>

>

EXAMEN_3 2025-2-2(CAP3&4) MARTES

> *restart*

1)

> $Sistema := diff(x(t), t) = -x(t) - 2\cdot y(t) + 1,\ diff(y(t), t) = 2\cdot x(t) + 3\cdot y(t) + 1$:
$Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\,x(t) = -x(t) - 2\,y(t) + 1$$

$$\frac{d}{dt}\,y(t) = 2\,x(t) + 3\,y(t) + 1 \qquad (1)$$

> $CondIni := x(0) = 0, y(0) = 0$

$$CondIni := x(0) = 0, y(0) = 0 \qquad (2)$$

> $with(inttrans)$:

> $SistTransLap := subs(CondIni, laplace(Sistema[1], t, s)), subs(CondIni, laplace(Sistema[2], t, s))$: $SistTransLap[1]; SistTransLap[2]$

$$s\,\mathcal{L}(x(t), t, s) = -\mathcal{L}(x(t), t, s) - 2\,\mathcal{L}(y(t), t, s) + \frac{1}{s}$$

$$s\,\mathcal{L}(y(t), t, s) = 2\,\mathcal{L}(x(t), t, s) + 3\,\mathcal{L}(y(t), t, s) + \frac{1}{s} \qquad (3)$$

> $VarTrans := solve(\{SistTransLap\}, \{laplace(x(t), t, s), laplace(y(t), t, s)\})$

$$VarTrans := \left\{\mathcal{L}(x(t), t, s) = \frac{s-5}{s\left(s^2 - 2\,s + 1\right)}, \mathcal{L}(y(t), t, s) = \frac{s+3}{s\left(s^2 - 2\,s + 1\right)}\right\} \qquad (4)$$

> $Solucion := invlaplace(VarTrans[1], s, t), invlaplace(VarTrans[2], s, t)$:
$expand(Solucion[1]); expand(Solucion[2])$

$$x(t) = -5 - 4\,e^t\,t + 5\,e^t$$

$$y(t) = 3 + 4\,e^t\,t - 3\,e^t \qquad (5)$$

> $ComprobarUno := simplify(eval(subs(Solucion[1], Solucion[2], lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))$

$$ComprobarUno := 0 = 0 \qquad (6)$$

> $ComprobarDos := simplify(eval(subs(Solucion[1], Solucion[2], lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))$

$$ComprobarDos := 0 = 0 \qquad (7)$$

> *restart*

2)

> $Ecua := t - 2\cdot f(t) = int((\exp(tau) - \exp(-tau))\cdot f(t - tau), tau = 0..t)$

$$Ecua := t - 2\,f(t) = \int_0^t \left(e^{\tau} - e^{-\tau}\right) f(t - \tau)\,d\tau \qquad (8)$$

> $with(inttrans)$:

> $TransLapEcua := laplace(Ecua, t, s)$

(9)

$$TransLapEcua := \frac{1}{s^2} - 2\,\mathcal{L}(f(t),t,s) = \frac{\mathcal{L}(f(t),t,s)}{s-1} - \frac{\mathcal{L}(f(t),t,s)}{1+s} \tag{9}$$

> $TransLapSol := simplify(isolate(TransLapEcua, laplace(f(t),t,s)))$

$$TransLapSol := \mathcal{L}(f(t),t,s) = \frac{s^2-1}{2\,s^4} \tag{10}$$

> $Sol := invlaplace(TransLapSol, s, t)$

$$Sol := f(t) = -\frac{t\,(t^2-6)}{12} \tag{11}$$

> $Ecua$

$$t - 2\,f(t) = \int_0^t \left(e^{\tau} - e^{-\tau}\right) f(t-\tau)\,d\tau \tag{12}$$

> $SolUno := f(t-tau) = -\frac{(t-tau)\cdot\left((t-tau)^2-6\right)}{12}$

$$SolUno := f(t-\tau) = -\frac{(t-\tau)\,\left((t-\tau)^2-6\right)}{12} \tag{13}$$

> $EcuaDos := t - 2\cdot f(t) = eval\left(subs\left(f(t-tau) = rhs(SolUno), \int_0^t \left(e^{\tau}-e^{-\tau}\right) f(t-\tau)\,d\tau\right)\right)$

$$EcuaDos := t - 2\,f(t) = \frac{t^3}{6} \tag{14}$$

> $SolFinal := isolate(EcuaDos, f(t))$

$$SolFinal := f(t) = -\frac{1}{12}\,t^3 + \frac{1}{2}\,t \tag{15}$$

> $ComprobarDos := simplify(eval(subs(f(t) = rhs(SolFinal), lhs(EcuaDos) - rhs(EcuaDos) = 0)))$

$$ComprobarDos := 0 = 0 \tag{16}$$

> $restart$

3)

> $Sistema := diff(x(t), t\$2) + 3\cdot diff(y(t),t) + 3\cdot y(t) = 0,\ diff(x(t), t\$2) + 3\cdot y(t) = t\cdot\exp(-t) : Sistema[1]; Sistema[2]$

$$\frac{d^2}{dt^2}\,x(t) + 3\,\frac{d}{dt}\,y(t) + 3\,y(t) = 0$$

$$\frac{d^2}{dt^2}\,x(t) + 3\,y(t) = t\,e^{-t} \tag{17}$$

> $CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0$

$$CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0 \tag{18}$$

> $with(inttrans) :$

> $SistTransLap := subs(CondIni, laplace(Sistema[1], t, s)), subs(CondIni, laplace(Sistema[2], t, s)) : SistTransLap[1]; SistTransLap[2]$

$$s^2\,\mathcal{L}(x(t),t,s) + 3\,s\,\mathcal{L}(y(t),t,s) + 3\,\mathcal{L}(y(t),t,s) = 0$$

$$s^2\,\mathcal{L}(x(t),t,s)+3\,\mathcal{L}(y(t),t,s)=\frac{1}{(1+s)^2} \tag{19}$$

> $VarTrans := solve(\{SistTransLap\},\{laplace(x(t),t,s),laplace(y(t),t,s)\})$

$$VarTrans := \left\{\mathcal{L}(x(t),t,s)=\frac{1}{(1+s)\,s^3},\ \mathcal{L}(y(t),t,s)=-\frac{1}{3\,s\,(s^2+2\,s+1)}\right\} \tag{20}$$

> $Solucion := invlaplace(VarTrans[1],s,t), invlaplace(VarTrans[2],s,t) : Solucion[1]; Solucion[2]$

$$x(t)=-e^{-t}+\frac{t^2}{2}-t+1$$

$$y(t)=-\frac{1}{3}+\frac{e^{-t}\,(1+t)}{3} \tag{21}$$

> $ComprobarUno := simplify(eval(subs(x(t)=rhs(Solucion[1]),y(t)=rhs(Solucion[2]),lhs(Sistema[1])-rhs(Sistema[1])=0)))$

$$ComprobarUno := 0=0 \tag{22}$$

> $ComprobarDos := simplify(eval(subs(x(t)=rhs(Solucion[1]),y(t)=rhs(Solucion[2]),lhs(Sistema[2])-rhs(Sistema[2])=0)))$

$$ComprobarDos := 0=0 \tag{23}$$

> $restart$

4)

> $Ecua := diff(u(x,y),x\$2)-3\cdot diff(u(x,y),y)=0$

$$Ecua := \frac{\partial^2}{\partial x^2}\,u(x,y)-3\,\frac{\partial}{\partial y}\,u(x,y)=0 \tag{24}$$

> alpha := 3

$$\alpha := 3 \tag{25}$$

> $EcuaDos := eval(subs(u(x,y)=F(x)\cdot G(y),Ecua))$

$$EcuaDos := \left(\frac{d^2}{dx^2}\,F(x)\right)G(y)-3\,F(x)\left(\frac{d}{dy}\,G(y)\right)=0 \tag{26}$$

> $$EcuaSeparada := \frac{\left(lhs(EcuaDos)+3\,F(x)\left(\frac{d}{dy}\,G(y)\right)\right)}{3\cdot F(x)\cdot G(y)} = \frac{\left(rhs(EcuaDos)+3\,F(x)\left(\frac{d}{dy}\,G(y)\right)\right)}{3\cdot F(x)\cdot G(y)}$$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2}\,F(x)}{3\,F(x)}=\frac{\frac{d}{dy}\,G(y)}{G(y)} \tag{27}$$

> $EcuaX := lhs(EcuaSeparada)=$ alpha

$$EcuaX := \frac{\frac{d^2}{dx^2}\,F(x)}{3\,F(x)}=3 \tag{28}$$

> $EcuaY := rhs(EcuaSeparada) = \text{alpha}$

$$EcuaY := \frac{\frac{d}{dy} G(y)}{G(y)} = 3 \tag{29}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 e^{-3x} + c_2 e^{3x} \tag{30}$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = c_1 e^{3y} \tag{31}$$

> $SolFinal := u(x,y) = rhs(SolX) \cdot subs(c_1 = 1, rhs(SolY))$

$$SolFinal := u(x,y) = \left(c_1 e^{-3x} + c_2 e^{3x}\right) e^{3y} \tag{32}$$

> $Ecua$

$$\frac{\partial^2}{\partial x^2} u(x,y) - 3 \frac{\partial}{\partial y} u(x,y) = 0 \tag{33}$$

> $Comprobar := simplify(eval(subs(u(x,y) = rhs(SolFinal), Ecua)))$

$$Comprobar := 0 = 0 \tag{34}$$

> $EcuaSeparadaDos := \frac{\left(lhs(EcuaDos) + 3 F(x) \left(\frac{d}{dy} G(y)\right)\right)}{F(x) \cdot G(y)}$

$$= \frac{\left(rhs(EcuaDos) + 3 F(x) \left(\frac{d}{dy} G(y)\right)\right)}{F(x) \cdot G(y)}$$

$$EcuaSeparadaDos := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{3 \left(\frac{d}{dy} G(y)\right)}{G(y)} \tag{35}$$

> $EcuaXX := lhs(EcuaSeparadaDos) = \text{alpha}$

$$EcuaXX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 3 \tag{36}$$

> $EcuaYY := \frac{rhs(EcuaSeparadaDos)}{3} = \frac{\text{alpha}}{3}$

$$EcuaYY := \frac{\frac{d}{dy} G(y)}{G(y)} = 1 \tag{37}$$

> $SolXX := dsolve(EcuaXX)$

$$SolXX := F(x) = c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x} \tag{38}$$

> $SolYY := dsolve(EcuaYY)$

$$SolYY := G(y) = c_1 e^{y} \tag{39}$$

> $SolFinalDos := u(x,y) = rhs(SolXX) \cdot subs(c_1 = 1, rhs(SolYY))$

(40)

$$SolFinalDos := u(x,y) = \left(c_1\, e^{\sqrt{3}\,x} + c_2\, e^{-\sqrt{3}\,x}\right) e^{y} \quad (40)$$

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> restart
5)
> f := (x + 2)·Heaviside(x + 2) − x·Heaviside(x) − 2· Heaviside(x) : plot(f, x = −2..2)
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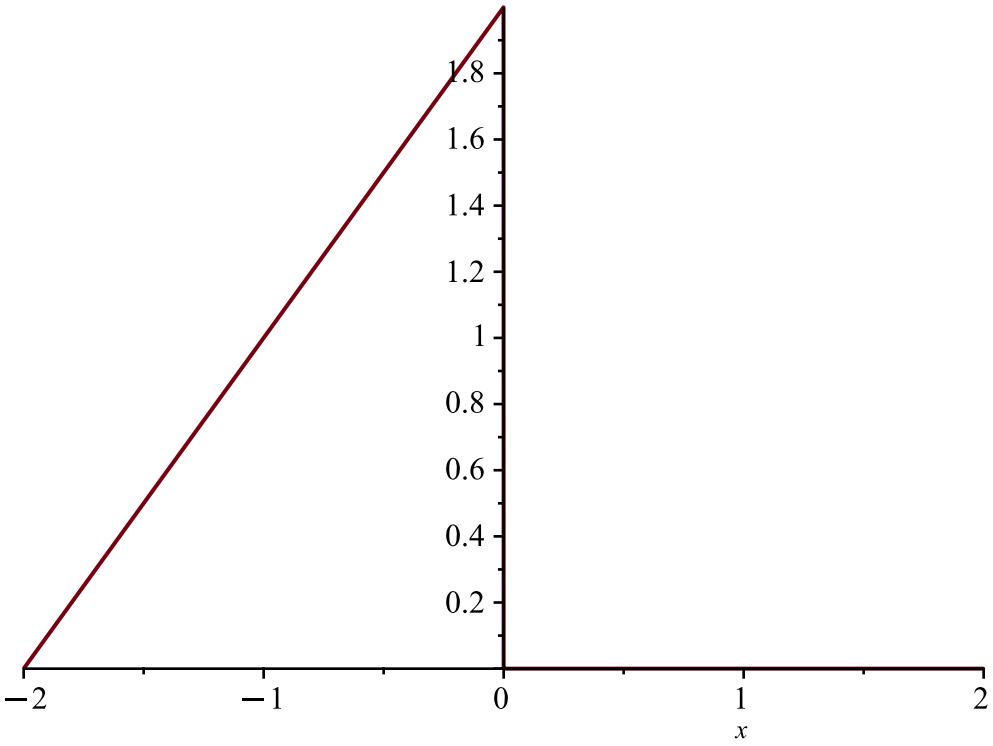


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> L := 2
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$$L := 2 \quad (41)$$

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> a[0] := (1/L)·int(f, x = −L..L)
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$$a_0 := 1 \quad (42)$$

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> a[n] := subs(cos(n·Pi) = (−1)^n, (1/L)·int(f·cos((n·Pi/L)·x), x = −L..L))
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$$a_n := \frac{-4\,(-1)^n + 4}{2\,n^2\,\pi^2} \quad (43)$$

```
> b[n] := subs(sin(n·Pi) = 0, (1/L)·int(f·sin((n·Pi/L)·x), x = −L..L))
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$$b_n := -\frac{2}{n\,\pi} \quad (44)$$

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> STF500 := a[0]/2 + sum(a[n]·cos((n·Pi/L)·x) + b[n]·sin((n·Pi/L)·x), n = 1..500) :
> plot(STF500, x = −2..2)
```

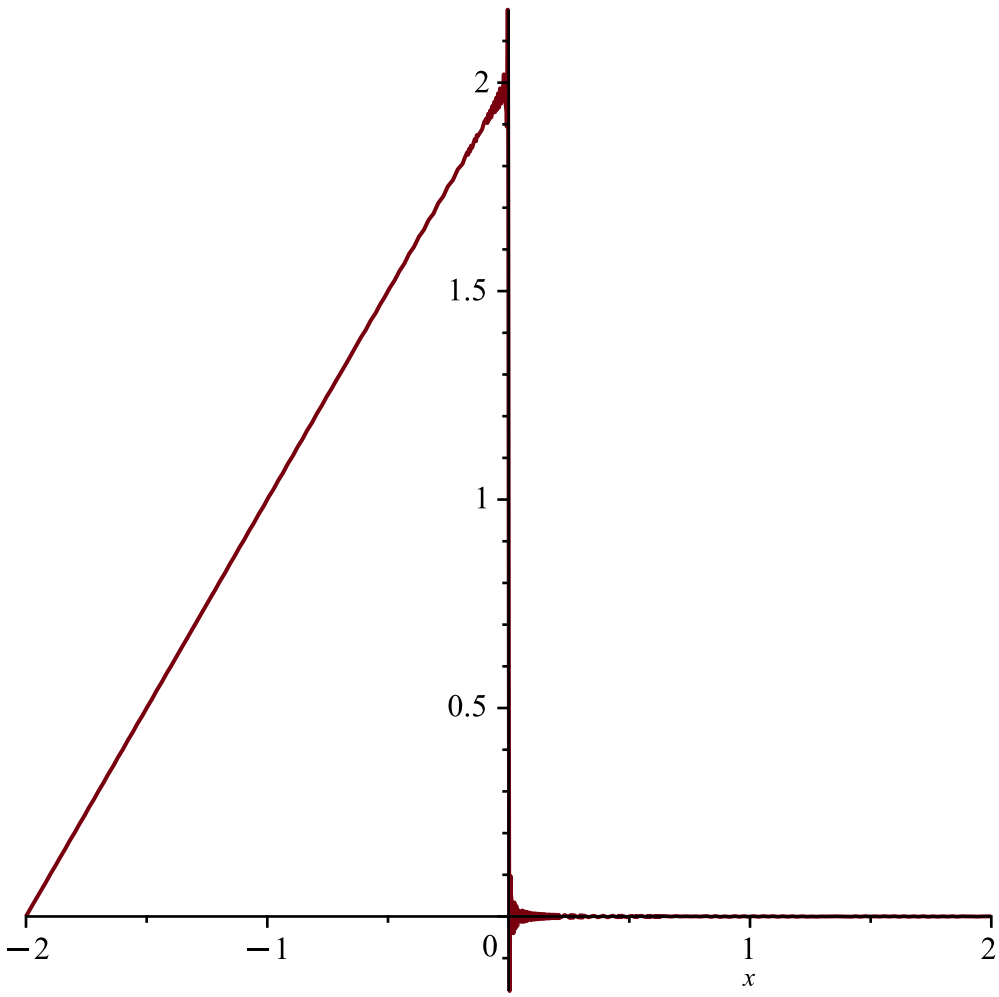


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> restart
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FIN EXAMEN

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