>

Eamen parcial 2026-1-1
tipo A
Solución

>

> *restart*

1) Muestre que la ecuación diferencial dada tiene por solución general presentada & obtenga la solución prticular que satisfaga la condición dada

>

> $Ec := y' = \frac{(y+x)}{(y-x)}$

$$Ec := \frac{d}{dx}\, y(x) = \frac{y(x)+x}{y(x)-x} \quad \textbf{(1)}$$

> $SolGral := x^2 - y(x)^2 + 2\cdot x\cdot y(x) = \_C1$

$$SolGral := x^2 - y(x)^2 + 2\,x\,y(x) = \_C1 \quad \textbf{(2)}$$

> $CondIni := y(-2) = 3$

$$CondIni := y(-2) = 3 \quad \textbf{(3)}$$

RESPUESTA

> $DerSolGral := simplify(isolate(diff(SolGral, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{y(x)+x}{y(x)-x} \quad \textbf{(4)}$$

> $Comprobar := simplify(rhs(Ec) - rhs(DerSolGral) = 0)$

$$Comprobar := 0 = 0 \quad \textbf{(5)}$$

> $Para := subs(x = -2, y(-2) = 3, SolGral)$

$$Para := -17 = \_C1 \quad \textbf{(6)}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$SolPart := x^2 - y(x)^2 + 2\,x\,y(x) = -17 \quad \textbf{(7)}$$

> $SolPartDos := isolate(SolPart, y(x))$

$$SolPartDos := y(x) = x - \sqrt{2\,x^2 + 17} \quad \textbf{(8)}$$

> $DerSolPart := simplify(isolate(diff(SolPart, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\, y(x) = \frac{y(x)+x}{y(x)-x} \quad \textbf{(9)}$$

> $ComprobarDos := simplify(rhs(Ec) - rhs(DerSolPart) = 0)$

$$ComprobarDos := 0 = 0 \quad \textbf{(10)}$$

> $ComprobarTres := subs(x = -2, y(-2) = 3, SolPart)$

$$ComprobarTres := -17 = -17 \quad \textbf{(11)}$$

FIN PROBLEMA 1

> *restart*

2) Sea la ecuación dada,
a) Obtenga, si es posible, un factor de integración que dependa de una sóla variable
b) Si obtiene un factor de integración, resuelva la ecuación diferencial

>

> $Ecua := y + (y + x - x\cdot y)\cdot y' = 0$

$$Ecua := y(x) + (y(x) + x - x\,y(x))\left(\frac{d}{dx}\,y(x)\right) = 0 \qquad \textbf{(12)}$$

RESPUESTA

> $with(DEtools):$

> $intfactor(Ecua)$

$$\frac{1}{e^{y(x)}} \qquad \textbf{(13)}$$

> $FactInt := \frac{1}{e^{y}}$

$$FactInt := \frac{1}{e^{y}} \qquad \textbf{(14)}$$

> $M := y$

$$M := y \qquad \textbf{(15)}$$

> $N := (y + x - x\cdot y)$

$$N := -x\,y + x + y \qquad \textbf{(16)}$$

> $MM := FactInt\cdot M$

$$MM := \frac{y}{e^{y}} \qquad \textbf{(17)}$$

> $NN := FactInt\cdot N$

$$NN := \frac{-x\,y + x + y}{e^{y}} \qquad \textbf{(18)}$$

> $ComprobarUno := simplify(diff(MM, y) - diff(NN, x)) = 0$

$$ComprobarUno := 0 = 0 \qquad \textbf{(19)}$$

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y\,x}{e^{y}} \qquad \textbf{(20)}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y\,x}{e^{y}} - \frac{y+1}{e^{y}} = \_C1 \qquad \textbf{(21)}$$

> $SolGralFinal := expand\left(\frac{y(x)\cdot x}{e^{y(x)}} - \frac{y(x)+1}{e^{y(x)}}\right) = \_C1$

$$SolGralFinal := \frac{y(x)\,x}{e^{y(x)}} - \frac{y(x)}{e^{y(x)}} - \frac{1}{e^{y(x)}} = \_C1 \qquad \textbf{(22)}$$

> $DerSolGralFinal := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGralFinal := \frac{d}{dx}\,y(x) = \frac{y(x)}{(-1+x)\,y(x) - x} \qquad \textbf{(23)}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

(24)

$$DerEcua := \frac{d}{dx}\, y(x) = -\frac{y(x)}{y(x) + x - x\, y(x)} \tag{24}$$

> $ComprobarDos := simplify(rhs(DerSolGralFinal) - rhs(DerEcua) = 0)$

$$ComprobarDos := 0 = 0 \tag{25}$$

FIN PROBLEMA 2

> $restart$

3) Resuelva el problema de valor inicial

> $Ecua := y' = \frac{(2\cdot x\cdot y - y^2)}{x^2}$

$$Ecua := \frac{d}{dx}\, y(x) = \frac{2\, x\, y(x) - y(x)^2}{x^2} \tag{26}$$

> $CondIni := y(1) = -1$

$$CondIni := y(1) = -1 \tag{27}$$

RESPUESTA

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[[\_homogeneous, class\ A], \_rational, \_Bernoulli] \tag{28}$$

> $EcuaDos := simplify(isolate(eval(subs(y(x) = x\cdot u(x), Ecua)), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx}\, u(x) = -\frac{u(x)\,(u(x) - 1)}{x} \tag{29}$$

> $EcuaTres := lhs(EcuaDos) - rhs(EcuaDos) = 0$

$$EcuaTres := \frac{d}{dx}\, u(x) + \frac{u(x)\,(u(x) - 1)}{x} = 0 \tag{30}$$

> $M := \frac{u\cdot (u - 1)}{x}$

$$M := \frac{u\,(u - 1)}{x} \tag{31}$$

> $N := 1$

$$N := 1 \tag{32}$$

> $P := \frac{1}{x}; Q := u\cdot(u - 1); R := 1; S := 1$

$$P := \frac{1}{x}$$

$$Q := u\,(u - 1)$$

$$R := 1$$

$$S := 1 \tag{33}$$

> $SolGralCero := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = \_C1$

$$SolGralCero := \ln(x) + \ln(u - 1) - \ln(u) = \_C1 \tag{34}$$

> $SolGral := subs\left(u = \frac{y(x)}{x}, SolGralCero\right)$

$$SolGral := \ln(x) + \ln\left(\frac{y(x)}{x} - 1\right) - \ln\left(\frac{y(x)}{x}\right) = \_C1 \quad \textbf{(35)}$$

> $SolGralDos := isolate(expand(simplify(\exp(lhs(SolGral)))) = \_C1, y(x))$

$$SolGralDos := y(x) = -\frac{x^2}{\_C1 - x} \quad \textbf{(36)}$$

> $Para := isolate(subs(x = 1, rhs(SolGralDos) = -1), \_C1)$

$$Para := \_C1 = 2 \quad \textbf{(37)}$$

> $SolPart := subs(\_C1 = rhs(Para), SolGralDos)$

$$SolPart := y(x) = -\frac{x^2}{2 - x} \quad \textbf{(38)}$$

> $Ecua$

$$\frac{d}{dx}\, y(x) = \frac{2\, x\, y(x) - y(x)^2}{x^2} \quad \textbf{(39)}$$

> $ComprobarTres := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarTres := 0 = 0 \quad \textbf{(40)}$$

FIN PROBLEMA 3

> $restart$

4) Sea el conjunto fundamental de soluciones dado de la ecuación diferencial homogénea
Obtener la solución general de la ecuación

diferencial no homogénea presentada

> $yy[1] := x;\, yy[2] := x^3$

$$yy_1 := x$$

$$yy_2 := x^3 \quad \textbf{(41)}$$

> $Ecua := x^2 \cdot y'' - 3 \cdot x \cdot y' + 3 \cdot y = 0$

$$Ecua := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - 3\, x \left(\frac{d}{dx}\, y(x)\right) + 3\, y(x) = 0 \quad \textbf{(42)}$$

> $EcuaNoHom := lhs(Ecua) = 2 \cdot x^4 \cdot \exp(x)$

$$EcuaNoHom := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - 3\, x \left(\frac{d}{dx}\, y(x)\right) + 3\, y(x) = 2\, x^4\, e^x \quad \textbf{(43)}$$

RESPUESTA

> $EcuaDos := expand\left(\frac{lhs(Ecua)}{x^2}\right) = 0$

$$EcuaDos := \frac{d^2}{dx^2}\, y(x) - \frac{3 \left(\frac{d}{dx}\, y(x)\right)}{x} + \frac{3\, y(x)}{x^2} = 0 \quad \textbf{(44)}$$

> $EcuaNoHomDos := expand\left(\frac{lhs(EcuaNoHom)}{x^2}\right) = \frac{rhs(EcuaNoHom)}{x^2}$

$$EcuaNoHomDos := \frac{d^2}{dx^2}\, y(x) - \frac{3\left(\frac{d}{dx}\, y(x)\right)}{x} + \frac{3\, y(x)}{x^2} = 2\, x^2\, e^x \quad \textbf{(45)}$$

> $Q := rhs(EcuaNoHomDos)$

$$Q := 2\, x^2\, e^x \quad \textbf{(46)}$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C2\, x^3 + \_C1\, x \quad \textbf{(47)}$$

> $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolHom), EcuaDos)))$

$$ComprobarUno := 0 = 0 \quad \textbf{(48)}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} x & x^3 \\ 1 & 3\, x^2 \end{bmatrix} \quad \textbf{(49)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 2\, x^2\, e^x \end{bmatrix} \quad \textbf{(50)}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} -x^2\, e^x & e^x \end{bmatrix} \quad \textbf{(51)}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -x^2\, e^x$$

$$Bprima := e^x \quad \textbf{(52)}$$

> $SolGral := y(x) = expand((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = 2\, x^2\, e^x - 2\, x\, e^x + x\, \_C1 + x^3\, \_C2 \quad \textbf{(53)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), EcuaNoHomDos)))$

$$Comprobar := 2\, x^2\, e^x = 2\, x^2\, e^x \quad \textbf{(54)}$$

FIN PROBLEMA 4

> $restart$

5) Obtenga la solución general de la ecuación diferencial

> $Ecua := y'' + 4 \cdot y = \csc(2\, x)$

$$Ecua := \frac{d^2}{dx^2}\, y(x) + 4\, y(x) = \csc(2\, x) \quad \textbf{(55)}$$

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) + 4\, y(x) = 0 \quad \textbf{(56)}$$

> $EcuaCarac := m^2 + 4 = 0$

$$EcuaCarac := m^2 + 4 = 0 \quad \textbf{(57)}$$

> $Q := rhs(Ecua)$

(58)

$$Q := \csc(2x) \tag{58}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2\,\mathrm{I},\ -2\,\mathrm{I} \tag{59}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1])\cdot x);\ yy[2] := \sin(\mathrm{Im}(Raiz[1])\cdot x)$

$$yy_1 := \cos(2x)$$

$$yy_2 := \sin(2x) \tag{60}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{bmatrix} \tag{61}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \csc(2x) \end{bmatrix} \tag{62}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{1}{2} & \frac{\cot(2x)}{2} \end{bmatrix} \tag{63}$$

> $Aprima := ParaVar[1]$

$$Aprima := -\frac{1}{2} \tag{64}$$

> $Bprima := ParaVar[2]$

$$Bprima := \frac{\cot(2x)}{2} \tag{65}$$

> $SolGral := y(x) = simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGral := y(x) = \left(-\frac{x}{2} + \_C1\right)\cos(2x) + \left(-\frac{\ln(\csc(2x)^2)}{8} + \_C2\right)\sin(2x) \tag{66}$$

> $Ecua$

$$\frac{d^2}{dx^2}\,y(x) + 4\,y(x) = \csc(2x) \tag{67}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))$

$$Comprobar := \csc(2x) = \csc(2x) \tag{68}$$

FIN RESPUESTA 5

> $restart$

FIN EXAMEN PARCIAL 2026-1-1

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