> restart

SERIE 2024-1-4
Solución

1) Determine una solución completa de la ecuación diferencial en derivadas parciales, para una constante de separación dada

>

> Ecua := 2·diff(z(x, y), x$2, y) − 2·diff(z(x, y), x, y) = z(x, y); alpha := 1

$$Ecua := 2\left(\frac{\partial^3}{\partial y\,\partial x^2} z(x,y)\right) - 2\left(\frac{\partial^2}{\partial y\,\partial x} z(x,y)\right) = z(x,y)$$

$$\alpha := 1 \quad \textbf{(1)}$$

RESPUESTA 1)

> EcuaSeparable := eval(subs(z(x, y) = F(x)·G(y), Ecua))

$$EcuaSeparable := 2\left(\frac{d^2}{dx^2} F(x)\right)\left(\frac{d}{dy} G(y)\right) - 2\left(\frac{d}{dx} F(x)\right)\left(\frac{d}{dy} G(y)\right) = F(x)\,G(y) \quad \textbf{(2)}$$

> EcuaSeparada := simplify(lhs(EcuaSeparable) / (2·diff(G(y), y)·F(x)) = rhs(EcuaSeparable) / (2·diff(G(y), y)·F(x)))

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = \frac{1}{2}\,\frac{G(y)}{\frac{d}{dy} G(y)} \quad \textbf{(3)}$$

> EcuaX := lhs(EcuaSeparada) = alpha

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = 1 \quad \textbf{(4)}$$

> EcuaY := rhs(EcuaSeparada) = alpha

$$EcuaY := \frac{1}{2}\,\frac{G(y)}{\frac{d}{dy} G(y)} = 1 \quad \textbf{(5)}$$

> SolX := dsolve(EcuaX)

$$SolX := F(x) = \_C1\, e^{\frac{1}{2}\left(\sqrt{5}+1\right)x} + \_C2\, e^{-\frac{1}{2}\left(\sqrt{5}-1\right)x} \quad \textbf{(6)}$$

> SolY := dsolve(EcuaY)

$$SolY := G(y) = \_C1\, e^{\frac{1}{2}y} \quad \textbf{(7)}$$

> SolGral := z(x, y) = subs(_C1 = 1, rhs(SolY))·rhs(SolX)

$$SolGral := z(x,y) = e^{\frac{1}{2}y}\left(\_C1\, e^{\frac{1}{2}\left(\sqrt{5}+1\right)x} + \_C2\, e^{-\frac{1}{2}\left(\sqrt{5}-1\right)x}\right) \quad \textbf{(8)}$$

FIN RESPUESTA 1)

> restart

2) Obtener la solución completa de la ecuación, considerando una constante de separación dada

> Ecua := t·diff(u(x, t), t) = x·diff(u(x, t), x); alpha := −1

$$Ecua := t\left(\frac{\partial}{\partial t}\,u(x,t)\right) = x\left(\frac{\partial}{\partial x}\,u(x,t)\right)$$

$$\alpha := -1 \qquad (9)$$

RESPUESTA 2)

> $EcuaSeparable := eval(subs(u(x,t) = F(x)\cdot G(t), Ecua))$

$$EcuaSeparable := t\,F(x)\left(\frac{d}{dt}\,G(t)\right) = x\left(\frac{d}{dx}\,F(x)\right)G(t) \qquad (10)$$

> $EcuaSeparada := \dfrac{lhs(EcuaSeparable)}{F(x)\cdot G(t)} = \dfrac{rhs(EcuaSeparable)}{F(x)\cdot G(t)}$

$$EcuaSeparada := \frac{t\left(\frac{d}{dt}\,G(t)\right)}{G(t)} = \frac{x\left(\frac{d}{dx}\,F(x)\right)}{F(x)} \qquad (11)$$

> $EcuaX := rhs(EcuaSeparada) = \text{alpha}$

$$EcuaX := \frac{x\left(\frac{d}{dx}\,F(x)\right)}{F(x)} = -1 \qquad (12)$$

> $EcuaT := lhs(EcuaSeparada) = \text{alpha}$

$$EcuaT := \frac{t\left(\frac{d}{dt}\,G(t)\right)}{G(t)} = -1 \qquad (13)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \frac{\_C1}{x} \qquad (14)$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = \frac{\_C1}{t} \qquad (15)$$

> $SolGral := u(x,t) = rhs(SolX)\cdot subs(\_C1 = 1, rhs(SolT))$

$$SolGral := u(x,t) = \frac{\_C1}{x\,t} \qquad (16)$$

FIN RESPUESTA 2)

> $restart$

3) Obtenga la Serie Trigonométrica de Fourier de la función f(x) en el intervalo dado

> $f := x + \text{Pi};\ -\text{Pi} \le x \le \text{Pi}$

$$f := x + \pi$$

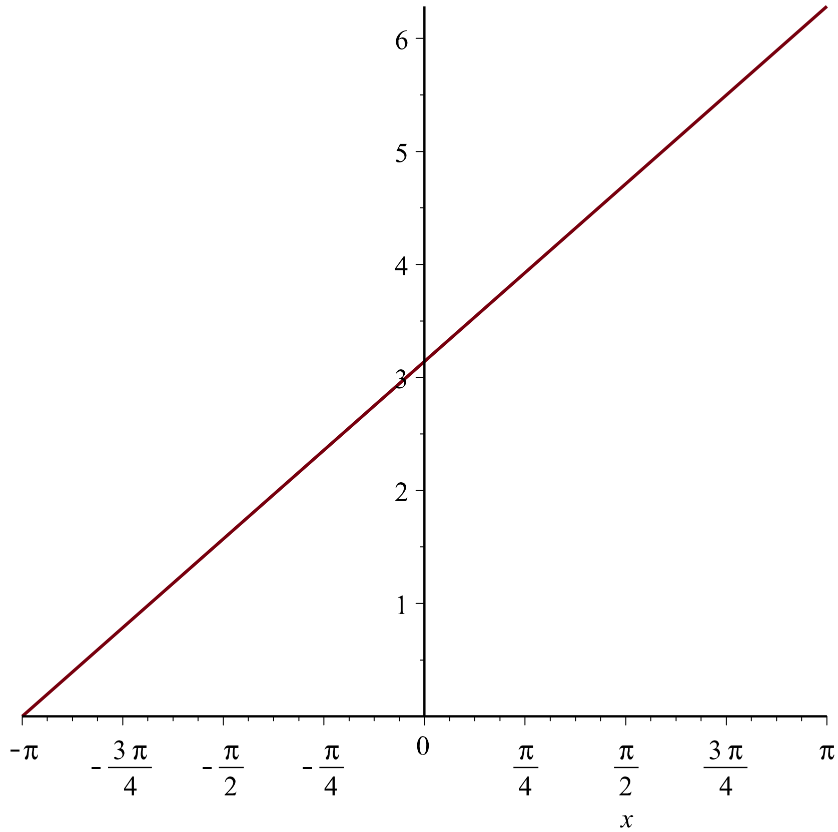
$$-\pi \le x \ \textbf{and}\ x \le \pi \qquad (17)$$

RESPUESTA 3)

> $L := \text{Pi}$

$$L := \pi \qquad (18)$$

> $plot(f, x = -L..L)$



> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := 2\pi \quad \textbf{(19)}$$

> $C := \frac{a[0]}{2}$

$$C := \pi \quad \textbf{(20)}$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$a_n := 0 \quad \textbf{(21)}$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

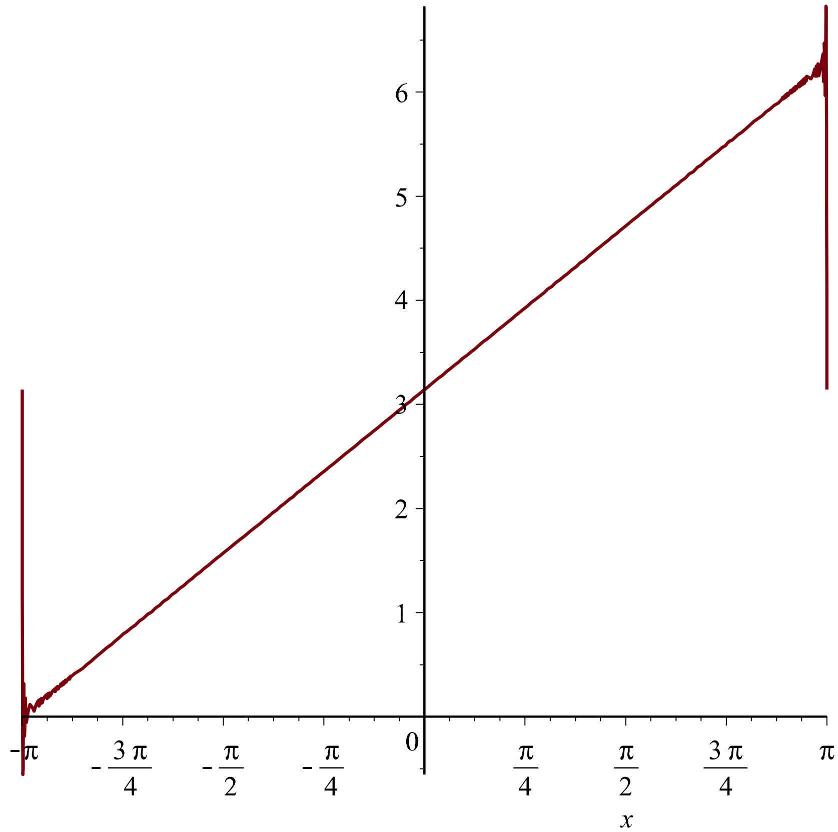
$$b_n := -\frac{2(-1)^n}{n} \quad \textbf{(22)}$$

> $STF := C + Sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..infinity\right)$

$$STF := \pi + \sum_{n=1}^{\infty}\left(-\frac{2(-1)^n \sin(n x)}{n}\right) \quad \textbf{(23)}$$

> $STF500 := C + sum\left(b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..500\right):$

> $plot(STF500, x = -L..L)$

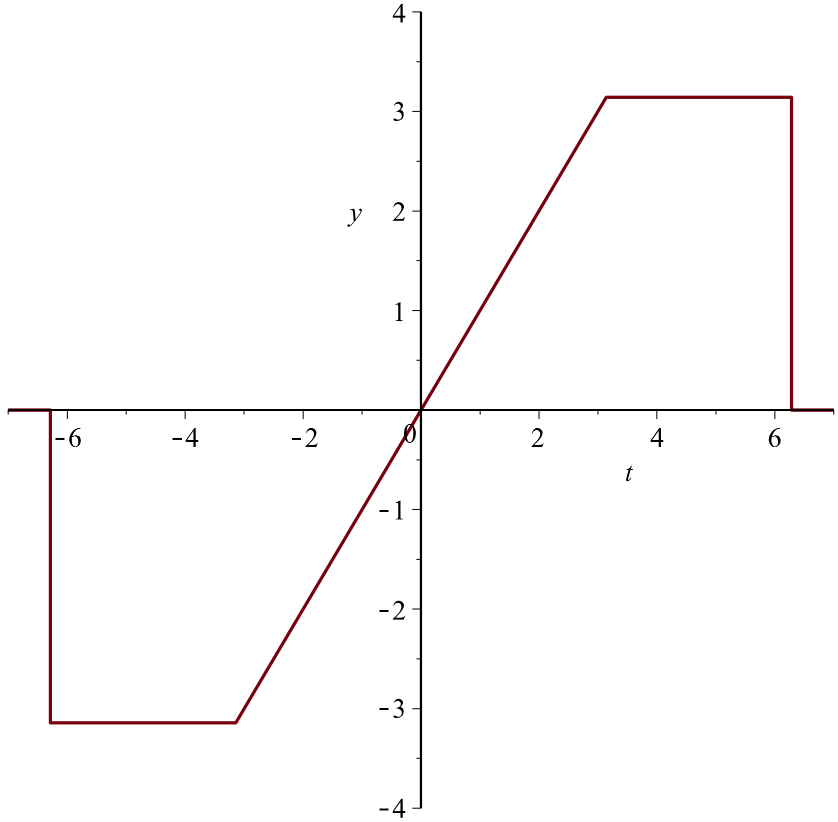


FIN RESPUESTA 3)

> *restart*

4) Obtenga ls Serie Trigonométrica de Fourier de la función cuya gráfica se muestra a continuación

> $f := -\mathrm{Pi}\cdot\mathrm{Heaviside}(t + 2\cdot\mathrm{Pi}) + (t + \mathrm{Pi})\cdot\mathrm{Heaviside}(t + \mathrm{Pi}) - (t - \mathrm{Pi})\cdot\mathrm{Heaviside}(t - \mathrm{Pi}) - \mathrm{Pi}\cdot\mathrm{Heaviside}(t - 2\cdot\mathrm{Pi}) : plot(f, t = -7..7, y = -4..4)$



RESPUESTA 4)

> $L := 2 \cdot \mathrm{Pi}$

$$L := 2\,\pi \qquad \textbf{(24)}$$

> $a[0] := \frac{1}{L} \cdot int(f, t = -L..L)$

$$a_0 := 0 \qquad \textbf{(25)}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$a_n := 0 \qquad \textbf{(26)}$$

> $b[n] := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

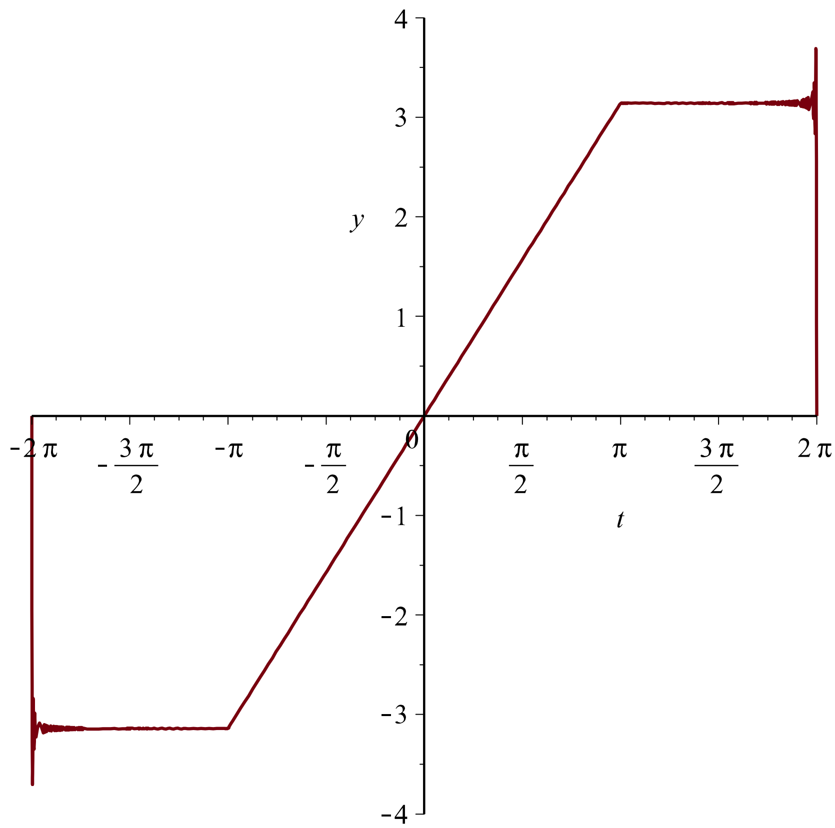
$$b_n := \frac{1}{2} \frac{-4\, n\, \pi \cos(n\, \pi) + 8 \sin\left(\frac{1}{2}\, n\, \pi\right)}{\pi\, n^2} \qquad \textbf{(27)}$$

> $STF := Sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..infinity\right)$

$$STF := \sum_{n=1}^{\infty} \frac{1}{2} \frac{\left(-4 n \pi \cos(n \pi) + 8 \sin\left(\frac{1}{2} n \pi\right)\right) \sin\left(\frac{1}{2} n t\right)}{\pi n^2} \quad \textbf{(28)}$$

> $STF500 := sum\left(b[n]\sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..500\right)$:

> $plot(STF500, t = -2 \cdot \mathrm{Pi}..2 \cdot \mathrm{Pi}, y = -4..4)$



FIN RESPUESTA 4)

> *restart*

5) Obtener la solución completa de la ecuación, considerando una constante de separación dada

> $Ecua := diff(u(x, y), x\$2) + 4 \cdot diff(u(x, y), x, y) + 4 \cdot diff(u(x, y), y) = 0;$ alpha := 1

$$Ecua := \frac{\partial^2}{\partial x^2} u(x, y) + 4 \left(\frac{\partial^2}{\partial y \, \partial x} u(x, y)\right) + 4 \left(\frac{\partial}{\partial y} u(x, y)\right) = 0$$

$$\alpha := 1 \quad \textbf{(29)}$$

RESPUESTA 5)

> $EcuaSeparable := simplify(eval(subs(u(x, y) = F(x) \cdot G(y), Ecua)))$

$$EcuaSeparable := \left(\frac{d^2}{dx^2} F(x)\right) G(y) + 4 \left(\frac{d}{dx} F(x)\right) \left(\frac{d}{dy} G(y)\right) + 4 F(x) \left(\frac{d}{dy} G(y)\right) \quad \textbf{(30)}$$

$$= 0$$

> EcuaSeparada

$$:= simplify\left(\frac{1}{\left(-4\left(\frac{d}{dx}F(x)\right)-4F(x)\right)\cdot G(y)}\left(lhs(EcuaSeparable) - \left(4\left(\frac{d}{dx}F(x)\right)\left(\frac{d}{dy}G(y)\right)+4F(x)\left(\frac{d}{dy}G(y)\right)\right)\right) = \frac{\left(rhs(EcuaSeparable) - \left(4\left(\frac{d}{dx}F(x)\right)\left(\frac{d}{dy}G(y)\right)+4F(x)\left(\frac{d}{dy}G(y)\right)\right)\right)}{\left(-4\left(\frac{d}{dx}F(x)\right)-4F(x)\right)\cdot G(y)}\right)$$

$$EcuaSeparada := -\frac{1}{4}\,\frac{\frac{d^2}{dx^2}F(x)}{\frac{d}{dx}F(x)+F(x)} = \frac{\frac{d}{dy}G(y)}{G(y)} \quad \textbf{(31)}$$

> EcuaX := lhs(EcuaSeparada) = alpha

$$EcuaX := -\frac{1}{4}\,\frac{\frac{d^2}{dx^2}F(x)}{\frac{d}{dx}F(x)+F(x)} = 1 \quad \textbf{(32)}$$

> EcuaY := rhs(EcuaSeparada) = alpha

$$EcuaY := \frac{\frac{d}{dy}G(y)}{G(y)} = 1 \quad \textbf{(33)}$$

> SolX := dsolve(EcuaX)

$$SolX := F(x) = \_C1\, e^{-2x} + \_C2\, e^{-2x}x \quad \textbf{(34)}$$

> SolY := dsolve(EcuaY)

$$SolY := G(y) = \_C1\, e^{y} \quad \textbf{(35)}$$

> SolGral := u(x, y) = rhs(SolX) · subs(_C1 = 1, rhs(SolY))

$$SolGral := u(x,y) = \left(\_C1\, e^{-2x} + \_C2\, e^{-2x}x\right) e^{y} \quad \textbf{(36)}$$

FIN RESPUESTA 5)

> restart

6) Obtener la solución completa de la ecuación diferencial en derivadas parciales, considerando una constante de separación positiva

> Ecua := diff(u(x, y), x) − diff(u(x, y), y) − u(x, y) = 0

$$Ecua := \frac{\partial}{\partial x}u(x,y) - \left(\frac{\partial}{\partial y}u(x,y)\right) - u(x,y) = 0 \quad \textbf{(37)}$$

RESPUESTA 6)

> EcuaSeparable := eval(subs(u(x, y) = F(x) · G(y), Ecua))

$$EcuaSeparable := \left(\frac{d}{dx}F(x)\right)G(y) - F(x)\left(\frac{d}{dy}G(y)\right) - F(x)\,G(y) = 0 \quad \textbf{(38)}$$

> $EcuaSeparada := \frac{\left(lhs(EcuaSeparable) - \left(-F(x)\left(\frac{d}{dy}G(y)\right) - F(x)\,G(y)\right)\right)}{F(x)\cdot G(y)} = simplify\left(\frac{\left(rhs(EcuaSeparable) - \left(-F(x)\left(\frac{d}{dy}G(y)\right) - F(x)\,G(y)\right)\right)}{F(x)\cdot G(y)}\right)$

$$EcuaSeparada := \frac{\frac{d}{dx}F(x)}{F(x)} = \frac{\frac{d}{dy}G(y) + G(y)}{G(y)} \quad \textbf{(39)}$$

> $EcuaX := lhs(EcuaSeparada) = \beta^2$

$$EcuaX := \frac{\frac{d}{dx}F(x)}{F(x)} = \beta^2 \quad \textbf{(40)}$$

> $EcuaY := rhs(EcuaSeparada) = \beta^2$

$$EcuaY := \frac{\frac{d}{dy}G(y) + G(y)}{G(y)} = \beta^2 \quad \textbf{(41)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1\, e^{\beta^2 x} \quad \textbf{(42)}$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = \_C1\, e^{(\beta - 1)(\beta + 1)y} \quad \textbf{(43)}$$

> $SolGral := u(x, y) = rhs(SolX)\cdot subs(\_C1 = 1, rhs(SolY))$

$$SolGral := u(x, y) = \_C1\, e^{\beta^2 x}\, e^{(\beta - 1)(\beta + 1)y} \quad \textbf{(44)}$$

FIN RESPUESTA 6)

> *restart*

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