>

Examen Ecuaciones Diferenciales
FINAL 2
Grupo 13
Semestre 2025-1

> *restart*

1) Sea la ecuación diferencial.....resuélvala

> $Ecua := y + (y + x - x\cdot y)\cdot y' = 0$

$$Ecua := y(x) + (y(x) + x - x\,y(x))\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{1}$$

> $with(DEtools):$

> $intfactor(Ecua)$

$$\frac{1}{e^{y(x)}} \tag{2}$$

> $FacInt := \frac{1}{e^{y}}$

$$FacInt := \frac{1}{e^{y}} \tag{3}$$

> $M := y$

$$M := y \tag{4}$$

> $N := y + x - x\cdot y$

$$N := -x\,y + x + y \tag{5}$$

> $diff(M, y) \neq diff(N, x)$

$$1 \neq -y + 1 \tag{6}$$

NO ES EXACTA

> $MM := M\cdot FacInt$

$$MM := \frac{y}{e^{y}} \tag{7}$$

> $NN := N\cdot FacInt$

$$NN := \frac{-x\,y + x + y}{e^{y}} \tag{8}$$

> $Comprobar := simplify(diff(MM, y) - diff(NN, x)) = 0$

$$Comprobar := 0 = 0 \tag{9}$$

YA ES EXACTA

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y\,x}{e^{y}} \tag{10}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y\,x}{e^{y}} - \frac{y + 1}{e^{y}} = \_C1 \tag{11}$$

> $SolFinal := \frac{y(x) \cdot x}{e^{y(x)}} - \frac{y(x)+1}{e^{y(x)}} = \_C1$

$$SolFinal := \frac{y(x)\, x}{e^{y(x)}} - \frac{y(x)+1}{e^{y(x)}} = \_C1 \qquad \textbf{(12)}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{y(x)}{(-1+x)\, y(x) - x} \qquad \textbf{(13)}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = -\frac{y(x)}{y(x) + x - x\, y(x)} \qquad \textbf{(14)}$$

> $ComprobarDos := simplify(rhs(DerEcua) - rhs(DerSolFinal)) = 0$

$$ComprobarDos := 0 = 0 \qquad \textbf{(15)}$$

> $restart$

2) Resuelva la ecuación diferencial

> $Ecua := y'' + y = \frac{1}{\cos(x)}$

$$Ecua := \frac{d^2}{dx^2}\, y(x) + y(x) = \frac{1}{\cos(x)} \qquad \textbf{(16)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) + y(x) = 0 \qquad \textbf{(17)}$$

> $Q := rhs(Ecua)$

$$Q := \frac{1}{\cos(x)} \qquad \textbf{(18)}$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \qquad \textbf{(19)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \mathrm{I}, -\mathrm{I} \qquad \textbf{(20)}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(x) \qquad \textbf{(21)}$$

> $yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_2 := \sin(x) \qquad \textbf{(22)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \qquad \textbf{(23)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{1}{\cos(x)} \end{bmatrix} \qquad \textbf{(24)}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} -\tan(x) & 1 \end{bmatrix} \quad (25)$$

> $Aprima := Para[1]$

$$Aprima := -\tan(x) \quad (26)$$

> $Bprima := Para[2]$

$$Bprima := 1 \quad (27)$$

> $SolGral := y(x) = expand((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = \cos(x)\ \ln(\cos(x)) + \cos(x)\ \_C1 + \sin(x)\ x + \sin(x)\ \_C2 \quad (28)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))$

$$Comprobar := \sec(x) = \sec(x) \quad (29)$$

> $restart$

3) Resolver el siguiente sistema de ecuaciones diferenciales con condiciones iniciales

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> $Sistema := diff(x[1](t), t) = x[1](t) + 9 \cdot x[2](t), diff(x[2](t), t) = x[1](t) + x[2](t) :$
$Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\ x_1(t) = x_1(t) + 9\ x_2(t)$$

$$\frac{d}{dt}\ x_2(t) = x_1(t) + x_2(t) \quad (30)$$

> $CondIni := x[1](0) = 1, x[2](0) = 1$

$$CondIni := x_1(0) = 1, x_2(0) = 1 \quad (31)$$

> $AA := array([[1, 9], [1, 1]])$

$$AA := \begin{bmatrix} 1 & 9 \\ 1 & 1 \end{bmatrix} \quad (32)$$

> $Xcero := array([1, 1])$

$$Xcero := \begin{bmatrix} 1 & 1 \end{bmatrix} \quad (33)$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{e^{-2\,t}}{2} + \frac{e^{4\,t}}{2} & \frac{3\ e^{4\,t}}{2} - \frac{3\ e^{-2\,t}}{2} \\ \frac{e^{4\,t}}{6} - \frac{e^{-2\,t}}{6} & \frac{e^{-2\,t}}{2} + \frac{e^{4\,t}}{2} \end{bmatrix} \quad (34)$$

> $SolPart := evalm(MatExp\ \&*\ Xcero) : x[1](t) = SolPart[1]; x[2](t) = SolPart[2]$

$$x_1(t) = -e^{-2\,t} + 2\ e^{4\,t}$$

$$x_2(t) = \frac{2\ e^{4\,t}}{3} + \frac{e^{-2\,t}}{3} \quad (35)$$

> $ComprobarUno := simplify(eval(subs(x[1](t) = SolPart[1], x[2](t) = SolPart[2],$
$Sistema[1])))$

(36)

$$ComprobarUno := 2\,e^{-2t} + 8\,e^{4t} = 2\,e^{-2t} + 8\,e^{4t} \tag{36}$$

> $ComprobarDos := simplify(eval(subs(x[1](t) = SolPart[1], x[2](t) = SolPart[2], Sistema[2])))$

$$ComprobarDos := -\frac{2\,e^{-2t}}{3} + \frac{8\,e^{4t}}{3} = -\frac{2\,e^{-2t}}{3} + \frac{8\,e^{4t}}{3} \tag{37}$$

> $restart$

4) Resuelva el problema de valor inicial

> $Ecua := diff(x(t), t\$2) + diff(x(t), t) - 6 \cdot x(t) = 30 \cdot \mathrm{Heaviside}(t - \mathrm{Pi})$

$$Ecua := \frac{d^2}{dt^2}\,x(t) + \frac{d}{dt}\,x(t) - 6\,x(t) = 30\,\mathrm{Heaviside}(t - \pi) \tag{38}$$

> $CondIni := x(0) = 0, \mathrm{D}(x)(0) = 0$

$$CondIni := x(0) = 0, \mathrm{D}(x)(0) = 0 \tag{39}$$

> $with(inttrans):$

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2\,\mathcal{L}(x(t), t, s) + s\,\mathcal{L}(x(t), t, s) - 6\,\mathcal{L}(x(t), t, s) = \frac{30\,e^{-s\pi}}{s} \tag{40}$$

> $SolTL := isolate(EcuaTL, laplace(x(t), t, s))$

$$SolTL := \mathcal{L}(x(t), t, s) = \frac{30\,e^{-s\pi}}{s\,(s^2 + s - 6)} \tag{41}$$

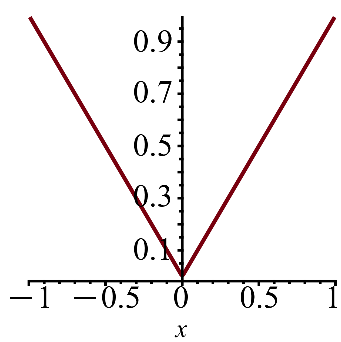
> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := x(t) = \mathrm{Heaviside}(t - \pi)\,\left(-5 + 2\,e^{-3t + 3\pi} + 3\,e^{2t - 2\pi}\right) \tag{42}$$

> $restart$

5) Desarrolle en serie de Fourier la función periódica, definida en el intervalo dado

> $f := \mathrm{abs}(x) : plot(f, x = -1..1)$



> $L := 1$

$$L := 1 \tag{43}$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := 1 \tag{44}$$

> $a[n] := subs\left(\cos(n \cdot \mathrm{Pi}) = (-1)^n, \sin(n \cdot \mathrm{Pi}) = 0, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

(45)

$$a_n := \frac{2\left(-1+(-1)^n\right)}{n^2\,\pi^2} \qquad \textbf{(45)}$$

> $b[n] := subs\left(\cos(n\cdot\mathrm{Pi}) = (-1)^n, \sin(n\cdot\mathrm{Pi}) = 0, \frac{1}{L}\cdot int\left(f\cdot\sin\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)\right)$

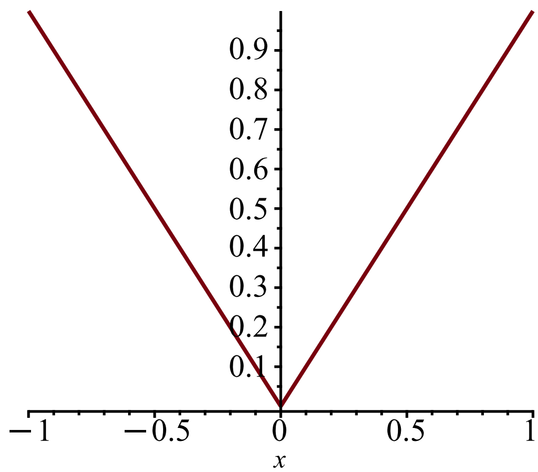
$$b_n := 0 \qquad \textbf{(46)}$$

> $STF := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), n = 1..\mathrm{infinity}\right)$

$$STF := \frac{1}{2} + \left(\sum_{n=1}^{\infty} \frac{2\left(-1+(-1)^n\right)\cos(n\,\pi\,x)}{n^2\,\pi^2}\right) \qquad \textbf{(47)}$$

> $STF500 := \frac{a[0]}{2} + sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), n = 1..500\right):$

> $plot(STF500, x = -1..1)$



> $restart$

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