

EXAMEN_3 2025-2-2(CAP3&4) JUEVES

> restart

1)

> Ecua := diff(x(t), t\$2) + 6*diff(x(t), t) + 9*x(t) = 0

$$Ecua := \frac{d^2}{dt^2} x(t) + 6 \frac{d}{dt} x(t) + 9 x(t) = 0 \quad (1)$$

> CondIni := x(0) = 20, D(x)(0) = 0

$$CondIni := x(0) = 20, D(x)(0) = 0 \quad (2)$$

> with(inttrans) :

> EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTransLap := s^2 \mathcal{L}(x(t), t, s) - 120 - 20s + 6s \mathcal{L}(x(t), t, s) + 9 \mathcal{L}(x(t), t, s) = 0 \quad (3)$$

> SolTransLap := isolate(EcuaTransLap, laplace(x(t), t, s))

$$SolTransLap := \mathcal{L}(x(t), t, s) = \frac{20s + 120}{s^2 + 6s + 9} \quad (4)$$

> SolPart := invlaplace(SolTransLap, s, t)

$$SolPart := x(t) = 20 e^{-3t} (1 + 3t) \quad (5)$$

> SolFinal := expand(SolPart)

$$SolFinal := x(t) = \frac{20}{(e^t)^3} + \frac{60t}{(e^t)^3} \quad (6)$$

> Ecua

$$\frac{d^2}{dt^2} x(t) + 6 \frac{d}{dt} x(t) + 9 x(t) = 0 \quad (7)$$

> ComprobarUno := simplify(eval(subs(x(t) = rhs(SolPart), Ecua)))

$$ComprobarUno := 0 = 0 \quad (8)$$

> ComprobarDos := simplify(subs(t = 0, SolPart))

$$ComprobarDos := x(0) = 20 \quad (9)$$

> ComprobarTres := D(x)(0) = simplify(subs(t = 0, rhs(diff(SolPart, t))))

$$ComprobarTres := D(x)(0) = 0 \quad (10)$$

> restart

2)

> Ecua := y(t) + int(tau*exp(2*tau)*y(t - tau), tau = 0..t) = t*exp(2*t)

$$Ecua := y(t) + \int_0^t \tau e^{2\tau} y(t - \tau) d\tau = t e^{2t} \quad (11)$$

> with(inttrans) :

> EcuaTransLap := laplace(Ecua, t, s)

$$(12)$$

$$EcuaTransLap := \mathcal{L}(y(t), t, s) + \frac{\mathcal{L}(y(t), t, s)}{(s-2)^2} = \frac{1}{(s-2)^2} \quad (12)$$

> SolTransLap := simplify(isolate(EcuaTransLap, laplace(y(t), t, s)))

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{1}{s^2 - 4s + 5} \quad (13)$$

> SolPart := invlaplace(SolTransLap, s, t)

$$SolPart := y(t) = \sin(t) e^{2t} \quad (14)$$

> Ecua

$$y(t) + \int_0^t \tau e^{2\tau} y(t-\tau) d\tau = t e^{2t} \quad (15)$$

> EcuaDos := lhs(Ecua) - \int_0^t \tau e^{2\tau} y(t-\tau) d\tau - t e^{2t} = rhs(Ecua) - \int_0^t \tau e^{2\tau} y(t-\tau) d\tau - t e^{2t}

$$EcuaDos := y(t) - t e^{2t} = - \left(\int_0^t \tau e^{2\tau} y(t-\tau) d\tau \right) \quad (16)$$

> SolUno := y(t - tau) = sin(t - tau) e^{2 \cdot (t - tau)}

$$SolUno := y(t - \tau) = \sin(t - \tau) e^{2t - 2\tau} \quad (17)$$

> SolFinal := isolate \left(y(t) - t e^{2t} = - \text{eval} \left(\text{subs} \left(y(t - \tau) = rhs(SolUno), \int_0^t \tau e^{2\tau} y(t - \tau) d\tau \right) \right), y(t) \right)

$$SolFinal := y(t) = \sin(t) e^{2t} - \cos(t)^2 e^{2t} t - \sin(t)^2 e^{2t} t + t e^{2t} \quad (18)$$

>

> restart

3)

> Sistema := diff(x(t), t\$2) + 3 \cdot diff(y(t), t) + 3 \cdot y(t) = 0, diff(x(t), t\$2) + 3 \cdot y(t) = t \cdot \exp(-t) : Sistema[1]; Sistema[2]

$$\frac{d^2}{dt^2} x(t) + 3 \frac{d}{dt} y(t) + 3 y(t) = 0$$

$$\frac{d^2}{dt^2} x(t) + 3 y(t) = t e^{-t} \quad (19)$$

> CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0

$$CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0 \quad (20)$$

> with(inttrans) :

> SistTransLap := subs(CondIni, laplace(Sistema[1], t, s)), subs(CondIni, laplace(Sistema[2], t, s)) : SistTransLap[1]; SistTransLap[2]

$$s^2 \mathcal{L}(x(t), t, s) + 3 s \mathcal{L}(y(t), t, s) + 3 \mathcal{L}(y(t), t, s) = 0$$

$$s^2 \mathcal{L}(x(t), t, s) + 3 \mathcal{L}(y(t), t, s) = \frac{1}{(1+s)^2} \quad (21)$$

$$\begin{aligned} &> \text{VarTrans} := \text{solve}(\{\text{SistTransLap}\}, \{\text{laplace}(x(t), t, s), \text{laplace}(y(t), t, s)\}) \\ &\quad \text{VarTrans} := \left\{ \mathcal{L}(x(t), t, s) = \frac{1}{(1+s)s^3}, \mathcal{L}(y(t), t, s) = -\frac{1}{3s(s^2 + 2s + 1)} \right\} \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{Solucion} := \text{invlaplace}(\text{VarTrans}[1], s, t), \text{invlaplace}(\text{VarTrans}[2], s, t) : \text{Solucion}[1]; \\ &\quad \text{Solucion}[2] \end{aligned}$$

$$\begin{aligned} x(t) &= \frac{t^2}{2} - e^{-t} + 1 - t \\ y(t) &= -\frac{1}{3} + \frac{e^{-t}(t+1)}{3} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{ComprobarUno} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{rhs}(\text{Solucion}[1]), y(t) = \text{rhs}(\text{Solucion}[2])), \\ &\quad \text{lhs}(\text{Sistema}[1]) - \text{rhs}(\text{Sistema}[1]) = 0))) \\ &\quad \text{ComprobarUno} := 0 = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{rhs}(\text{Solucion}[1]), y(t) = \text{rhs}(\text{Solucion}[2])), \\ &\quad \text{lhs}(\text{Sistema}[2]) - \text{rhs}(\text{Sistema}[2]) = 0))) \\ &\quad \text{ComprobarDos} := 0 = 0 \end{aligned} \quad (25)$$

> restart

4)

$$\begin{aligned} &> \text{Ecua} := x^2 \cdot \text{diff}(u(x, y), x\$2) + \text{diff}(u(x, y), y\$2) = 0 \\ &\quad \text{Ecua} := x^2 \left(\frac{\partial^2}{\partial x^2} u(x, y) \right) + \frac{\partial^2}{\partial y^2} u(x, y) = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{alpha} := 0 \end{aligned}$$

$$\alpha := 0 \quad (27)$$

$$\begin{aligned} &> \text{EcuaDos} := \text{eval}(\text{subs}(u(x, y) = F(x) \cdot G(y), \text{Ecua})) \\ &\quad \text{EcuaDos} := x^2 \left(\frac{d^2}{dx^2} F(x) \right) G(y) + F(x) \left(\frac{d^2}{dy^2} G(y) \right) = 0 \end{aligned} \quad (28)$$

$$\begin{aligned} &> \text{EcuaSeparada} := \frac{\text{lhs}(\text{EcuaDos}) - F(x) \left(\frac{d^2}{dy^2} G(y) \right)}{F(x) \cdot G(y)} \\ &\quad = \frac{\text{rhs}(\text{EcuaDos}) - F(x) \left(\frac{d^2}{dy^2} G(y) \right)}{F(x) \cdot G(y)} \\ &\quad \text{EcuaSeparada} := \frac{x^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = -\frac{\frac{d^2}{dy^2} G(y)}{G(y)} \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{EcuaX} := \text{lhs}(\text{EcuaSeparada}) = \text{alpha} \end{aligned}$$

$$\text{EcuaX} := \frac{x^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = 0 \quad (30)$$

$$\begin{aligned} &> \text{EcuaY} := \text{rhs}(\text{EcuaSeparada}) = \text{alpha} \end{aligned}$$

$$EcuaY := -\frac{\frac{d^2}{dy^2} G(y)}{G(y)} = 0 \quad (31)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 x + c_2 \quad (32)$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = c_1 y + c_2 \quad (33)$$

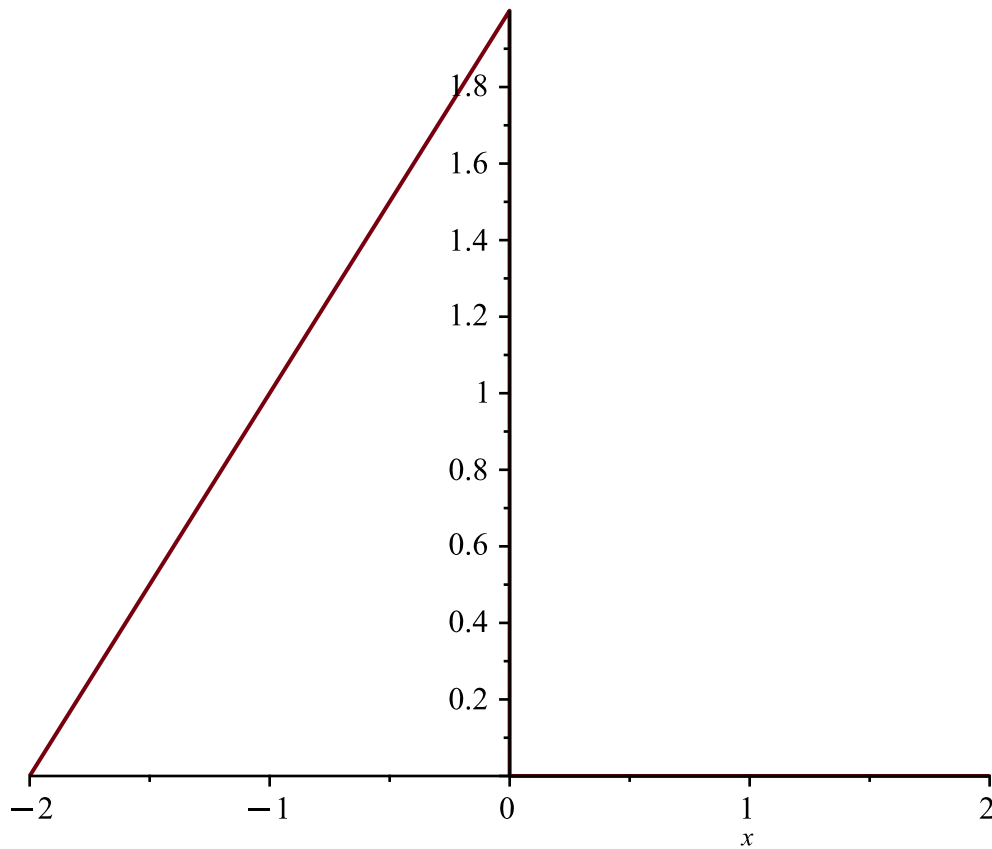
> $SolGral := u(x, y) = rhs(SolX) \cdot subs(c_1 = 1, c_2 = 1, rhs(SolY))$

$$SolGral := u(x, y) = (c_1 x + c_2) (1 + y) \quad (34)$$

> $restart$

5)

> $f := (x + 2) \cdot Heaviside(x + 2) - x \cdot Heaviside(x) - 2 \cdot Heaviside(x) : plot(f, x = -2..2)$



> $L := 2$

$$L := 2 \quad (35)$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := 1 \quad (36)$$

> $a[n] := subs\left(\cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$(37)$$

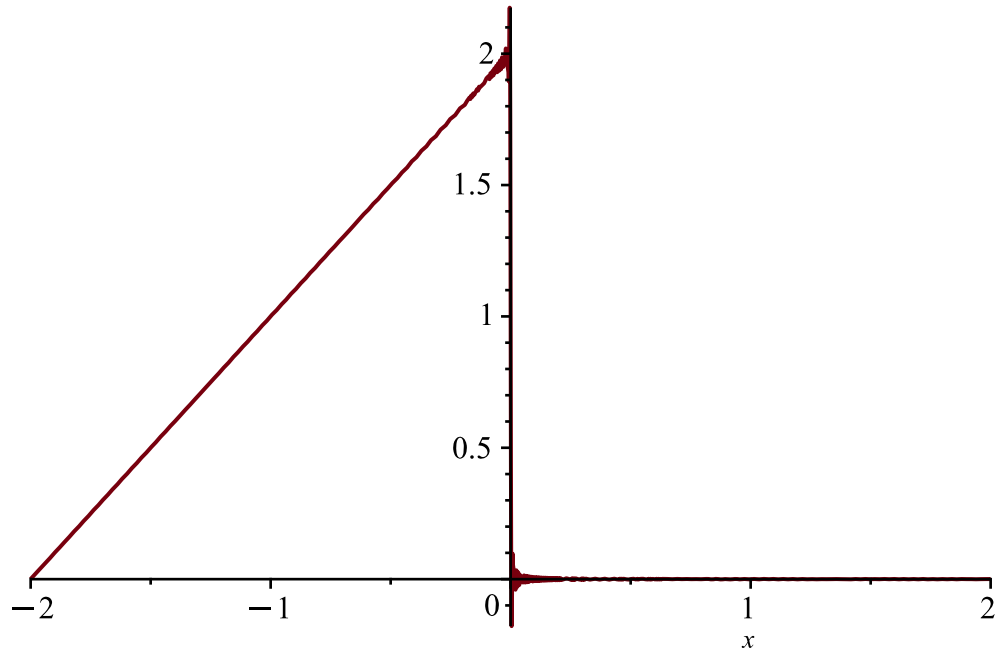
$$a_n := \frac{4 - 4(-1)^n}{2n^2\pi^2} \quad (37)$$

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> b[n] := subs(sin(n·Pi) = 0, 1/L · int(f·sin(n·Pi/L · x), x = -L..L))
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$$b_n := -\frac{2}{n\pi} \quad (38)$$

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> STF500 := a[0]/2 + sum(a[n]·cos(n·Pi/L · x) + b[n]·sin(n·Pi/L · x), n = 1..500) :
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> plot(STF500, x = -2..2)
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> restart
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FIN EXAMEN
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