 0; y(0) = -3; y\left(\frac{1}{2}\pi\right) = 3; y\left(\frac{3}{2}\pi\right) = 9$

$$\frac{d^3}{dx^3}\,y(x) + \frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) + y(x) = 0$$

$$y(0) = -3$$

$$y\left(\frac{1}{2}\pi\right) = 3 \quad (19)$$

$$y\left(\frac{3}{2}\pi\right)=9 \tag{19}$$

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SOLUCIÓN 3a)

> $Ecuacion := \frac{d^3}{dx^3}y(x)+\frac{d^2}{dx^2}y(x)+\frac{d}{dx}y(x)+y(x)=0$

$$Ecuacion := \frac{d^3}{dx^3}y(x)+\frac{d^2}{dx^2}y(x)+\frac{d}{dx}y(x)+y(x)=0 \tag{20}$$

> $EcuaCarac := m^3+m^2+m+1=0$

$$EcuaCarac := m^3+m^2+m+1=0 \tag{21}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1, I, -I \tag{22}$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \cos(\mathrm{Im}(Raiz[2])\cdot x); yy[3] := \sin(\mathrm{Im}(Raiz[2])\cdot x)$

$$yy_1 := e^{-x}$$

$$yy_2 := \cos(x)$$

$$yy_3 := \sin(x) \tag{23}$$

> $SolGral := y(x) = \_C1\cdot yy[1]+\_C2\cdot yy[2]+\_C3\cdot yy[3]$

$$SolGral := y(x) = \_C1\, e^{-x}+\_C2\cos(x)+\_C3\sin(x) \tag{24}$$

> $EcuaUno := simplify(subs(x=0, rhs(SolGral)=-3))$

$$EcuaUno := \_C1+\_C2=-3 \tag{25}$$

> $EcuaDos := simplify\left(subs\left(x=\frac{Pi}{2}, rhs(SolGral)=3\right)\right)$

$$EcuaDos := \_C1\, e^{-\frac{1}{2}\pi}+\_C3=3 \tag{26}$$

> $EcuaTres := simplify\left(subs\left(x=\frac{3\cdot Pi}{2}, rhs(SolGral)=9\right)\right)$

$$EcuaTres := \_C1\, e^{-\frac{3}{2}\pi}-\_C3=9 \tag{27}$$

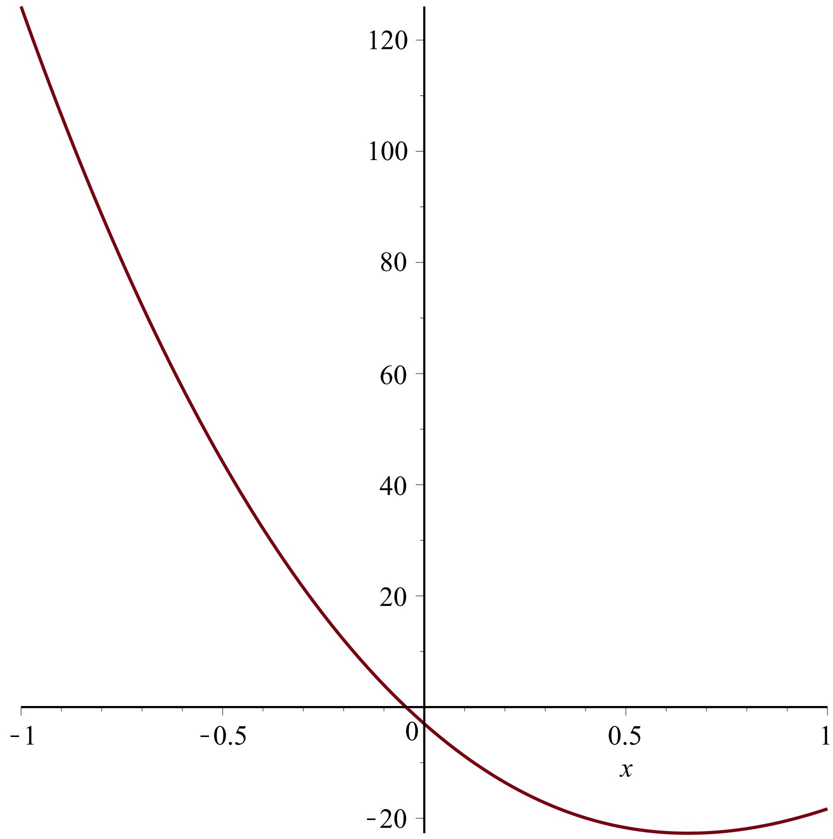
> $Para := solve(\{EcuaUno, EcuaDos, EcuaTres\})$

$$Para := \left\{\_C1=\frac{12}{e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}}, \_C2=-\frac{3\left(e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}+4\right)}{e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}}, \_C3=\frac{3\left(e^{-\frac{3}{2}\pi}-3\,e^{-\frac{1}{2}\pi}\right)}{e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}}\right\} \tag{28}$$

> $SolPart := subs(\_C1=rhs(Para[1]), \_C2=rhs(Para[2]), \_C3=rhs(Para[3]), SolGral) :$
> $evalf(\%, 3)$

$$y(x) = 55.3\, e^{-1.x}-58.2\cos(x)-8.49\sin(x) \tag{29}$$

> $plot(rhs(SolPart), x=-1..1)$



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> *restart* :

b) CON CONDICIONES INICIALES

> $\frac{d^2}{dt^2} x(t) - 7\left(\frac{d}{dt} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2; x(1) = 2; D(x)(1) =-2$

$$\frac{d^2}{dt^2} x(t) - 7\left(\frac{d}{dt} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2$$
$$x(1) = 2$$
$$D(x)(1) = -2 \qquad \textbf{(30)}$$

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SOLUCIÓN 3b)

> $Ecua := \frac{d^2}{dt^2} x(t) - 7\left(\frac{d}{dt} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2$

$$Ecua := \frac{d^2}{dt^2} x(t) - 7\left(\frac{d}{dt} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2 \qquad \textbf{(31)}$$

> $EcuaHom := lhs(Ecua) = 0; Q := rhs(Ecua)$

$$EcuaHom := \frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = 0$$

$$Q := \cos(3t) + t^2 \quad \textbf{(32)}$$

> $EcuaCarac := m^2 - 7 \cdot m + 12 = 0$

$$EcuaCarac := m^2 - 7m + 12 = 0 \quad \textbf{(33)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 4, 3 \quad \textbf{(34)}$$

> $xx[1] := \exp(Raiz[1] \cdot t); xx[2] := \exp(Raiz[2] \cdot t)$

$$xx_1 := e^{4t}$$

$$xx_2 := e^{3t} \quad \textbf{(35)}$$

> $SolGralHom := x(t) = \_C1 \cdot xx[1] + \_C2 \cdot xx[2]$

$$SolGralHom := x(t) = \_C1\, e^{4t} + \_C2\, e^{3t} \quad \textbf{(36)}$$

> $SolGralNoHom := x(t) = AA \cdot xx[1] + BB \cdot xx[2]$

$$SolGralNoHom := x(t) = AA\, e^{4t} + BB\, e^{3t} \quad \textbf{(37)}$$

> $with(linalg):$

> $WW := wronskian([xx[1], xx[2]], t)$

$$WW := \begin{bmatrix} e^{4t} & e^{3t} \\ 4\, e^{4t} & 3\, e^{3t} \end{bmatrix} \quad \textbf{(38)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \cos(3t) + t^2 \end{bmatrix} \quad \textbf{(39)}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} e^{-4t} (\cos(3t) + t^2) & -e^{-3t} (\cos(3t) + t^2) \end{bmatrix} \quad \textbf{(40)}$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := e^{-4t} (\cos(3t) + t^2)$$

$$Bprima := -e^{-3t} (\cos(3t) + t^2) \quad \textbf{(41)}$$

> $AA := simplify(int(Aprima, t) + \_C10)$

$$AA := \frac{1}{800} \left(384 \cos(t)^2 \sin(t) - 512 \cos(t)^3 - 96 \sin(t) + 384 \cos(t) + 800\, \_C10\, e^{4t} - 200 t^2 - 100 t - 25\right) e^{-4t} \quad \textbf{(42)}$$

> $BB := simplify(int(Bprima, t) + \_C20)$

$$BB := -\frac{1}{54} \left(36 \cos(t)^2 \sin(t) - 36 \cos(t)^3 - 9 \sin(t) + 27 \cos(t) - 54\, \_C20\, e^{3t} - 18 t^2 - 12 t - 4\right) e^{-3t} \quad \textbf{(43)}$$

> $SolGralNoHom$

$$x(t) = \frac{1}{800} \left(384 \cos(t)^2 \sin(t) - 512 \cos(t)^3 - 96 \sin(t) + 384 \cos(t) + 800\, \_C10\, e^{4t} \right. \quad \textbf{(44)}$$

$$- 200\, t^2 - 100\, t - 25\big)\, e^{-4t}\, e^{4t} - \frac{1}{54}\, \big(36 \cos(t)^2 \sin(t) - 36 \cos(t)^3 - 9 \sin(t) + 27 \cos(t) - 54\, \_C20\, e^{3t} - 18\, t^2 - 12\, t - 4\big)\, e^{-3t}\, e^{3t}$$

> *EcuaUno* := *subs*(*t* = 1, *rhs*(*SolGralNoHom*) = 2) : *evalf*(%, 2)

$$0.24 + 53.\ \_C10 + 21.\ \_C20 = 2. \qquad \textbf{(45)}$$

> *EcuaDos* := *subs*(*t* = 1, *rhs*(*diff*(*SolGralNoHom*, *t*)) = -2) : *evalf*(%, 2)

$$0.42 + 220.\ \_C10 + 61.\ \_C20 = -2. \qquad \textbf{(46)}$$

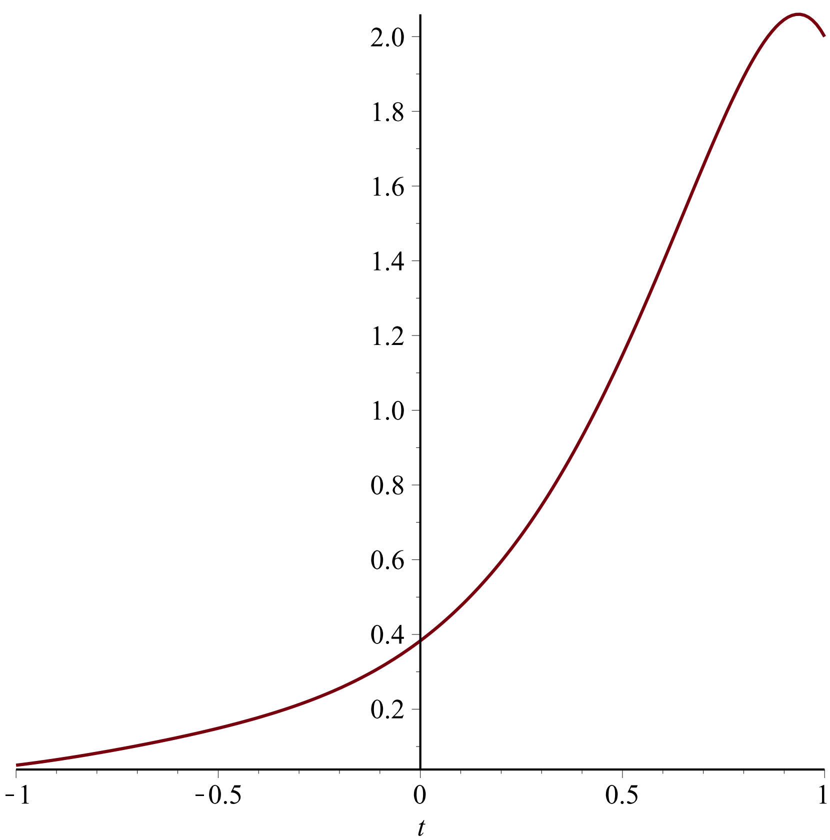
> *Param* := *solve*({*EcuaUno*, *EcuaDos*}) : *evalf*(%, 2)

$$\{\_C10 = -0.15,\ \_C20 = 0.52\} \qquad \textbf{(47)}$$

> *SolPart* := *simplify*(*subs*(_C10 = *rhs*(*Param*[1]), _C20 = *rhs*(*Param*[2]), *SolGralNoHom*)) : *evalf*(%, 2)

$$x(t) = -0.0069\, e^{3. + 4.t} + 0.0087\, e^{4. + 3.t} - 0.18 \cos(t)^2 \sin(t) + 0.027 \cos(t)^3 + 0.083\, t^2 + 0.046 \sin(t) - 0.020 \cos(t) + 0.097\, t + 0.042 \qquad \textbf{(48)}$$

> *plot*(*rhs*(*SolPart*), *t* = -1 ..1)



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> *restart* :

c) CON CONDICIONES INICIALES

> $\frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x}; y(0) = -5; D(y)(0) = 8$

$$\frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x}$$
$$y(0) = -5$$
$$D(y)(0) = 8 \quad \textbf{(49)}$$

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SOLUCIÓN 3c)

> $Ecua := \frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x}$

$$Ecua := \frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x} \quad \textbf{(50)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 0 \quad \textbf{(51)}$$

> $Q := rhs(Ecua)$

$$Q := 3 e^{2x} \quad \textbf{(52)}$$

> $EcuaCarac := m^2 + 2m + 2 = 0$

$$EcuaCarac := m^2 + 2m + 2 = 0 \quad \textbf{(53)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1 + I, -1 - I \quad \textbf{(54)}$$

> $yy[1] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \cos(\mathrm{Im}(Raiz[1]) \cdot x); yy[2] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := e^{-x} \cos(x)$$
$$yy_2 := e^{-x} \sin(x) \quad \textbf{(55)}$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGralHom := y(x) = \_C1\, e^{-x} \cos(x) + \_C2\, e^{-x} \sin(x) \quad \textbf{(56)}$$

> $SolGralNoHom := y(x) = AA \cdot yy[1] + BB \cdot yy[2]$

$$SolGralNoHom := y(x) = AA\, e^{-x} \cos(x) + BB\, e^{-x} \sin(x) \quad \textbf{(57)}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-x} \cos(x) & e^{-x} \sin(x) \\ -e^{-x} \cos(x) - e^{-x} \sin(x) & -e^{-x} \sin(x) + e^{-x} \cos(x) \end{bmatrix} \quad \textbf{(58)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 3 e^{2x} \end{bmatrix} \quad \textbf{(59)}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \left[\begin{array}{cc} -3\, e^{3x} \sin(x) & 3\, e^{3x} \cos(x) \end{array}\right] \quad (60)$$

> $Aprima := Para[1];\ Bprima := Para[2]$

$$Aprima := -3\, e^{3x} \sin(x)$$

$$Bprima := 3\, e^{3x} \cos(x) \quad (61)$$

> $AA := simplify(int(Aprima, x) + \_C10) : evalf(\%, 2)$

$$0.30\, e^{3.x} \cos(x) - 0.90\, e^{3.x} \sin(x) + \_C10 \quad (62)$$

> $BB := simplify(int(Bprima, x) + \_C20) : evalf(\%, 2)$

$$0.90\, e^{3.x} \cos(x) + 0.30\, e^{3.x} \sin(x) + \_C20 \quad (63)$$

> $SolGralNoHom$

$$y(x) = \left(\frac{3}{10} e^{3x} \cos(x) - \frac{9}{10} e^{3x} \sin(x) + \_C10\right) e^{-x} \cos(x) + \left(\frac{9}{10} e^{3x} \cos(x) + \frac{3}{10} e^{3x} \sin(x) + \_C20\right) e^{-x} \sin(x) \quad (64)$$

> $EcuaUno := subs(x = 0, rhs(SolGralNoHom) = -5) : evalf(\%, 2)$

$$0.30 + \_C10 = -5. \quad (65)$$

> $EcuaDos := subs(x = 0, rhs(diff(SolGralNoHom, x)) = 8) : evalf(\%, 2)$

$$0.60 - 1.\ \_C10 + \_C20 = 8. \quad (66)$$

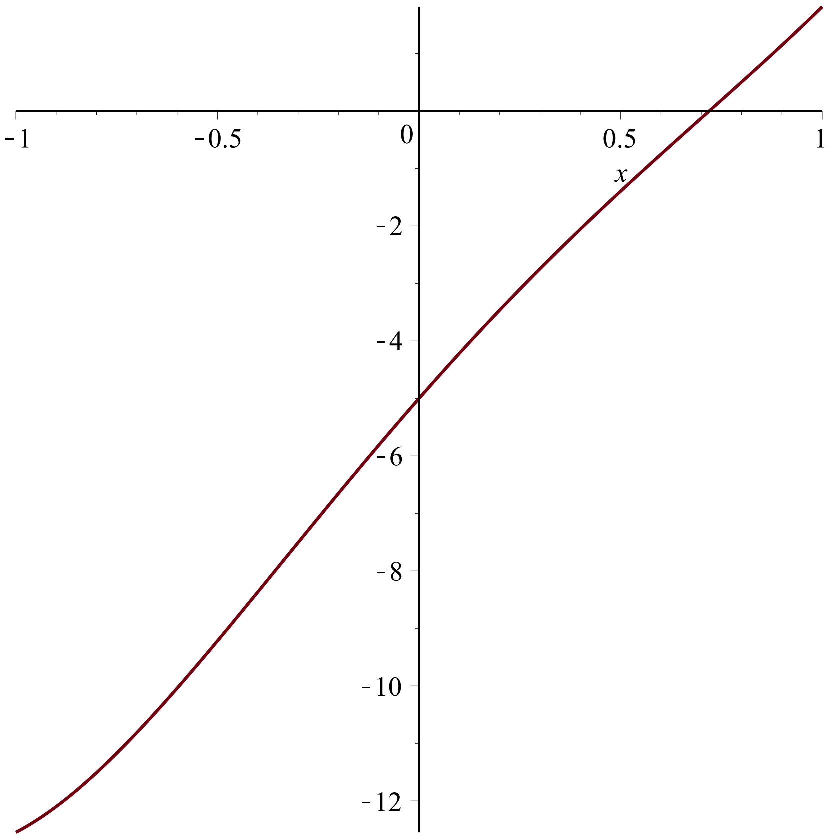
> $Param := solve(\{EcuaUno, EcuaDos\}) : evalf(\%, 2)$

$$\{\_C10 = -5.3, \_C20 = 2.1\} \quad (67)$$

> $SolPart := simplify(subs(\_C10 = rhs(Param[1]), \_C20 = rhs(Param[2]), SolGralNoHom)) : evalf(\%, 2)$

$$y(x) = 0.10\, e^{-1.x} \left(3.\, e^{3.x} - 53.\cos(x) + 21.\sin(x)\right) \quad (68)$$

> $plot(rhs(SolPart), x = -1..1)$



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<<<<<<<<<<<<<<<<<< FIN PREGUNTA 3 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> *restart* :

4) DADO EL SIGUIENTE PROBLEMA DE CONDICIONES INICIALES & UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**

> $\frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-3t} \cos(2\,t);\; y(0) = -2;\; D(y)(0) = 0;\; D^{(2)}(y)(0) = 7;\; D^{(3)}(y)(0) = -5$

$$\frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-3t} \cos(2\,t)$$
$$y(0) = -2$$
$$D(y)(0) = 0$$
$$D^{(2)}(y)(0) = 7$$
$$D^{(3)}(y)(0) = -5 \qquad \textbf{(69)}$$

a) OBTENER SU SOLUCIÓN PARTICULAR

b) GRAFICAR EL RESULTADO DEL INCISO a) EN UN INTERVALO 0 < t < 1

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SOLUCION 4a)

> $Ecua := \frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\,y(t) = 5\,e^{-3\,t}\cos(2\,t)$

$$Ecua := \frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\,y(t) = 5\,e^{-3\,t}\cos(2\,t) \quad \textbf{(70)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\,y(t) = 0 \quad \textbf{(71)}$$

> $Q := rhs(Ecua)$

$$Q := 5\,e^{-3\,t}\cos(2\,t) \quad \textbf{(72)}$$

> $EcuaCarac := m^4 + 5\cdot m^2 - 4 = 0$

$$EcuaCarac := m^4 + 5\,m^2 - 4 = 0 \quad \textbf{(73)}$$

> $Raiz := solve(EcuaCarac) : evalf(\%, 2)$

$$2.4\,I,\ -2.4\,I,\ 0.85,\ -0.85 \quad \textbf{(74)}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot t) : evalf(\%, 2);\ yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot t) : evalf(\%, 2);\ yy[3] := \exp(Raiz[3]\cdot t) : evalf(\%, 2);\ yy[4] := \exp(Raiz[4]\cdot t) : evalf(\%, 2)$

$$\cos(2.4\,t)$$
$$\sin(2.4\,t)$$
$$e^{0.85\,t}$$
$$e^{-0.85\,t} \quad \textbf{(75)}$$

> $SolGralHom := y(t) = \_C1\cdot yy[1] + \_C2\cdot yy[2] + \_C3\cdot yy[3] + \_C4\cdot yy[4] : evalf(\%, 2)$

$$y(t) = \_C1\cos(2.4\,t) + \_C2\sin(2.4\,t) + \_C3\,e^{0.85\,t} + \_C4\,e^{-0.85\,t} \quad \textbf{(76)}$$

> $SolGralNoHom := y(t) = AA\cdot yy[1] + BB\cdot yy[2] + DD\cdot yy[3] + EE\cdot yy[4] : evalf(\%, 2)$

$$y(t) = AA\cos(2.4\,t) + BB\sin(2.4\,t) + DD\,e^{0.85\,t} + EE\,e^{-0.85\,t} \quad \textbf{(77)}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2], yy[3], yy[4]], t) : evalf(\%, 2)$

$$\begin{bmatrix} \cos(2.4\,t) & \sin(2.4\,t) & e^{0.85\,t} & e^{-0.85\,t} \\ -2.4\sin(2.4\,t) & 2.4\cos(2.4\,t) & 0.85\,e^{0.85\,t} & -0.85\,e^{-0.85\,t} \\ -5.8\cos(2.4\,t) & -5.8\sin(2.4\,t) & 0.75\,e^{0.85\,t} & 0.75\,e^{-0.85\,t} \\ 13.\sin(2.4\,t) & -13.\cos(2.4\,t) & 0.59\,e^{0.85\,t} & -0.59\,e^{-0.85\,t} \end{bmatrix} \quad \textbf{(78)}$$

> $BB := array([0, 0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 0 & 5\,e^{-3\,t}\cos(2\,t) \end{bmatrix} \quad \textbf{(79)}$$

> $Para := simplify(linsolve(WW, BB)) : evalf(\%, 2)$

$$[0.31\sin(2.4\,t)\,e^{-3.\,t}\cos(2.\,t),\ -0.31\cos(2.4\,t)\,e^{-3.\,t}\cos(2.\,t),\ 0.46\,e^{-3.8\,t}\cos(2.\,t),\ -0.46\,e^{-2.2\,t}\cos(2.\,t)] \quad \textbf{(80)}$$

> $Aprima := Para[1] : evalf(\%, 2); Bprima := Para[2] : evalf(\%, 2); Dprima := Para[3] : evalf(\%, 2); Eprima := Para[4] : evalf(\%, 2)$

$$0.31\sin(2.4\,t)\,e^{-3.\,t}\cos(2.\,t)$$
$$-0.31\cos(2.4\,t)\,e^{-3.\,t}\cos(2.\,t)$$
$$0.46\,e^{-3.8\,t}\cos(2.\,t)$$
$$-0.46\,e^{-2.2\,t}\cos(2.\,t) \quad \textbf{(81)}$$

> $AA := simplify(int(Aprima, t) + \_C10) : evalf(\%, 2); BB := simplify(int(Bprima, t) + \_C20) : evalf(\%, 2); DD := simplify(int(Dprima, t) + \_C30) : evalf(\%, 2); EE := simplify(int(Eprima, t) + \_C40) : evalf(\%, 2)$

$$-0.0069\cos(0.40\,t)\,e^{-3.\,t} - 0.025\cos(4.4\,t)\,e^{-3.\,t} - 0.017\sin(4.4\,t)\,e^{-3.\,t} - 0.054\sin(0.40\,t)\,e^{-3.\,t} + 1.0\,\_C10$$
$$-0.0072\sin(0.40\,t)\,e^{-3.\,t} - 0.026\sin(4.4\,t)\,e^{-3.\,t} + 1.0\,\_C20 + 0.054\cos(0.40\,t)\,e^{-3.\,t} + 0.018\cos(4.4\,t)\,e^{-3.\,t}$$
$$-0.091\,e^{-3.8\,t}\cos(2.\,t) + 0.050\,e^{-3.8\,t}\sin(2.\,t) + 1.0\,\_C30$$
$$0.12\,e^{-2.2\,t}\cos(2.\,t) - 0.11\,e^{-2.2\,t}\sin(2.\,t) + 1.2\,\_C40 \quad \textbf{(82)}$$

> $SolGralNoHom : evalf(\%, 2)$

$$y(t) = -0.0000049\,\left(1400.\cos(0.40\,t)\,e^{-3.\,t} + 5100.\cos(4.4\,t)\,e^{-3.\,t} + 3500.\sin(4.4\,t)\,e^{-3.\,t} + 11000.\sin(0.40\,t)\,e^{-3.\,t} - 2.1\,10^{5}\,\_C10\right)\cos(2.4\,t) + 0.0000050\,\left(-1400.\sin(0.40\,t)\,e^{-3.\,t} - 5100.\sin(4.4\,t)\,e^{-3.\,t} + 2.1\,10^{5}\,\_C20 + 11000.\cos(0.40\,t)\,e^{-3.\,t} + 3500.\cos(4.4\,t)\,e^{-3.\,t}\right)\sin(2.4\,t) - 0.00038\,\left(240.\,e^{-3.8\,t}\cos(2.\,t) - 130.\,e^{-3.8\,t}\sin(2.\,t) - 2600.\,\_C30\right)e^{0.85\,t} - 0.00084\,\left(-140.\,e^{-2.2\,t}\cos(2.\,t) + 130.\,e^{-2.2\,t}\sin(2.\,t) - 1400.\,\_C40\right)e^{-0.85\,t} \quad \textbf{(83)}$$

> $EcuaUno := subs(t=0, rhs(SolGralNoHom) = -2) : evalf(\%, 2)$

$$1.0\,\_C10 + 1.0\,\_C30 + 1.2\,\_C40 = -2. \quad \textbf{(84)}$$

> $EcuaDos := subs(t=0, rhs(diff(SolGralNoHom, t)) = 0) : evalf(\%, 2)$

$$-0.01 + 2.5\,\_C20 + 0.84\,\_C30 - 1.0\,\_C40 = 0. \quad \textbf{(85)}$$

> $EcuaTres := subs(t=0, rhs(diff(SolGralNoHom, t\$2)) = 7) : evalf(\%, 2)$

$$0.28 - 5.8\,\_C10 + 0.73\,\_C30 + 0.85\,\_C40 = 7. \quad \textbf{(86)}$$

> $EcuaCuatro := subs(t=0, rhs(diff(SolGralNoHom, t\$3)) = -5) : evalf(\%, 2)$

$$-1.5 - 15.\,\_C20 + 0.63\,\_C30 - 0.75\,\_C40 = -5. \quad \textbf{(87)}$$

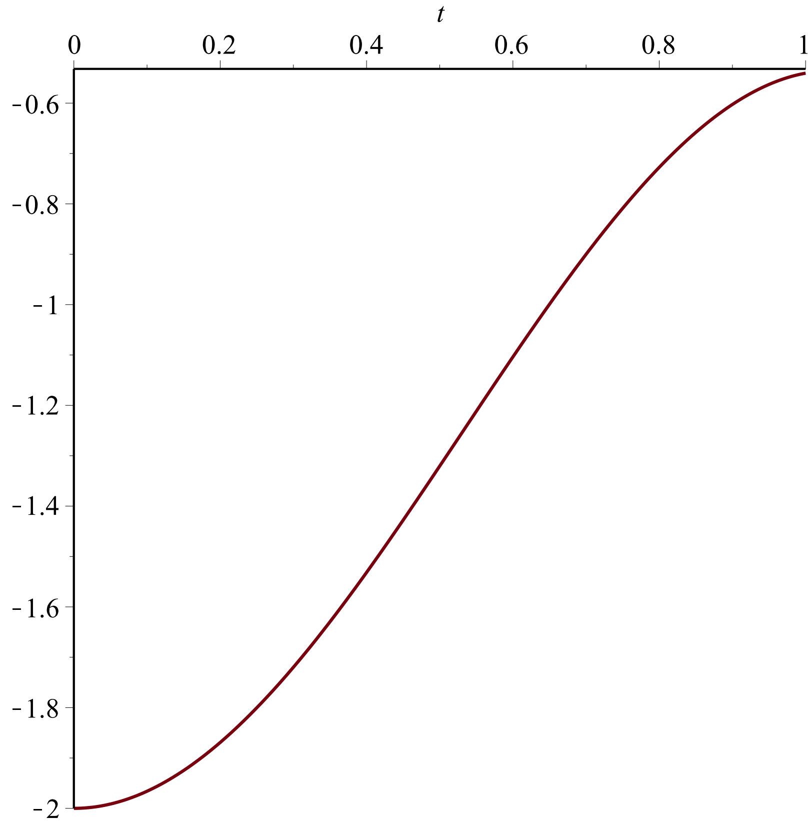
> $Param := solve(\{EcuaUno, EcuaDos, EcuaTres, EcuaCuatro\}) : evalf(\%, 2)$

$$\{\_C10 = -1.3,\ \_C20 = 0.27,\ \_C30 = -1.1,\ \_C40 = -0.28\} \quad \textbf{(88)}$$

> $SolPart := simplify(subs(\_C10 = rhs(Param[1]), \_C20 = rhs(Param[2]), \_C30 = rhs(Param[3]), \_C40 = rhs(Param[4]), SolGralNoHom)) : evalf(\%, 2)$

$$y(t) = -0.020\cos(0.40\,t)\,e^{-3.\,t}\cos(2.4\,t) - 0.020\,e^{-3.\,t}\cos(4.4\,t)\cos(2.4\,t) - 0.054\,e^{-3.\,t}\sin(4.4\,t)\cos(2.4\,t) - 0.054\,e^{-3.\,t}\sin(0.40\,t)\cos(2.4\,t) - 0.015\,e^{-3.\,t}\sin(4.4\,t)\sin(2.4\,t) - 0.020\,e^{-3.\,t}\sin(0.40\,t)\sin(2.4\,t) + 0.049\cos(0.40\,t)\,e^{-3.\,t}\sin(2.4\,t) + 0.059\,e^{-3.\,t}\cos(4.4\,t)\sin(2.4\,t) - 0.49\,e^{0.85\,t} + 0.49\,e^{-0.85\,t} + 0.24\sin(2.4\,t) - 1.5\cos(2.4\,t) - 0.049\,e^{-3.\,t}\sin(2.\,t) \quad \textbf{(89)}$$

> $plot(rhs(SolPart), t = 0..1)$



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<<<<<<<<<<<<<<<<<<<<<<FIN PREGUNTA 4 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> *restart*

5) SI SABEMOS QUE LA SOLUCIÓN GENERAL

> $y(x) = \frac{C_1}{x^2} + C_2 x$

$$y(x) = \frac{C_1}{x^2} + C_2 x \qquad \textbf{(90)}$$

SATISFACE LA ECUACIÓN DIFERENCIAL ***HOMOGÉNEA*** *SIGUIENTE*

> $-2\,y(x) + \left(\frac{d^2}{dx^2}\,y(x)\right)x^2 + 2\,x\left(\frac{d}{dx}\,y(x)\right) = 0$

$$-2\,y(x) + \left(\frac{d^2}{dx^2}\,y(x)\right)x^2 + 2\,x\left(\frac{d}{dx}\,y(x)\right) = 0 \qquad \textbf{(91)}$$

(92)

OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN NO HOMOGÉNEA UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**

> $-2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2$

$$-2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2 \quad (93)$$

>

SOLUCIÓN 5

> $SolGral := y(x)=\frac{C_1}{x^2}+C_2\,x$

$$SolGral := y(x)=\frac{C_1}{x^2}+C_2\,x \quad (94)$$

> $EcuaHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=0$

$$EcuaHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=0 \quad (95)$$

> $EcuaNoHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2$

$$EcuaNoHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2 \quad (96)$$

> $EcuaHomStandard := expand\left(\frac{lhs(EcuaHom)}{x^2}\right)=0$

$$EcuaHomStandard := -\frac{2\,y(x)}{x^2}+\frac{d^2}{dx^2}\,y(x)+\frac{2\left(\frac{d}{dx}\,y(x)\right)}{x}=0 \quad (97)$$

> $EcuaNoHomStandard := expand\left(\frac{lhs(EcuaNoHom)}{x^2}\right)=expand\left(\frac{rhs(EcuaNoHom)}{x^2}\right)$

$$EcuaNoHomStandard := -\frac{2\,y(x)}{x^2}+\frac{d^2}{dx^2}\,y(x)+\frac{2\left(\frac{d}{dx}\,y(x)\right)}{x}=32 \quad (98)$$

> $Q := rhs(EcuaNoHomStandard)$

$$Q := 32 \quad (99)$$

> $ComprobarUno := simplify(eval(subs(y(x)=rhs(SolGral), EcuaHomStandard)))$

$$ComprobarUno := 0=0 \quad (100)$$

> $SolNoHom := y(x)=\frac{AA}{x^2}+BB\cdot x$

$$SolNoHom := y(x)=\frac{AA}{x^2}+BB\,x \quad (101)$$

> $yy[1] := \frac{1}{x^2}; yy[2] := x$

$$yy_1 := \frac{1}{x^2}$$
$$yy_2 := x \qquad (102)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \frac{1}{x^2} & x \\ -\frac{2}{x^3} & 1 \end{bmatrix} \qquad (103)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 32 \end{bmatrix} \qquad (104)$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} -\frac{32}{3} x^3 & \frac{32}{3} \end{bmatrix} \qquad (105)$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -\frac{32}{3} x^3$$
$$Bprima := \frac{32}{3} \qquad (106)$$

> $AA := simplify(int(Aprima, x) + \_C1)$

$$AA := -\frac{8}{3} x^4 + \_C1 \qquad (107)$$

> $BB := simplify(int(Bprima, x) + \_C2)$

$$BB := \frac{32}{3} x + \_C2 \qquad (108)$$

> $SolFinal := expand(SolNoHom)$

$$SolFinal := y(x) = 8 x^2 + \frac{\_C1}{x^2} + x \_C2 \qquad (109)$$

> $comprobarDos := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(EcuaNoHom) - rhs(EcuaNoHom) = 0)))$

$$comprobarDos := 0 = 0 \qquad (110)$$

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<<<<<<<<<<<<<<<<<<<<<<<FIN PREGUNTA 5 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

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FINAL SERIE 2

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