>

Examen Ecuaciones Diferenciales
FINAL
Grupo 13
Semestre 2025-1
Fila A

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1a) Resuelva el problema de valor inicial

> $Ecua := y' = \frac{(x \cdot y^2 - 1)}{1 - x^2 \cdot y}$

$$Ecua := \frac{d}{dx}\, y(x) = \frac{x\, y(x)^2 - 1}{1 - x^2\, y(x)} \quad \textbf{(1)}$$

> $CondIni := y(0) = 1$

$$CondIni := y(0) = 1 \quad \textbf{(2)}$$

>

RESPUESTA

> $EcuaDos := -rhs(Ecua) + lhs(Ecua) = 0$

$$EcuaDos := -\frac{x\, y(x)^2 - 1}{1 - x^2\, y(x)} + \frac{d}{dx}\, y(x) = 0 \quad \textbf{(3)}$$

> $EcuaTres := -x\, y(x)^2 + 1 + (1 - x^2\, y(x)) \cdot diff(y(x), x) = 0$

$$EcuaTres := -x\, y(x)^2 + 1 + (1 - x^2\, y(x)) \left(\frac{d}{dx}\, y(x)\right) = 0 \quad \textbf{(4)}$$

> with(DEtools) :

> odeadvisor(EcuaTres)

[_exact, _rational, [_Abel, 2nd type, class B]] **(5)**

> $M := -x\, y^2 + 1$

$$M := -x\, y^2 + 1 \quad \textbf{(6)}$$

> $N := 1 - x^2\, y$

$$N := -x^2\, y + 1 \quad \textbf{(7)}$$

> $IntMx := int(M, x)$

$$IntMx := -\frac{1}{2}\, x^2\, y^2 + x \quad \textbf{(8)}$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGral := -\frac{1}{2}\, x^2\, y^2 + x + y = \_C1 \quad \textbf{(9)}$$

> $SolGralFinal := -\frac{1}{2}\, y(x)^2\, x^2 + x + y(x) = \_C1$

$$SolGralFinal := -\frac{y(x)^2\, x^2}{2} + x + y(x) = \_C1 \quad \textbf{(10)}$$

> $DerSolFinal := isolate(diff(SolGralFinal, x), diff(y(x), x))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{x\, y(x)^2 - 1}{1 - x^2\, y(x)} \tag{11}$$

> $Ecua$

$$\frac{d}{dx}\, y(x) = \frac{x\, y(x)^2 - 1}{1 - x^2\, y(x)} \tag{12}$$

> $Comprobar := rhs(Ecua) - rhs(DerSolFinal) = 0$

$$Comprobar := 0 = 0 \tag{13}$$

> $Parametro := subs(y(x) = 1, x = 0, SolGralFinal)$

$$Parametro := 1 = \_C1 \tag{14}$$

> $SolPart := subs(\_C1 = lhs(Parametro), SolGralFinal)$

$$SolPart := -\frac{y(x)^2\, x^2}{2} + x + y(x) = 1 \tag{15}$$

> $restart$

2a) Resuelva la ecuación diferencial

> $Ecua := y'' + y = 3 \cdot \sec(x) + 1$

$$Ecua := \frac{d^2}{dx^2}\, y(x) + y(x) = 3 \sec(x) + 1 \tag{16}$$

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RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) + y(x) = 0 \tag{17}$$

> $Q := rhs(Ecua)$

$$Q := 3 \sec(x) + 1 \tag{18}$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \tag{19}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \mathrm{I}, -\mathrm{I} \tag{20}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x); yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(x)$$

$$yy_2 := \sin(x) \tag{21}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \tag{22}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 3 \sec(x) + 1 \end{bmatrix} \tag{23}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \left[\begin{array}{cc} -3\tan(x)-\sin(x) & 3+\cos(x)\end{array}\right] \quad (24)$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -3\tan(x)-\sin(x)$$

$$Bprima := 3+\cos(x) \quad (25)$$

> $SolGral := y(x) = expand((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGral := y(x) = \cos(x)^2 + 3\cos(x)\ln(\cos(x)) + \cos(x)\ \_C1 + 3\sin(x)\,x + \sin(x)^2 + \sin(x)\ \_C2 \quad (26)$$

> $SolFinal := simplify(SolGral)$

$$SolFinal := y(x) = (\_C2 + 3x)\sin(x) + 1 + 3\cos(x)\ln(\cos(x)) + \cos(x)\ \_C1 \quad (27)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (28)$$

> $restart$

3a) Resolver

> $Ecua := diff(y(t), t\$2) + y(t) = 4\cdot \mathrm{Dirac}(t - 2\cdot \mathrm{Pi})$

$$Ecua := \frac{d^2}{dt^2}\,y(t) + y(t) = 4\,\mathrm{Dirac}(t - 2\pi) \quad (29)$$

> $CondIni := y(0) = 1, \mathrm{D}(y)(0) = 1$

$$CondIni := y(0) = 1, \mathrm{D}(y)(0) = 1 \quad (30)$$

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RESPUESTA

> $with(inttrans):$

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2\,\mathcal{L}(y(t), t, s) - 1 - s + \mathcal{L}(y(t), t, s) = 4\,\mathrm{e}^{-2s\pi} \quad (31)$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{4\,\mathrm{e}^{-2s\pi} + s + 1}{s^2 + 1} \quad (32)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \cos(t) + \sin(t)\,(1 + 4\,\mathrm{Heaviside}(t - 2\pi)) \quad (33)$$

> $CondIniUno := simplify(subs(t = 0, SolPart))$

$$CondIniUno := y(0) = 1 \quad (34)$$

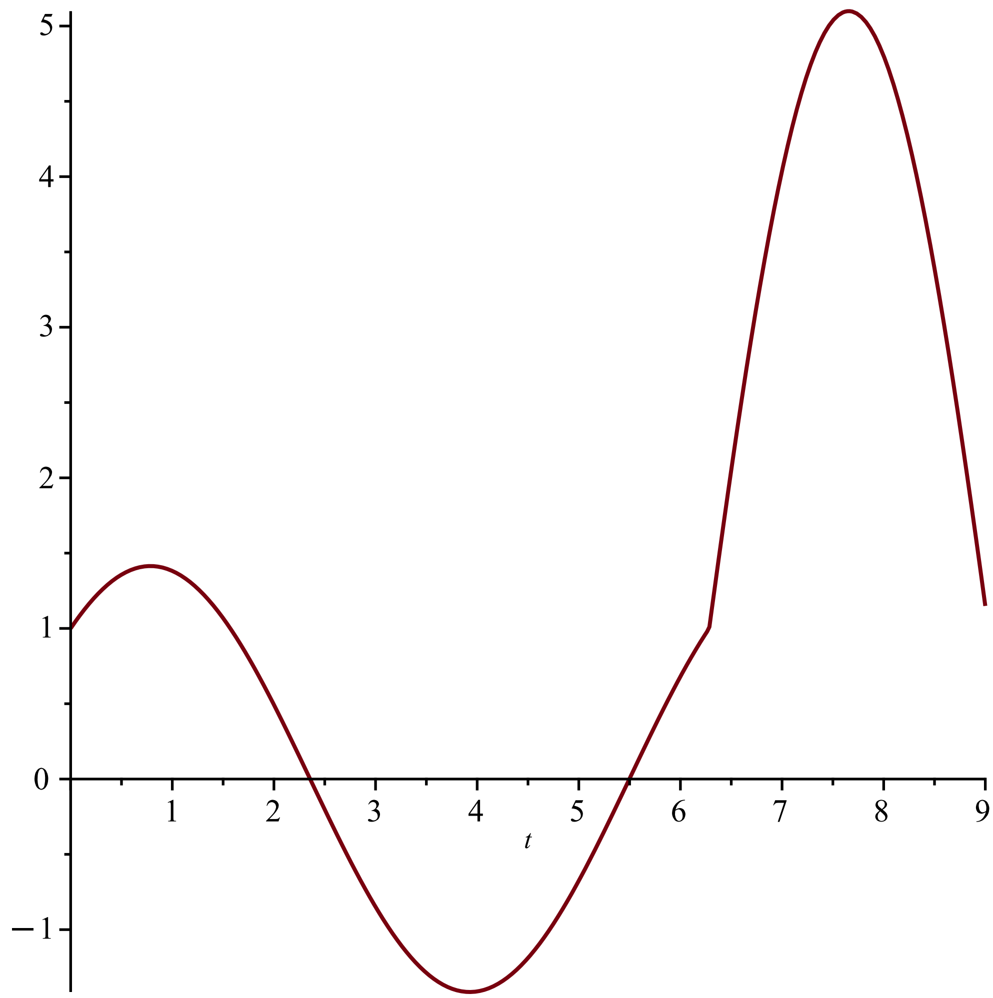
> $CondIniDos := \mathrm{D}(y)(0) = simplify(rhs(subs(t = 0, diff(SolPart, t))))$

$$CondIniDos := \mathrm{D}(y)(0) = 1 \quad (35)$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (36)$$

> $plot(rhs(SolPart), t = 0..9)$



> *restart*

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> *restart*

4a) Determine el sistema equivalente y resuélvalo de

> $Ecua := y'' + 4\cdot y' + 4\cdot y = 8$

$$Ecua := \frac{d^2}{dx^2}\, y(x) + 4\, \frac{d}{dx}\, y(x) + 4\, y(x) = 8 \tag{37}$$

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RESPUESTA

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> $Sistema := diff(y[1](x), x) = y[2](x), diff(y[2](x), x) = -4\cdot y[1](x) - 4\cdot y[2](x) + 8 :$
> $Sistema[1]; Sistema[2]$

$$\frac{d}{dx}\, y_1(x) = y_2(x)$$

$$\frac{d}{dx}\, y_2(x) = -4\, y_1(x) - 4\, y_2(x) + 8 \tag{38}$$

> $AA := array([[0, 1], [-4, -4]])$

$$AA := \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \quad \textbf{(39)}$$

> $Xcero := array([\_C1, \_C2])$

$$Xcero := \begin{bmatrix} \_C1 & \_C2 \end{bmatrix} \quad \textbf{(40)}$$

> $BB := array([0, 8])$

$$BB := \begin{bmatrix} 0 & 8 \end{bmatrix} \quad \textbf{(41)}$$

> $with(linalg):$

> $MatExp := exponential(AA, x)$

$$MatExp := \begin{bmatrix} e^{-2x} + 2x\,e^{-2x} & x\,e^{-2x} \\ -4x\,e^{-2x} & e^{-2x} - 2x\,e^{-2x} \end{bmatrix} \quad \textbf{(42)}$$

> $SolHom := evalm(MatExp \&* Xcero) : y[1](x) = simplify(SolHom[1]); y[2](x) = simplify(SolHom[2])$

$$y_1(x) = e^{-2x}\,(2x\,\_C1 + \_C2\,x + \_C1)$$

$$y_2(x) = ((-2x+1)\,\_C2 - 4x\,\_C1)\,e^{-2x} \quad \textbf{(43)}$$

> $ComprobarUno := simplify(subs(x=0, y[1](x) = SolHom[1]))$

$$ComprobarUno := y_1(0) = \_C1 \quad \textbf{(44)}$$

> $ComprobarDos := simplify(subs(x=0, y[2](x) = SolHom[2]))$

$$ComprobarDos := y_2(0) = \_C2 \quad \textbf{(45)}$$

> $MatExpTau := map(rcurry(eval, x='x - \mathrm{tau}'), MatExp)$

$$MatExpTau := \begin{bmatrix} e^{-2x+2\tau} + 2\,(x-\tau)\,e^{-2x+2\tau} & (x-\tau)\,e^{-2x+2\tau} \\ -4\,(x-\tau)\,e^{-2x+2\tau} & e^{-2x+2\tau} - 2\,(x-\tau)\,e^{-2x+2\tau} \end{bmatrix} \quad \textbf{(46)}$$

> $ProdTau := evalm(MatExpTau \&* BB)$

$$ProdTau := \begin{bmatrix} 8\,(x-\tau)\,e^{-2x+2\tau} & 8\,e^{-2x+2\tau} - 16\,(x-\tau)\,e^{-2x+2\tau} \end{bmatrix} \quad \textbf{(47)}$$

> $SolNoHom := simplify(map(int, ProdTau, \mathrm{tau} = 0..x)) : y[1](x) = SolNoHom[1]; y[2](x) = SolNoHom[2]$

$$y_1(x) = 2 + (-4x - 2)\,e^{-2x}$$

$$y_2(x) = 8x\,e^{-2x} \quad \textbf{(48)}$$

> $ComprobarCinco := simplify(subs(x=0, y[1](x) = SolNoHom[1]))$

$$ComprobarCinco := y_1(0) = 0 \quad \textbf{(49)}$$

> $ComprobarSeis := simplify(subs(x=0, y[2](x) = SolNoHom[2]))$

$$ComprobarSeis := y_2(0) = 0 \quad \textbf{(50)}$$

> $SolFinal[1] := y[1](x) = simplify(SolHom[1] + SolNoHom[1]); SolFinal[2] := y[2](x) = simplify(SolHom[2] + SolNoHom[2])$

$$SolFinal_1 := y_1(x) = 2 + ((2\,\_C1 + \_C2 - 4)\,x + \_C1 - 2)\,e^{-2x}$$

$$SolFinal_2 := y_2(x) = e^{-2x}\,(-4x\,\_C1 - 2\,\_C2\,x + \_C2 + 8x) \quad \textbf{(51)}$$

> *Sistema*[1]

$$\frac{d}{dx}\, y_1(x) = y_2(x) \tag{52}$$

> *ComprobarTres* := *simplify*(*eval*(*subs*(*y*[1](*x*) = *rhs*(*SolFinal*[1]), *y*[2](*x*) = *rhs*(*SolFinal*[2]), *lhs*(*Sistema*[1]) − *rhs*(*Sistema*[1]) = 0)))

$$ComprobarTres := 0 = 0 \tag{53}$$

> *Sistema*[2]

$$\frac{d}{dx}\, y_2(x) = -4\, y_1(x) - 4\, y_2(x) + 8 \tag{54}$$

> *ComprobarCuatro* := *simplify*(*eval*(*subs*(*y*[1](*x*) = *rhs*(*SolFinal*[1]), *y*[2](*x*) = *rhs*(*SolFinal*[2]), *lhs*(*Sistema*[2]) − *rhs*(*Sistema*[2]) = 0)))

$$ComprobarCuatro := 0 = 0 \tag{55}$$

> *restart*

5a)Obtener la solución completa para constante positiva

> *Ecua* := *diff*(*u*(*x*, *t*), *x*$2) + *diff*(*u*(*x*, *t*), *x*, *t*) = 4·*t*·*diff*(*u*(*x*, *t*), *x*)

$$Ecua := \frac{\partial^2}{\partial x^2}\, u(x,t) + \frac{\partial^2}{\partial t\partial x}\, u(x,t) = 4\, t \left(\frac{\partial}{\partial x}\, u(x,t)\right) \tag{56}$$

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RESPUESTA

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> *EcuaSeparable* := *eval*(*subs*(*u*(*x*, *t*) = *F*(*x*) * *G*(*t*), *Ecua*))

$$EcuaSeparable := \left(\frac{d^2}{dx^2}\, F(x)\right) G(t) + \left(\frac{d}{dx}\, F(x)\right)\left(\frac{d}{dt}\, G(t)\right) = 4\, t \left(\frac{d}{dx}\, F(x)\right) G(t) \tag{57}$$

$$> EcuaSeparada := \frac{\left(lhs(EcuaSeparable) - \left(\frac{d}{dx}\, F(x)\right)\left(\frac{d}{dt}\, G(t)\right)\right)}{\frac{d}{dx}\, F(x)\cdot G(t)} = simplify\left(\frac{\left(rhs(EcuaSeparable) - \left(\frac{d}{dx}\, F(x)\right)\left(\frac{d}{dt}\, G(t)\right)\right)}{\frac{d}{dx}\, F(x)\cdot G(t)}\right)$$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2}\, F(x)}{\frac{d}{dx}\, F(x)} = \frac{4\, G(t)\, t - \frac{d}{dt}\, G(t)}{G(t)} \tag{58}$$

> *EcuaX* := *lhs*(*EcuaSeparada*) = β^2

$$EcuaX := \frac{\frac{d^2}{dx^2}\, F(x)}{\frac{d}{dx}\, F(x)} = \beta^2 \tag{59}$$

> $EcuaT := rhs(EcuaSeparada) = \beta^2$

$$EcuaT := \frac{4\,G(t)\,t - \frac{d}{dt}\,G(t)}{G(t)} = \beta^2 \tag{60}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 + c_2\,e^{\beta^2 x} \tag{61}$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = c_1\,e^{-t\,(\beta^2 - 2\,t)} \tag{62}$$

> $SolGral := u(x,t) = rhs(SolX) \cdot subs(c_1 = 1, rhs(SolT))$

$$SolGral := u(x,t) = \left(c_1 + c_2\,e^{\beta^2 x}\right) e^{-t\,(\beta^2 - 2\,t)} \tag{63}$$

> $Ecua$

$$\frac{\partial^2}{\partial x^2}\,u(x,t) + \frac{\partial^2}{\partial t\,\partial x}\,u(x,t) = 4\,t\left(\frac{\partial}{\partial x}\,u(x,t)\right) \tag{64}$$

> $Comprobar := simplify(eval(subs(u(x,t) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \tag{65}$$

> $restart$

FIN EXAMEN

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