

FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE CIENCIAS APLICADAS
MATEMÁTICAS APLICADAS
ECUACIONES DIFERENCIALES
SEMESTRE 2025 – 1 GRUPO 13
PRIMER EXAMEN PARCIAL
TIPO "B"
SOLUCIÓN

> restart

1)

> Ecua := x·sqrt(1 + y²) + y·sqrt(1 + x²)·y'=0

$$Ecua := x \sqrt{1 + y(x)^2} + y(x) \sqrt{x^2 + 1} \left(\frac{d}{dx} y(x) \right) = 0 \quad (1)$$

> CondIni := y(0) = 1

$$CondIni := y(0) = 1 \quad (2)$$

RESPUESTA

> with(DEtools):

> odeadvisor(Ecua)

$$[_separable] \quad (3)$$

> M := x·sqrt(1 + y²)

$$M := x \sqrt{y^2 + 1} \quad (4)$$

> N := y·sqrt(1 + x²)

$$N := y \sqrt{x^2 + 1} \quad (5)$$

> P := x; Q := sqrt(1 + y²); R := sqrt(1 + x²); S := y

$$P := x$$

$$Q := \sqrt{y^2 + 1}$$

$$R := \sqrt{x^2 + 1}$$

$$S := y$$

(6)

> SolGral := int($\frac{P}{R}, x$) + int($\frac{S}{Q}, y$) = _C1

$$SolGral := \sqrt{x^2 + 1} + \sqrt{y^2 + 1} = _C1 \quad (7)$$

> Para := subs(x=0, y=1, SolGral)

$$Para := 1 + \sqrt{2} = _C1 \quad (8)$$

> SolPart := subs(_C1=lhs(Para), SolGral)

$$SolPart := \sqrt{x^2 + 1} + \sqrt{y^2 + 1} = 1 + \sqrt{2} \quad (9)$$

> SolPartFinal := sqrt(x² + 1) + sqrt(y(x)² + 1) = 1 + sqrt(2)

$$SolPartFinal := \sqrt{x^2 + 1} + \sqrt{1 + y(x)^2} = 1 + \sqrt{2} \quad (10)$$

> DerSolPart := isolate(diff(SolPartFinal, x), diff(y(x), x))

$$DerSolPart := \frac{d}{dx} y(x) = -\frac{x \sqrt{1 + y(x)^2}}{y(x) \sqrt{x^2 + 1}} \quad (11)$$

> DerEcua := isolate(Ecua, diff(y(x), x))

$$DerEcua := \frac{d}{dx} y(x) = -\frac{x \sqrt{1 + y(x)^2}}{y(x) \sqrt{x^2 + 1}} \quad (12)$$

> Comprobar := simplify(rhs(DerEcua) - rhs(DerSolPart)) = 0

$$Comprobar := 0 = 0 \quad (13)$$

> restart

2)

> Ecua := 4·x² - x·y + y² + (x² - x·y + 4·y²)·y' = 0

$$Ecua := 4x^2 - xy(x) + y(x)^2 + (x^2 - xy(x) + 4y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (14)$$

>

RESPUESTA

> with(DEtools) :

> odeadvisor(Ecua)

$$[_{homogeneous}, class A], _{rational}, _{dAlembert}] \quad (15)$$

LA ECUACIÓN ES DE COEFICIENTES HOMOGENEOS

> EcuaDos := simplify(isolate(eval(subs(y(x) = u(x)·x, Ecua)), diff(u(x), x)))

$$EcuaDos := \frac{d}{dx} u(x) = \frac{-4u(x)^3 - 4}{(1 - u(x) + 4u(x)^2)x} \quad (16)$$

> odeadvisor(EcuaDos)

$$[_{separable}] \quad (17)$$

> EcuaTres := lhs(EcuaDos)·x = rhs(EcuaDos)·x

$$EcuaTres := \left(\frac{d}{dx} u(x) \right) x = \frac{-4u(x)^3 - 4}{1 - u(x) + 4u(x)^2} \quad (18)$$

> M := - \left(\frac{-4u^3 - 4}{1 - u + 4u^2} \right)

$$M := -\frac{-4u^3 - 4}{4u^2 - u + 1} \quad (19)$$

> N := x

$$N := x \quad (20)$$

> P := 1; Q := M; R := N; S := 1

$$P := 1$$

$$Q := -\frac{-4u^3 - 4}{4u^2 - u + 1}$$

$$R := x$$

$$S := 1$$

(21)

$$> \text{SolGralUno} := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, u\right) = _CI$$

$$\text{SolGralUno} := \ln(x) + \frac{\ln(u^2 - u + 1)}{4} + \frac{\ln(u + 1)}{2} = _CI$$

(22)

$$> \text{SolGralDos} := \text{simplify}(\exp(\text{lhs}(\text{SolGralUno}))) = _CI$$

$$\text{SolGralDos} := x (u^2 - u + 1)^{1/4} \sqrt{u + 1} = _CI$$

(23)

$$> \text{SolGralTres} := \text{expand}\left(\text{subs}\left(u = \frac{y(x)}{x}, \text{SolGralDos}\right)\right)$$

$$\text{SolGralTres} := x \left(\frac{y(x)^2}{x^2} - \frac{y(x)}{x} + 1 \right)^{1/4} \sqrt{\frac{y(x)}{x} + 1} = _CI$$

(24)

$$> \text{SolGralCuatro} := \text{expand}\left(\text{subs}\left(u = \frac{y(x)}{x}, \text{SolGralUno}\right)\right)$$

$$\text{SolGralCuatro} := \ln(x) + \frac{\ln\left(\frac{y(x)^2}{x^2} - \frac{y(x)}{x} + 1\right)}{4} + \frac{\ln\left(\frac{y(x)}{x} + 1\right)}{2} = _CI$$

(25)

$$> \text{DerSolGral} := \text{simplify}(\text{isolate}(\text{diff}(\text{SolGralTres}, x), \text{diff}(y(x), x)))$$

$$\text{DerSolGral} := \frac{d}{dx} y(x) = \frac{-4x^2 + xy(x) - y(x)^2}{x^2 - xy(x) + 4y(x)^2}$$

(26)

$$> \text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x))$$

$$\text{DerEcua} := \frac{d}{dx} y(x) = \frac{-4x^2 + xy(x) - y(x)^2}{x^2 - xy(x) + 4y(x)^2}$$

(27)

$$> \text{Comprobar} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolGral})) = 0$$

$$\text{Comprobar} := 0 = 0$$

(28)

$$> \text{intfactor}(\text{Ecua})$$

$$\frac{1}{(y(x) + x) (y(x)^2 - xy(x) + x^2)}$$

(29)

$$> FI := \frac{1}{(y + x) (y^2 - xy + x^2)}$$

$$FI := \frac{1}{(x + y) (x^2 - xy + y^2)}$$

(30)

$$> M := 4x^2 - xy + y^2$$

$$M := 4x^2 - xy + y^2$$

(31)

$$> N := x^2 - xy + 4y^2$$

$$N := x^2 - xy + 4y^2$$

(32)

$$> MM := \text{simplify}(M \cdot FI)$$

(33)

$$MM := \frac{4x^2 - xy + y^2}{(x+y)(x^2 - xy + y^2)} \quad (33)$$

> NN := simplify(N·FI)

$$NN := \frac{x^2 - xy + 4y^2}{(x+y)(x^2 - xy + y^2)} \quad (34)$$

> CompruebaUno := simplify(diff(MM, y) - diff(NN, x)) = 0

$$\text{CompruebaUno} := 0 = 0 \quad (35)$$

> IntMMx := int(MM, x)

$$\text{IntMMx} := \ln(x^2 - xy + y^2) + 2 \ln(x+y) \quad (36)$$

> SolucionGralDos := IntMMx + int((NN - diff(IntMMx, y)), y) = _C10

$$\text{SolucionGralDos} := \ln(x^2 - xy + y^2) + 2 \ln(x+y) = _C10 \quad (37)$$

> SolucionGralTres := ln(x^2 - xy(x) + y(x)^2) + 2 ln(x + y(x)) = _C10

$$\text{SolucionGralTres} := \ln(y(x)^2 - xy(x) + x^2) + 2 \ln(y(x) + x) = _C10 \quad (38)$$

> Ecua

$$4x^2 - xy(x) + y(x)^2 + (x^2 - xy(x) + 4y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (39)$$

> DerEcua

$$\frac{d}{dx} y(x) = \frac{-4x^2 + xy(x) - y(x)^2}{x^2 - xy(x) + 4y(x)^2} \quad (40)$$

> DerSolucionGral := isolate(diff(SolucionGralTres, x), diff(y(x), x))

$$\text{DerSolucionGral} := \frac{d}{dx} y(x) = \frac{-4x^2 + xy(x) - y(x)^2}{x^2 - xy(x) + 4y(x)^2} \quad (41)$$

> CompruebaDos := simplify(rhs(DerEcua) - rhs(DerSolucionGral)) = 0

$$\text{CompruebaDos} := 0 = 0 \quad (42)$$

> SolGralCuatro

$$\ln(x) + \frac{\ln\left(\frac{y(x)^2}{x^2} - \frac{y(x)}{x} + 1\right)}{4} + \frac{\ln\left(\frac{y(x)}{x} + 1\right)}{2} = _C1 \quad (43)$$

> SolucionGralTres

$$\ln(y(x)^2 - xy(x) + x^2) + 2 \ln(y(x) + x) = _C10 \quad (44)$$

> restart

3)

> Ecua := 3·x^2 - 2·x - y + (2·y - x + 3·y^2)·y' = 0

$$\text{Ecua} := 3x^2 - 2x - y(x) + (2y(x) - x + 3y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (45)$$

> CondIni := y(4) = 2

$$\text{CondIni} := y(4) = 2 \quad (46)$$

>

RESPUESTA

$$\begin{aligned} &> \text{with(DEtools)} : \\ &> \text{odeadvisor(Ecua)} \\ & \qquad \qquad \qquad [_{\text{exact}}, _{\text{rational}}] \end{aligned} \tag{47}$$

$$\begin{aligned} &> M := 3 \cdot x^2 - 2 \cdot x - y \\ & \qquad \qquad \qquad M := 3 x^2 - 2 x - y \end{aligned} \tag{48}$$

$$\begin{aligned} &> N := 2 \cdot y - x + 3 \cdot y^2 \\ & \qquad \qquad \qquad N := 3 y^2 - x + 2 y \end{aligned} \tag{49}$$

$$\begin{aligned} &> \text{diff}(M, y) = \text{diff}(N, x) \\ & \qquad \qquad \qquad -1 = -1 \end{aligned} \tag{50}$$

ES EXACTA

$$\begin{aligned} &> \text{IntMx} := \text{int}(M, x) \\ & \qquad \qquad \qquad \text{IntMx} := x^3 - x^2 - x y \end{aligned} \tag{51}$$

$$\begin{aligned} &> \text{SolGral} := \text{IntMx} + \text{int}((N - \text{diff}(\text{IntMx}, y)), y) = _C1 \\ & \qquad \qquad \qquad \text{SolGral} := x^3 + y^3 - x^2 - x y + y^2 = _C1 \end{aligned} \tag{52}$$

$$\begin{aligned} &> \text{Para} := \text{subs}(x=4, y=2, \text{SolGral}) \\ & \qquad \qquad \qquad \text{Para} := 52 = _C1 \end{aligned} \tag{53}$$

$$\begin{aligned} &> \text{SolPart} := \text{subs}(_C1 = \text{lhs}(\text{Para}), \text{SolGral}) \\ & \qquad \qquad \qquad \text{SolPart} := x^3 + y^3 - x^2 - x y + y^2 = 52 \end{aligned} \tag{54}$$

$$\begin{aligned} &> \text{SolPartFinal} := x^3 + y(x)^3 - x^2 - x y(x) + y(x)^2 = 52 \\ & \qquad \qquad \qquad \text{SolPartFinal} := x^3 + y(x)^3 - x^2 - x y(x) + y(x)^2 = 52 \end{aligned} \tag{55}$$

$$\begin{aligned} &> \text{DerSolPart} := \text{isolate}(\text{diff}(\text{SolPartFinal}, x), \text{diff}(y(x), x)) \\ & \qquad \qquad \qquad \text{DerSolPart} := \frac{d}{dx} y(x) = \frac{-3 x^2 + 2 x + y(x)}{2 y(x) - x + 3 y(x)^2} \end{aligned} \tag{56}$$

$$\begin{aligned} &> \text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x)) \\ & \qquad \qquad \qquad \text{DerEcua} := \frac{d}{dx} y(x) = \frac{-3 x^2 + 2 x + y(x)}{2 y(x) - x + 3 y(x)^2} \end{aligned} \tag{57}$$

$$\begin{aligned} &> \text{ComprobarUno} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolPart})) = 0 \\ & \qquad \qquad \qquad \text{ComprobarUno} := 0 = 0 \end{aligned} \tag{58}$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{subs}(x=4, y=2, \text{SolPart}) \\ & \qquad \qquad \qquad \text{ComprobarDos} := 52 = 52 \end{aligned} \tag{59}$$

> restart

4)

$$\begin{aligned} &> \text{Ecua} := x + \sin(x) + \sin(y) + \cos(y) \cdot y' = 0 \\ & \qquad \qquad \qquad \text{Ecua} := x + \sin(x) + \sin(y(x)) + \cos(y(x)) \left(\frac{d}{dx} y(x) \right) = 0 \end{aligned} \tag{60}$$

>

RESPUESTA

> with(DEtools) :

> odeadvisor(Ecua)

$$[y=_G(x,y')] \quad (61)$$

> FI := intfactor(Ecua)

$$FI := e^x \quad (62)$$

> M := x + sin(x) + sin(y)

$$M := x + \sin(x) + \sin(y) \quad (63)$$

> N := cos(y)

$$N := \cos(y) \quad (64)$$

> Comprobar := diff(M, y) ≠ diff(N, x)

$$Comprobar := \cos(y) \neq 0 \quad (65)$$

NO ES EXACTA

> MM := expand(M·FI)

$$MM := e^x x + e^x \sin(x) + e^x \sin(y) \quad (66)$$

> NN := N·FI

$$NN := e^x \cos(y) \quad (67)$$

> ComprobarDos := diff(MM, y) = diff(NN, x)

$$ComprobarDos := e^x \cos(y) = e^x \cos(y) \quad (68)$$

> EcuaDos := e^x x + e^x sin(x) + e^x sin(y(x)) + e^x cos(y(x)) · diff(y(x), x) = 0

$$EcuaDos := e^x x + e^x \sin(x) + e^x \sin(y(x)) + e^x \cos(y(x)) \left(\frac{d}{dx} y(x) \right) = 0 \quad (69)$$

> odeadvisor(EcuaDos)

$$[_{exact}] \quad (70)$$

> IntMMx := int(MM, x)

$$IntMMx := e^x x - e^x - \frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2} + e^x \sin(y) \quad (71)$$

> SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = _CI

$$SolGral := e^x x - e^x - \frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2} + e^x \sin(y) = c_1 \quad (72)$$

> SolGralFinal := 2 · (e^x x - e^x - $\frac{e^x \cos(x)}{2}$ + $\frac{e^x \sin(x)}{2}$ + e^x sin(y(x))) = _CI

$$SolGralFinal := 2 e^x x - 2 e^x - e^x \cos(x) + e^x \sin(x) + 2 e^x \sin(y(x)) = c_1 \quad (73)$$

> DerEcua := isolate(Ecua, diff(y(x), x))

$$DerEcua := \frac{d}{dx} y(x) = \frac{-\sin(x) - x - \sin(y(x))}{\cos(y(x))} \quad (74)$$

> DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))

$$DerSolGral := \frac{d}{dx} y(x) = -(x + \sin(x) + \sin(y(x))) \sec(y(x)) \quad (75)$$

> ComprobarUno := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0

$$ComprobarUno := 0 = 0 \quad (76)$$

> restart

| 5)

> $Ecua := y' + \cos(x) \cdot y = \sin(x) \cdot \cos(x)$

$$Ecua := \frac{d}{dx} y(x) + \cos(x) y(x) = \sin(x) \cos(x) \quad (77)$$

>

| RESPUESTA

> $p := \cos(x)$

$$p := \cos(x) \quad (78)$$

> $q := \sin(x) \cdot \cos(x)$

$$q := \sin(x) \cos(x) \quad (79)$$

> $SolGral := y(x) = \text{simplify}(_CI \cdot \exp(-\text{int}(p, x)) + \exp(-\text{int}(p, x)) \cdot \text{int}(\exp(\text{int}(p, x)) \cdot q, x))$

$$SolGral := y(x) = \sin(x) + _CI e^{-\sin(x)} - 1 \quad (80)$$

> $Comprobar := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(SolGral), \text{lhs}(Ecua) - \text{rhs}(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (81)$$

> restart

>

>