>

EXAMEN_5 2025-2-F1

> restart

1)

> Ecua := diff(r(theta), theta) + r(theta)·cot(theta) = cos(theta)

$$Ecua := \frac{d}{d\theta}\, r(\theta) + r(\theta)\cot(\theta) = \cos(\theta) \quad \textbf{(1)}$$

RESPUESTA

> P := cot(theta); Q := cos(theta)

$$P := \cot(\theta)$$
$$Q := \cos(\theta) \quad \textbf{(2)}$$

> ExpMenosPtehta := exp(int(−P, theta))

$$ExpMenosPtehta := \frac{1}{\sin(\theta)} \quad \textbf{(3)}$$

> ExpMasPtehta := exp(int(P, theta))

$$ExpMasPtehta := \sin(\theta) \quad \textbf{(4)}$$

> SolGral := r(theta) = _C1·ExpMenosPtehta + ExpMenosPtehta·int(ExpMasPtehta·Q, theta)

$$SolGral := r(\theta) = \frac{\_C1}{\sin(\theta)} + \frac{\sin(\theta)}{2} \quad \textbf{(5)}$$

> Ecua

$$\frac{d}{d\theta}\, r(\theta) + r(\theta)\cot(\theta) = \cos(\theta) \quad \textbf{(6)}$$

> Comprobar := simplify(eval(subs(r(theta) = rhs(SolGral), lhs(Ecua) − rhs(Ecua) = 0)))

$$Comprobar := 0 = 0 \quad \textbf{(7)}$$

> restart

2)

> $Ecua := \left(\frac{3\cdot x}{(x^2+y^2)} + 6\cdot x\cdot y\right) + \left(\frac{3\cdot y}{(x^2+y^2)} + 3\cdot x^2\right)\cdot y' = 0$

$$Ecua := \frac{3\,x}{x^2+y(x)^2} + 6\,x\,y(x) + \left(\frac{3\,y(x)}{x^2+y(x)^2} + 3\,x^2\right)\left(\frac{d}{dx}\, y(x)\right) = 0 \quad \textbf{(8)}$$

> with(DEtools) :

> odeadvisor(Ecua)

$$[\_exact, \_rational] \quad \textbf{(9)}$$

> $M := \frac{3\,x}{x^2+y^2} + 6\,x\,y$

$$M := \frac{3\,x}{x^2+y^2} + 6\,x\,y \quad \textbf{(10)}$$

> $N := \left(\frac{3\,y}{x^2+y^2} + 3\,x^2\right)$

$$N := \frac{3\,y}{x^2+y^2} + 3\,x^2 \quad \textbf{(11)}$$

> $IntMx := int(M, x)$

$$IntMx := \frac{3\ln(x^2+y^2)}{2} + 3\,x^2\,y \quad \textbf{(12)}$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGral := \frac{3\ln(x^2+y^2)}{2} + 3\,x^2\,y = \_C1 \quad \textbf{(13)}$$

> $SolGralDos := lhs(SolGral)\cdot 2 = \_C1$

$$SolGralDos := 3\ln(x^2+y^2) + 6\,x^2\,y = \_C1 \quad \textbf{(14)}$$

> $SolFinal := 3\ln(x^2+y(x)^2) + 6\,x^2\,y(x) = \_C1$

$$SolFinal := 3\ln(x^2+y(x)^2) + 6\,x^2\,y(x) = \_C1 \quad \textbf{(15)}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = -\frac{2\,x\left(x^2\,y(x) + y(x)^3 + \frac{1}{2}\right)}{x^4 + x^2\,y(x)^2 + y(x)} \quad \textbf{(16)}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\,y(x) = -\frac{2\,x\left(x^2\,y(x) + y(x)^3 + \frac{1}{2}\right)}{x^4 + x^2\,y(x)^2 + y(x)} \quad \textbf{(17)}$$

> $Comprobar := rhs(DerSolFinal) - rhs(DerEcua) = 0$

$$Comprobar := 0 = 0 \quad \textbf{(18)}$$

> $restart$

3)

> $Ecua := y'' - \frac{2\cdot x}{(x^2+1)}\cdot y' + \frac{2}{(x^2+1)}\cdot y = x^2 + 1$

$$Ecua := \frac{d^2}{dx^2}\,y(x) - \frac{2\,x\left(\frac{d}{dx}\,y(x)\right)}{x^2+1} + \frac{2\,y(x)}{x^2+1} = x^2 + 1 \quad \textbf{(19)}$$

> $Q := rhs(Ecua)$

$$Q := x^2 + 1 \quad \textbf{(20)}$$

> $SolHom := y(x) = \_C1\cdot(x^2-1) + \_C2\cdot x$

$$SolHom := y(x) = \_C1\,(x^2-1) + \_C2\,x \quad \textbf{(21)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) - \frac{2\,x\left(\frac{d}{dx}\,y(x)\right)}{x^2+1} + \frac{2\,y(x)}{x^2+1} = 0 \quad \textbf{(22)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolHom), EcuaHom)))$

$$Comprobar := 0 = 0 \quad \textbf{(23)}$$

> $SolNoHom := y(x) = A\,(x^2-1) + B\,x$

$$SolNoHom := y(x) = A\,(x^2-1) + B\,x \qquad (24)$$

> $yy[1] := x^2-1; yy[2] := x$

$$yy_1 := x^2 - 1$$
$$yy_2 := x \qquad (25)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} x^2-1 & x \\ 2\,x & 1 \end{bmatrix} \qquad (26)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x^2+1 \end{bmatrix} \qquad (27)$$

> $RR := linsolve(WW, BB)$

$$RR := \begin{bmatrix} x & -x^2+1 \end{bmatrix} \qquad (28)$$

> $Aprima := RR[1]$

$$Aprima := x \qquad (29)$$

> $Bprima := RR[2]$

$$Bprima := -x^2+1 \qquad (30)$$

> $SolGralNoHom := y(x) = expand((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGralNoHom := y(x) = \frac{1}{6}\,x^4 + \frac{1}{2}\,x^2 + \_C1\,x^2 - \_C1 + \_C2\,x \qquad (31)$$

>

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), Ecua)))$

$$Comprobar := x^2+1 = x^2+1 \qquad (32)$$

> $restart$

4)

> $Ecua := diff(y(t), t) + int(y(\mathrm{tau}), \mathrm{tau} = 0..t) = \cos(t)$

$$Ecua := \frac{\mathrm{d}}{\mathrm{d}t}\,y(t) + \int_0^t y(\tau)\,\mathrm{d}\tau = \cos(t) \qquad (33)$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \qquad (34)$$

por transformada de Laplace

> $with(inttrans):$

> $EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTransLap := s\,\mathcal{L}(y(t), t, s) + \frac{\mathcal{L}(y(t), t, s)}{s} = \frac{s}{s^2+1} \qquad (35)$$

> $SolTransLap := isolate(EcuaTransLap, laplace(y(t), t, s))$

(36)

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{s}{(s^2+1)\left(s+\frac{1}{s}\right)} \quad \textbf{(36)}$$

> *SolPart* := *invlaplace*(*SolTransLap*, *s*, *t*)

$$SolPart := y(t) = \frac{\sin(t)}{2} + \frac{t\cos(t)}{2} \quad \textbf{(37)}$$

> $SolEsp := y(\mathrm{tau}) = \frac{\sin(\mathrm{tau})}{2} + \frac{(\mathrm{tau})\cdot\cos(\mathrm{tau})}{2}$

$$SolEsp := y(\tau) = \frac{\sin(\tau)}{2} + \frac{\tau\cos(\tau)}{2} \quad \textbf{(38)}$$

> *IntF* := *eval*(*subs*(*y*(tau) = *rhs*(*SolEsp*), *int*(*y*(tau), tau = 0 ..*t*)))

$$IntF := \frac{t\sin(t)}{2} \quad \textbf{(39)}$$

> *EcuaDos* := *lhs*(*Ecua*) − *diff*(*y*(*t*), *t*) = *rhs*(*Ecua*) − *diff*(*y*(*t*), *t*)

$$EcuaDos := \int_0^t y(\tau)\,\mathrm{d}\tau = \cos(t) - \frac{\mathrm{d}}{\mathrm{d}t}\,y(t) \quad \textbf{(40)}$$

> *EcuaTres* := *IntF* = *rhs*(*EcuaDos*)

$$EcuaTres := \frac{t\sin(t)}{2} = \cos(t) - \frac{\mathrm{d}}{\mathrm{d}t}\,y(t) \quad \textbf{(41)}$$

> *Comprobar* := *eval*(*subs*(*y*(*t*) = *rhs*(*SolPart*), *EcuaTres*))

$$Comprobar := \frac{t\sin(t)}{2} = \frac{t\sin(t)}{2} \quad \textbf{(42)}$$

> *restart*

5)

> $F := \frac{(s^2+4\cdot s+10)}{(s^2+4\cdot s+13)\cdot(s+2)}$

$$F := \frac{s^2+4\,s+10}{(s^2+4\,s+13)\,(s+2)} \quad \textbf{(43)}$$

> *with*(*inttrans*) :

> *invlaplace*(*F*, *s*, *t*)

$$\frac{\mathrm{e}^{-2\,t}\,(2+\cos(3\,t))}{3} \quad \textbf{(44)}$$

> *restart*

6)

> *Sistema* := *diff*(*u*(*t*), *t*) = 2·*u*(*t*) + *v*(*t*), *diff*(*v*(*t*), *t*) = −3·*u*(*t*) + *v*(*t*) : *Sistema*[1]; *Sistema*[2]

$$\frac{\mathrm{d}}{\mathrm{d}t}\,u(t) = 2\,u(t) + v(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,v(t) = -3\,u(t) + v(t) \quad \textbf{(45)}$$

> $CondIni := u(0) = 1, v(0) = 2$

$$CondIni := u(0) = 1, v(0) = 2 \quad (46)$$

> $with(inttrans):$

> $SistTrans[1] := subs(CondIni, laplace(Sistema[1], t, s))$

$$SistTrans_1 := s\,\mathcal{L}(u(t), t, s) - 1 = 2\,\mathcal{L}(u(t), t, s) + \mathcal{L}(v(t), t, s) \quad (47)$$

> $SistTrans[2] := subs(CondIni, laplace(Sistema[2], t, s))$

$$SistTrans_2 := s\,\mathcal{L}(v(t), t, s) - 2 = -3\,\mathcal{L}(u(t), t, s) + \mathcal{L}(v(t), t, s) \quad (48)$$

> $SolTrans := solve(\{SistTrans[1], SistTrans[2]\}, \{laplace(u(t), t, s), laplace(v(t), t, s)\})$

$$SolTrans := \left\{\mathcal{L}(u(t), t, s) = \frac{s+1}{s^2 - 3\,s + 5}, \mathcal{L}(v(t), t, s) = \frac{2\,s - 7}{s^2 - 3\,s + 5}\right\} \quad (49)$$

> $SolPart[1] := simplify(invlaplace(SolTrans[1], s, t))$

$$SolPart_1 := u(t) = \frac{e^{\frac{3\,t}{2}}\left(11\cos\left(\frac{\sqrt{11}\,t}{2}\right) + 5\sqrt{11}\sin\left(\frac{\sqrt{11}\,t}{2}\right)\right)}{11} \quad (50)$$

> $SolPart[2] := simplify(invlaplace(SolTrans[2], s, t))$

$$SolPart_2 := v(t) = 2\,e^{\frac{3\,t}{2}}\left(-\frac{4\sqrt{11}\sin\left(\frac{\sqrt{11}\,t}{2}\right)}{11} + \cos\left(\frac{\sqrt{11}\,t}{2}\right)\right) \quad (51)$$

> $ComprobarUno := simplify(subs(t=0, SolPart[1]))$

$$ComprobarUno := u(0) = 1 \quad (52)$$

> $ComprobarDos := simplify(subs(t=0, SolPart[2]))$

$$ComprobarDos := v(0) = 2 \quad (53)$$

> $CondIni$

$$u(0) = 1, v(0) = 2 \quad (54)$$

> $ComprobarTres := simplify(eval(subs(u(t) = rhs(SolPart[1]), v(t) = rhs(SolPart[2]), lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))$

$$ComprobarTres := 0 = 0 \quad (55)$$

> $ComprobarCuatro := simplify(eval(subs(u(t) = rhs(SolPart[1]), v(t) = rhs(SolPart[2]), lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))$

$$ComprobarCuatro := 0 = 0 \quad (56)$$

> $restart$

7)

> $Ecua := diff(u(x, t), t) = 9 \cdot diff(u(x, t), x\$2)$

$$Ecua := \frac{\partial}{\partial t}\,u(x, t) = 9\,\frac{\partial^2}{\partial x^2}\,u(x, t) \quad (57)$$

> $Sep := \text{alpha} = 9$

$$Sep := \alpha = 9 \quad (58)$$

> $EcuaDos := eval(subs(u(x, t) = F(x) \cdot G(t), Ecua))$

$$EcuaDos := F(x)\left(\frac{d}{dt}\,G(t)\right) = 9\left(\frac{d^2}{dx^2}\,F(x)\right)G(t) \quad (59)$$

> $EcuaSep := \frac{lhs(EcuaDos)}{F(x)\cdot G(t)} = \frac{rhs(EcuaDos)}{F(x)\cdot G(t)}$

$$EcuaSep := \frac{\frac{d}{dt} G(t)}{G(t)} = \frac{9\left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} \quad (60)$$

> $EcuaX := rhs(EcuaSep) = 9$

$$EcuaX := \frac{9\left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = 9 \quad (61)$$

> $EcuaT := lhs(EcuaSep) = 9$

$$EcuaT := \frac{\frac{d}{dt} G(t)}{G(t)} = 9 \quad (62)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 e^{x} + c_2 e^{-x} \quad (63)$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = c_1 e^{9t} \quad (64)$$

> $SolFinal := u(x, t) = rhs(SolX)\cdot(subs(c_1 = 1, rhs(SolT)))$

$$SolFinal := u(x, t) = \left(c_1 e^{x} + c_2 e^{-x}\right) e^{9t} \quad (65)$$

> $Comprobar := simplify(eval(subs(u(x, t) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (66)$$

> $EcuaSepDos := \frac{lhs(EcuaDos)}{9\cdot F(x)\cdot G(t)} = \frac{rhs(EcuaDos)}{9\cdot F(x)\cdot G(t)}$

$$EcuaSepDos := \frac{\frac{d}{dt} G(t)}{9\,G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \quad (67)$$

> $EcuaXX := rhs(EcuaSepDos) = 9$

$$EcuaXX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 9 \quad (68)$$

> $EcuaTT := lhs(EcuaSepDos) = 9$

$$EcuaTT := \frac{\frac{d}{dt} G(t)}{9\,G(t)} = 9 \quad (69)$$

> $SolXX := dsolve(EcuaXX)$

$$SolXX := F(x) = c_1 e^{-3x} + c_2 e^{3x} \quad (70)$$

> $SolTT := dsolve(EcuaTT)$

$$SolTT := G(t) = c_1 e^{81t} \quad (71)$$

> $SolFinalDos := u(x,t) = rhs(SolXX) \cdot (subs(c_1 = 1, rhs(SolTT)))$

$$SolFinalDos := u(x,t) = \left(c_1 e^{-3x} + c_2 e^{3x}\right) e^{81 t} \quad (72)$$

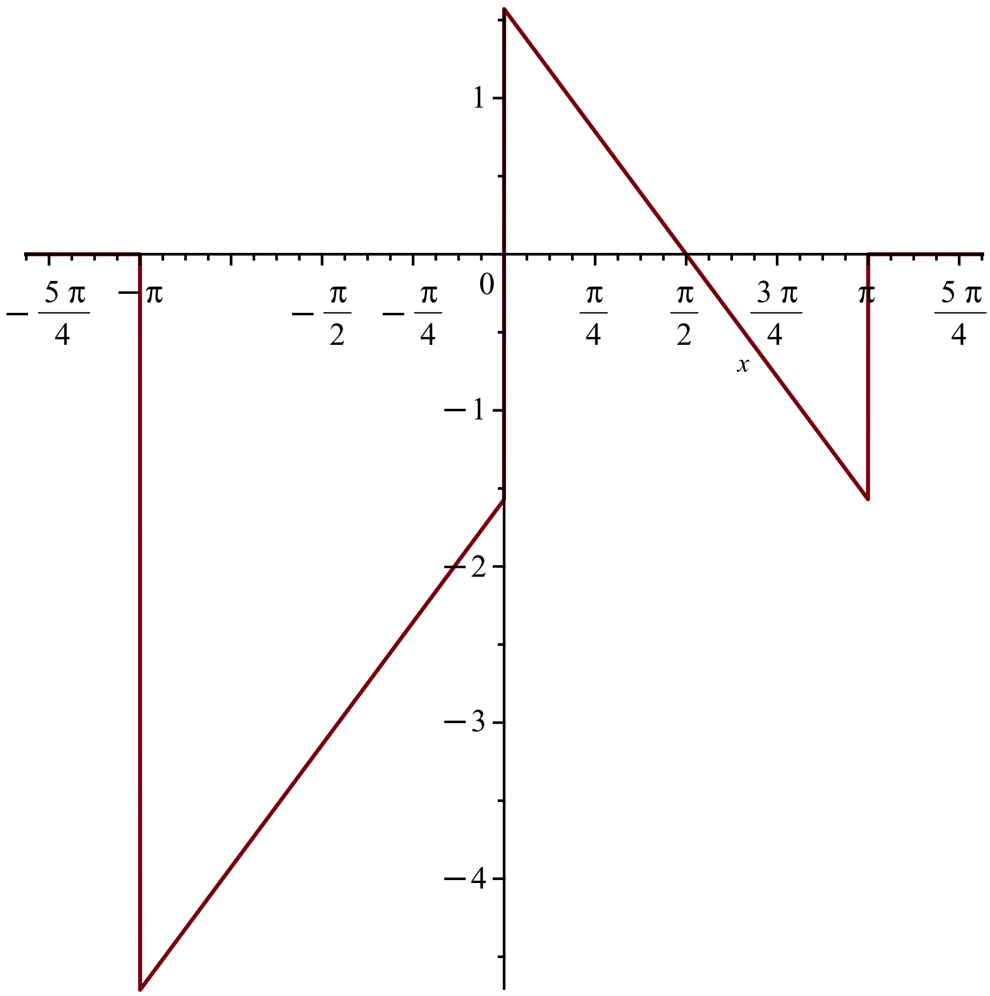
> $ComprobarDos := simplify(eval(subs(u(x,t) = rhs(SolFinalDos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \quad (73)$$

> $restart$

8)

> $f := piecewise\left(x < -\text{Pi}, 0, x < 0, x - \frac{\text{Pi}}{2}, x \le \text{Pi}, \frac{\text{Pi}}{2} - x\right) : plot(f, x = -\text{Pi} - 1 .. \text{Pi} + 1)$



> $L := \text{Pi}$

$$L := \pi \quad (74)$$

> $with(linalg):$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L .. L)$

$$a_0 := -\pi \quad (75)$$

> $evalf(a[0])$

$$-3.141592654 \quad (76)$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} x\right), x = -L..L\right)\right)$

$$a_n := -\frac{-2 + 2\,(-1)^n}{\pi\, n^2} \tag{77}$$

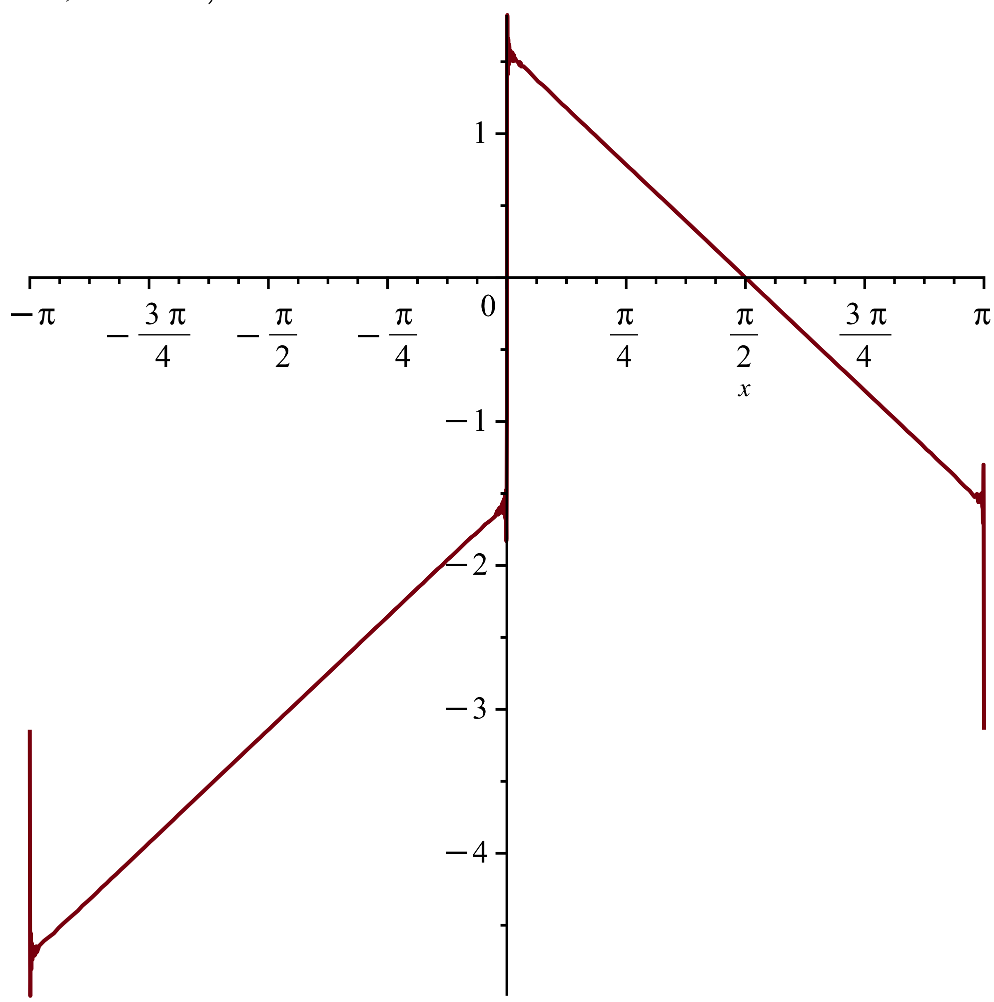
>

> $b[n] := simplify\left(subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} x\right), x = -L..L\right)\right)\right)$

$$b_n := \frac{1 - (-1)^n}{n} \tag{78}$$

> $STF1000 := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..1000\right)$:

> $plot(STF1000, x = -L..L)$



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