>

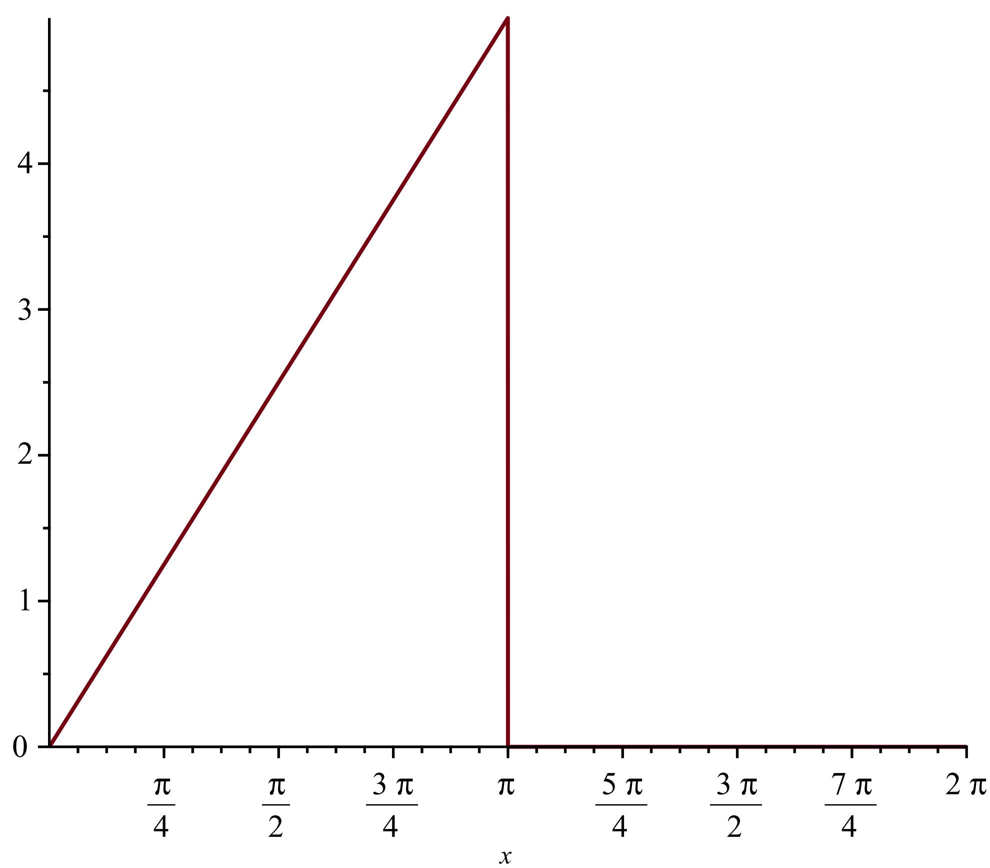
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE CIENCIAS APLICADAS
MATEMÁTICAS APLICADAS
ECUACIONES DIFERENCIALES
SEMESTRE 2025 − 1 GRUPO 13
TERCER EXAMEN PARCIAL
TEMAS 4
SOLUCIÓN (1)

Noviembre de 2024

> *restart*

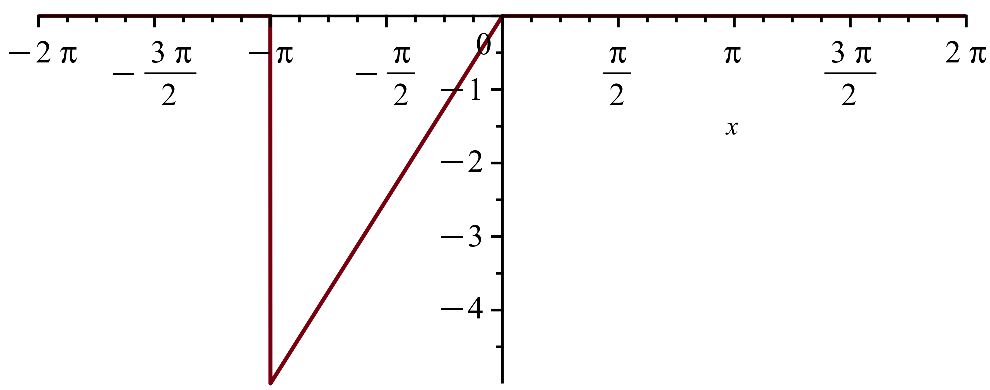
1) Obtener los cuatro primeros términos de la serie seno de Fourier de la función f en el intervalo de (0, 2*Pi)

> $f := \frac{5}{\mathrm{Pi}} \cdot \mathrm{Heaviside}(x)\cdot(x) - \frac{5}{\mathrm{Pi}}\cdot(x-\mathrm{Pi})\cdot\mathrm{Heaviside}(x-\mathrm{Pi}) - 5\cdot\mathrm{Heaviside}(x-\mathrm{Pi})$:
> $plot(f, x = 0\,..2\cdot\mathrm{Pi}, scaling = CONSTRAINED)$



> $g := -5\cdot\mathrm{Heaviside}(x+\mathrm{Pi}) + \frac{5}{\mathrm{Pi}}\cdot(x+\mathrm{Pi})\cdot\mathrm{Heaviside}(x+\mathrm{Pi}) - \frac{5}{\mathrm{Pi}}\cdot(x)\cdot\mathrm{Heaviside}(x)$:

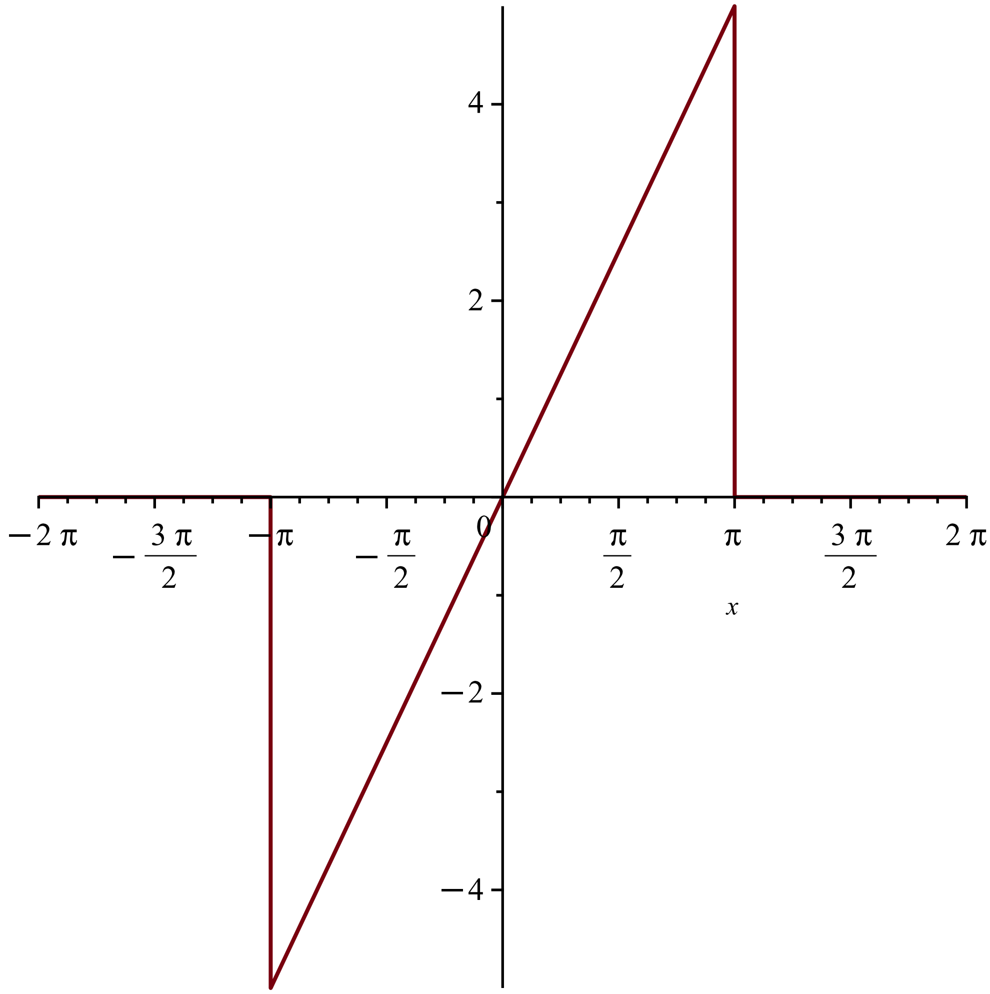
$plot(g, x = -2 \cdot \mathrm{Pi}..2 \cdot \mathrm{Pi}, scaling = CONSTRAINED)$



> $h := f + g$

$$h := -\frac{5\ (x - \pi)\ \mathrm{Heaviside}(x - \pi)}{\pi} - 5\ \mathrm{Heaviside}(x - \pi) - 5\ \mathrm{Heaviside}(x + \pi) + \frac{5\ (x + \pi)\ \mathrm{Heaviside}(x + \pi)}{\pi} \qquad \textbf{(1)}$$

> $plot(h, x = -2 \cdot \mathrm{Pi}..2 \cdot \mathrm{Pi})$



> $L := 2\cdot\mathrm{Pi}$

$$L := 2\,\pi \qquad (2)$$

> $a[0] := \frac{1}{L}\cdot int(h, x = -L..L)$

$$a_0 := 0 \qquad (3)$$

> $a[n] := \frac{1}{L}\cdot int\left(h\cdot\cos\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)$

$$a_n := 0 \qquad (4)$$

> $b[n] := \frac{1}{L}\cdot int\left(h\cdot\sin\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), x = -L..L\right)$

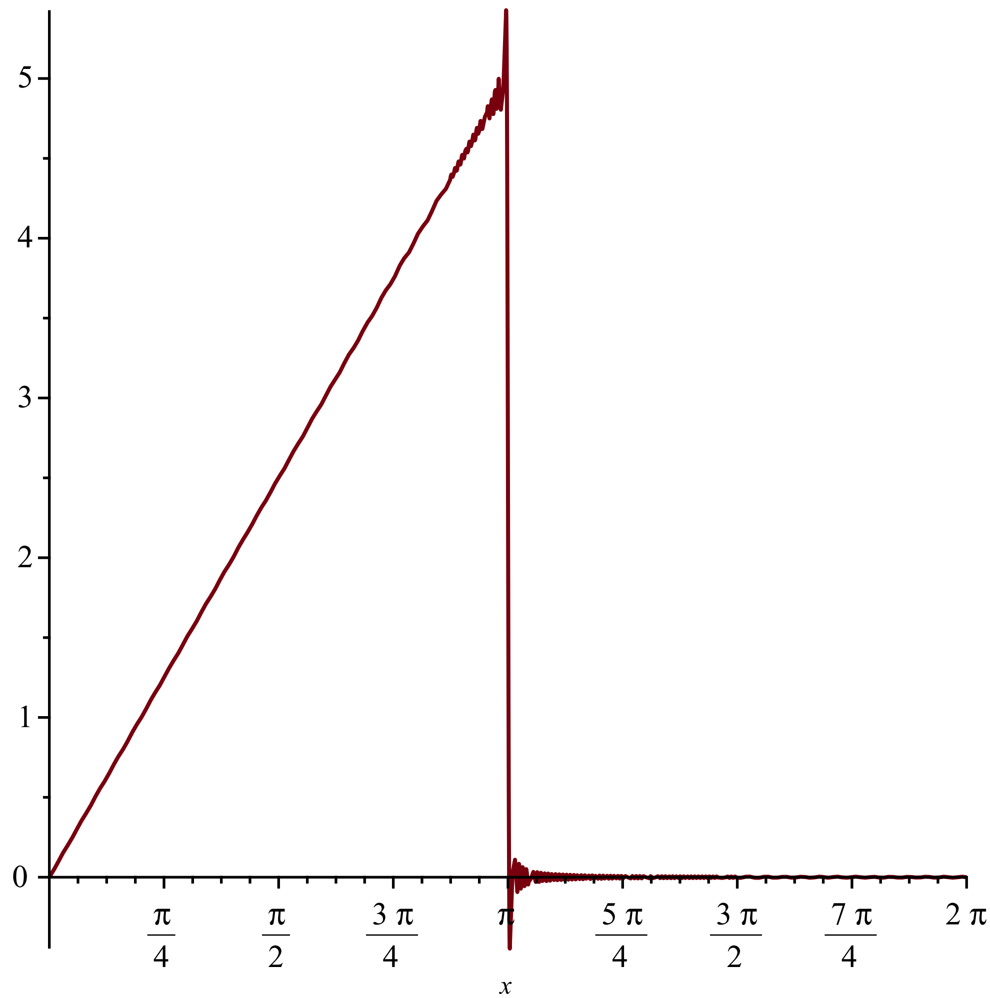
$$b_n := \frac{\frac{2\left(-20\,n\cos\left(\frac{n\,\pi}{2}\right)\pi + 40\sin\left(\frac{n\,\pi}{2}\right)\right)}{\pi\,n^2} + \frac{20\,n\cos\left(\frac{n\,\pi}{2}\right)\pi - 40\sin\left(\frac{n\,\pi}{2}\right)}{\pi\,n^2}}{2\,\pi} \qquad (5)$$

> $STF := Sum\left(b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), n = 1..\mathrm{infinity}\right)$

$$STF := \sum_{n=1}^{\infty} \frac{1}{2\pi}\left(\left(\frac{2\left(-20\, n \cos\left(\frac{n\pi}{2}\right)\pi + 40 \sin\left(\frac{n\pi}{2}\right)\right)}{\pi n^2} + \frac{20\, n \cos\left(\frac{n\pi}{2}\right)\pi - 40 \sin\left(\frac{n\pi}{2}\right)}{\pi n^2}\right) \sin\left(\frac{n x}{2}\right)\right) \tag{6}$$

> $STF500 := sum\left(b[n]\cdot \sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..500\right):$

> $plot(STF500, x = 0..2\cdot \mathrm{Pi})$



> $STFcuatro := sum\left(b[n]\cdot \sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..4\right)$

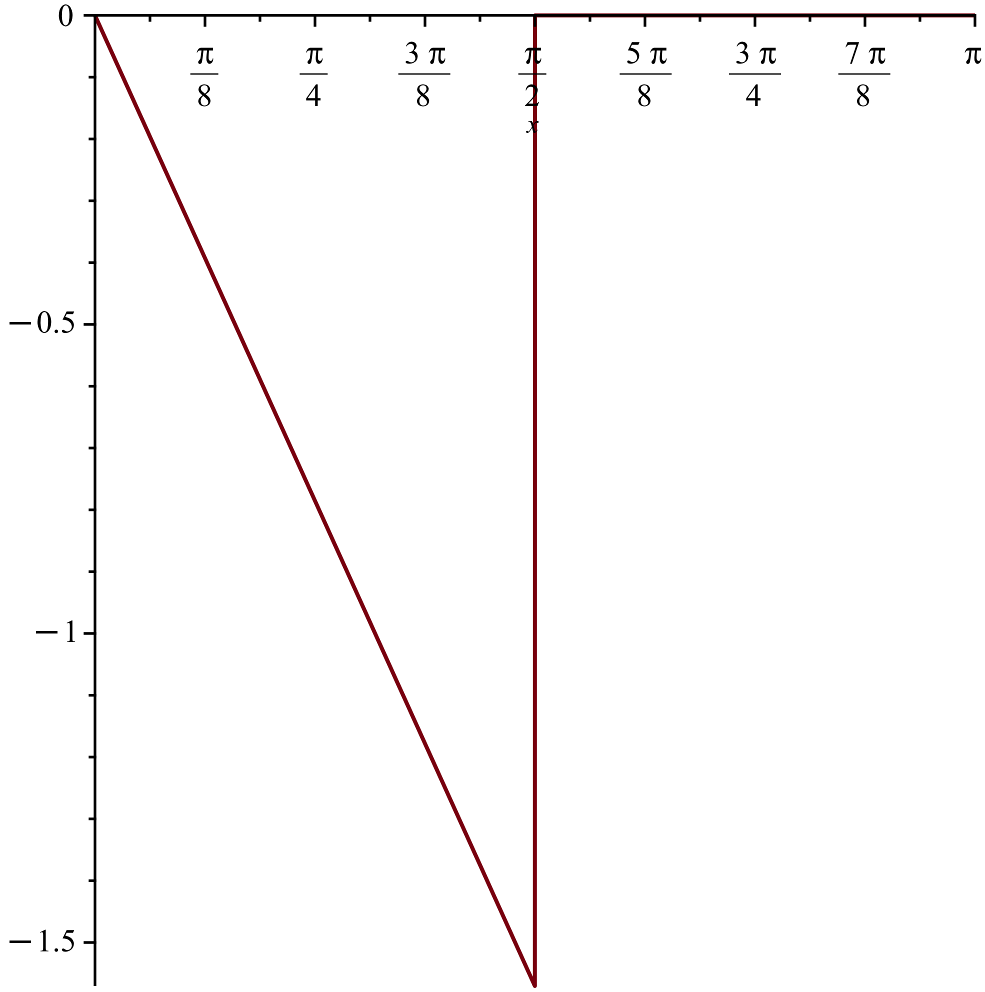
$$STFcuatro := \frac{20 \sin\left(\frac{x}{2}\right)}{\pi^2} + \frac{5 \sin(x)}{\pi} - \frac{20 \sin\left(\frac{3x}{2}\right)}{9\pi^2} - \frac{5 \sin(2x)}{2\pi} \tag{7}$$

>

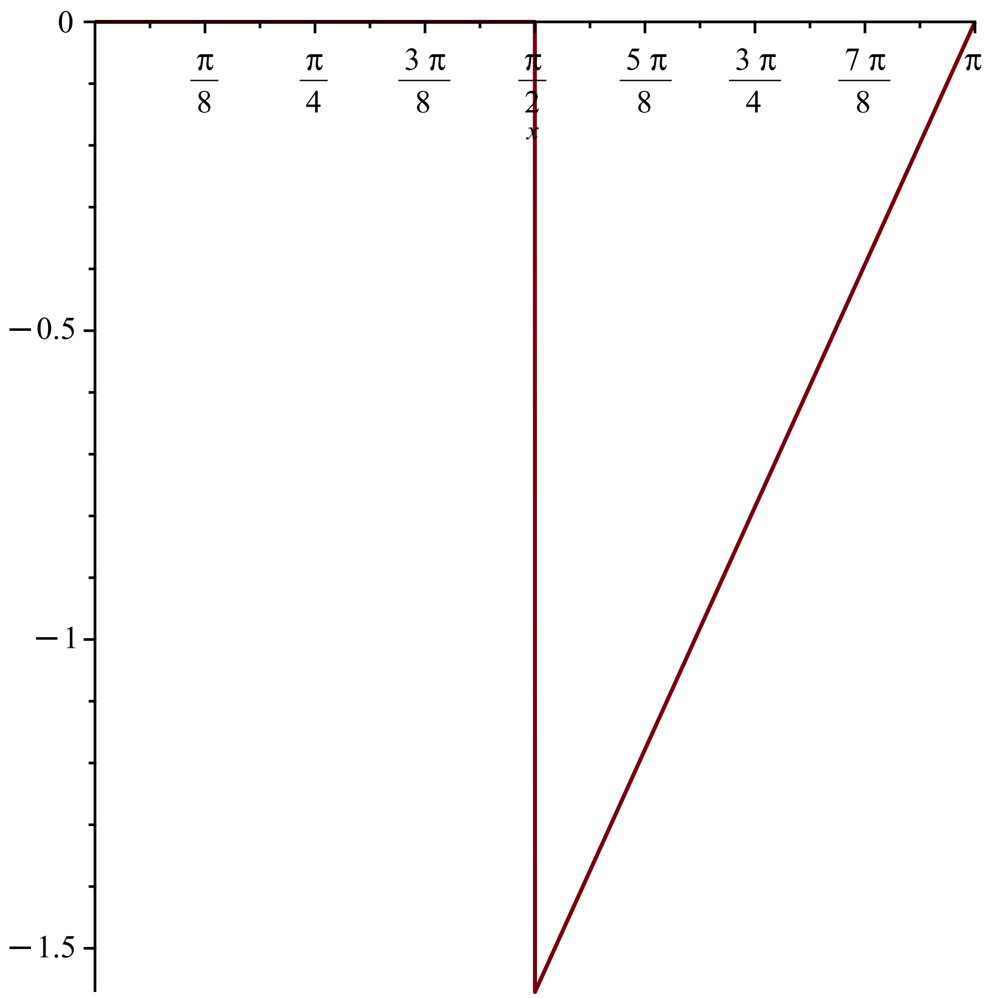
> *restart*

2) Obtener la serie de Fourier de la función

> $g := -x + (x)\cdot\text{Heaviside}\left(x - \frac{\text{Pi}}{2}\right) : plot(g, x = 0..\text{Pi})$

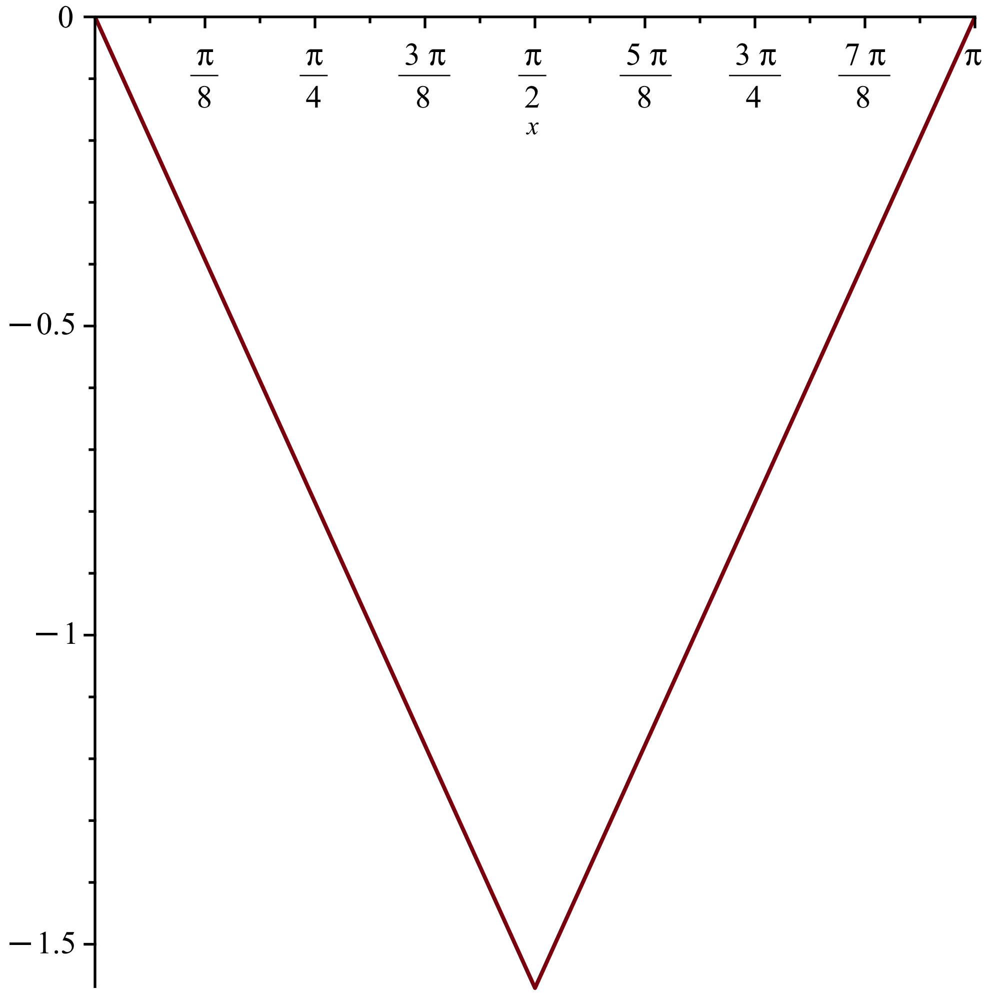


> $h := (x - \text{Pi})\cdot\text{Heaviside}\left(x - \frac{\text{Pi}}{2}\right) : plot(h, x = 0..\text{Pi})$



> $f := g + h; plot(f, x = 0 ..\mathrm{Pi})$

$$f := -x + x\,\mathrm{Heaviside}\left(x - \frac{\pi}{2}\right) + (x - \pi)\,\mathrm{Heaviside}\left(x - \frac{\pi}{2}\right)$$



> $L := \frac{\mathrm{Pi}}{2}$

$$L := \frac{\pi}{2} \qquad (8)$$

> $a[0] := \frac{1}{L} \cdot int(f, x = 0 ..\mathrm{Pi})$

$$a_0 := -\frac{\pi}{2} \qquad (9)$$

> $a[n] := \frac{1}{L} \cdot \left(subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \sin(2 \cdot n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x \right), x = 0 ..\mathrm{Pi} \right) \right) \right)$

$$a_n := \frac{2 \left(\frac{\cos(2\,n\,\pi) - 2\,(-1)^n}{4\,n^2} + \frac{1}{4\,n^2} \right)}{\pi} \qquad (10)$$

> $b[n] := \frac{1}{L} \cdot \left(subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \sin(2 \cdot n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x \right), x \right.\right.\right.$

$= 0 ..\mathrm{Pi})))$

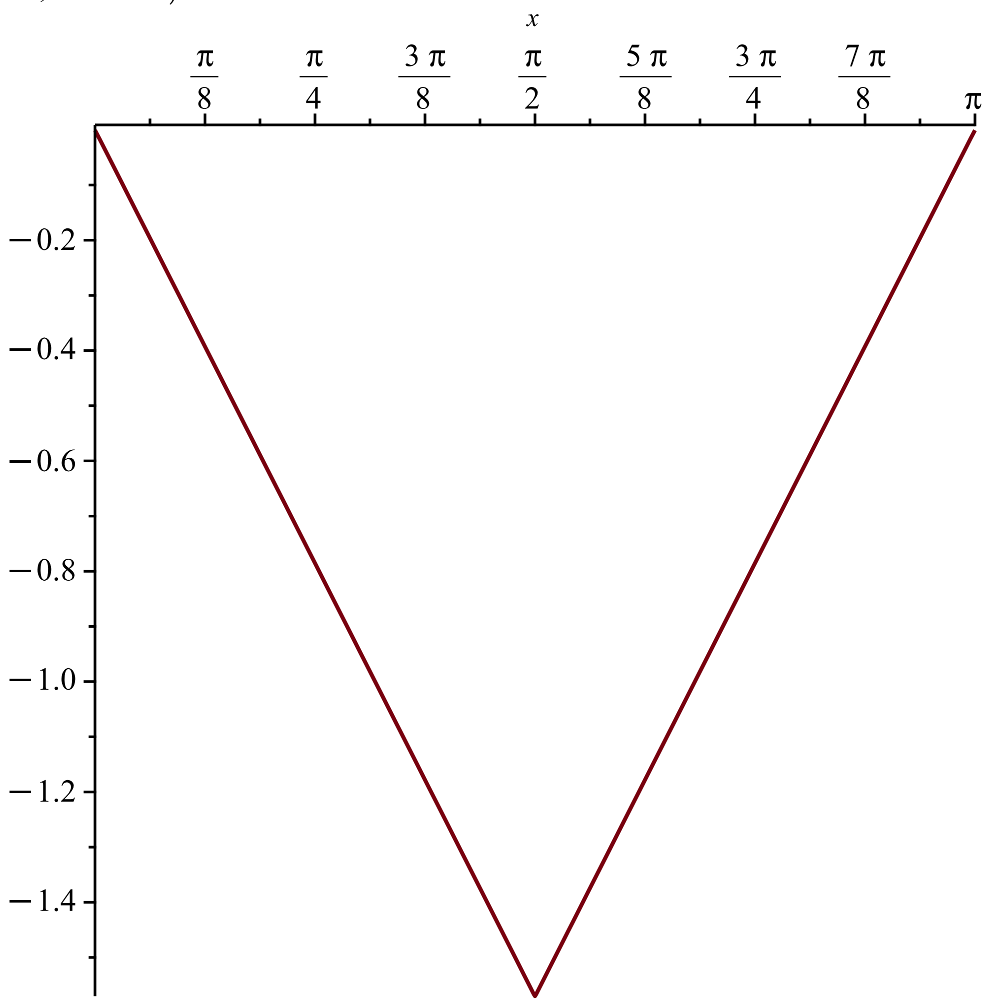
$$b_n := 0 \qquad \textbf{(11)}$$

> $STF := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1 ..\mathrm{infinity}\right)$

$$STF := -\frac{\pi}{4} + \left(\sum_{n=1}^{\infty} \frac{2\left(\frac{\cos(2\,n\,\pi) - 2\,(-1)^n}{4\,n^2} + \frac{1}{4\,n^2}\right)\cos(2\,n\,x)}{\pi}\right) \qquad \textbf{(12)}$$

> $STF500 := \frac{a[0]}{2} + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1 ..500\right):$

> $plot(STF500, x = 0 ..\mathrm{Pi})$



> $restart$

3) Resolver con una constante de separación negativa

> $Ecua := s\cdot diff(u(s, t), s) = t\cdot diff(u(s, t), t)$

$$Ecua := s\left(\frac{\partial}{\partial s}\, u(s, t)\right) = t\left(\frac{\partial}{\partial t}\, u(s, t)\right) \qquad \textbf{(13)}$$

> $EcuaSep := eval(subs(u(s, t) = F(s)\cdot G(t), Ecua))$

(14)

$$EcuaSep := s\left(\frac{d}{ds}F(s)\right)G(t) = t\,F(s)\left(\frac{d}{dt}G(t)\right) \quad \textbf{(14)}$$

> $EcuaSeparada := \dfrac{lhs(EcuaSep)}{F(s)\cdot G(t)} = \dfrac{rhs(EcuaSep)}{F(s)\cdot G(t)}$

$$EcuaSeparada := \frac{s\left(\frac{d}{ds}F(s)\right)}{F(s)} = \frac{t\left(\frac{d}{dt}G(t)\right)}{G(t)} \quad \textbf{(15)}$$

> $EcuaS := lhs(EcuaSeparada) = -\beta^2$

$$EcuaS := \frac{s\left(\frac{d}{ds}F(s)\right)}{F(s)} = -\beta^2 \quad \textbf{(16)}$$

> $EcuaT := rhs(EcuaSeparada) = -\beta^2$

$$EcuaT := \frac{t\left(\frac{d}{dt}G(t)\right)}{G(t)} = -\beta^2 \quad \textbf{(17)}$$

> $SolS := dsolve(EcuaS)$

$$SolS := F(s) = c_1\, s^{-\beta^2} \quad \textbf{(18)}$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = c_1\, t^{-\beta^2} \quad \textbf{(19)}$$

> $SolGral := u(s,t) = rhs(SolS)\cdot subs\big(c_1 = 1, rhs(SolT)\big)$

$$SolGral := u(s,t) = c_1\, s^{-\beta^2}\, t^{-\beta^2} \quad \textbf{(20)}$$

> $comprobar := simplify(eval(subs(u(s,t) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$comprobar := 0 = 0 \quad \textbf{(21)}$$

> $restart$

>