Capítulo II: La ecuación diferencial LINEAL

$$y = C_1 x + \frac{C_2}{x} + x^2 + 8$$

EDO (2)

$\boxed{\text{para cada EDO hay una y sólo una SG.}}$

$$EcuacionElegante := -y(x) + x\left(\frac{d}{dx}y(x)\right) + \left(\frac{d^2}{dx^2}y(x)\right)x^2 = 3x^2 - 8$$

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = Q(x)$$

$a_0(x) = x^2 \qquad a_1(x) = x \qquad a_2(x) = -1 \qquad Q(x) = 3x^2 - 8$

EDO(n)L

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \dots + a_n(x)\,y = Q(x)$$

Según sus coeficientes

EDO(n)L

- coeficientes variables $\quad \forall_i \; a_i(x) \neq a_i$
- coeficientes constantes. $\quad \forall_i \; a_i(x) = a_i \quad a_i \in \mathbb{R}$

Según su homogeneidad

EDO(n)L

- No-homogénea $\quad Q(x) \neq 0.$
- Homogénea $\quad Q(x) = 0$

+ 1^er^ orden coeficientes constantes homogenea

$$\frac{dy}{dx} + a_1 y = 0 \quad \text{normalizada}$$

$$\frac{dy}{dx} = -a_1 y$$

$$\frac{dy}{y} = -a_1 dx \quad \text{variables separadas}$$

$$\int \frac{dy}{y} = -a_1 \int dx$$

$$Ly + k_1 = -a_1 (x + k_2)$$

$$Ly = -a_1 x + (-a_1 k_2 - k_1)$$

$$y = e^{(-a_1 x + (-a_1 k_2 - k_1))}$$

$$y = e^{(-a_1 k_2 - k_1)} \cdot e^{-a_1 x}$$

$$\boxed{y = c_1 e^{-a_1 x}} \Leftrightarrow \boxed{\frac{dy}{dx} + a_1 y = 0}$$

EDO(1) L CV H.

$$a_0(x)\frac{dy}{dx} + a_1(x)y = 0$$

normalizar

$$\frac{dy}{dx} + \frac{a_1(x)}{a_0(x)}y = \frac{0}{a_0(x)}$$

$$\frac{dy}{dx} + p(x)y = 0$$

$$\frac{dy}{dx} = -p(x)y$$

$$\frac{dy}{y} = -p(x)dx$$

$$\int\frac{dy}{y} = -\int p(x)dx$$

$$Ly + k_1 = \left[-\int p(x)dx\right] + k_2$$

$$Ly = \left[-\int p(x)dx\right] + (k_2 - k_1)$$

$$y = e^{\left(\left[-\int p(x)dx\right] + (k_2 - k_1)\right)}$$

$$y = e^{(k_2-k_1)}e^{-\int p(x)dx}$$

$$\boxed{y = c_1 e^{-\int p(x)dx}} \Leftrightarrow \boxed{\frac{dy}{dx} + p(x)y = 0}$$

$$\frac{dy}{dx} + a_1 y = 0 \qquad y = C_1 e^{-a_1 x}$$

$$\frac{dy}{dx} + p(x) y = 0 \qquad y = C_1 e^{-\int p(x)dx}$$

$$y e^{\int p(x)dx} = C_1 \qquad y = \frac{C_1}{e^{\int p(x)dx}}$$

$$\boxed{F(x, y) = C}$$

$$\frac{d}{dx} F(x, y) = 0$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0$$

$$y e^{\int p(x)\,dx} = C_1$$

$$\frac{d}{dx}\left[y e^{\int p(x)\,dx} \right] = 0$$

$$y \frac{\partial}{\partial x}\left(e^{\int p(x)\,dx} \right) + \left(e^{\int p(x)\,dx} \frac{\partial}{\partial y} y \right) \cdot \frac{dy}{dx} = 0$$

$$y\left(e^{\int p(x)\,dx} \cdot p(x) \right) + e^{\int p(x)\,dx} \cdot \frac{dy}{dx} = 0$$

$$e^{\int p(x)\,dx} \left(p(x)\, y + \frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} + p(x)\, y = 0$$

$$\frac{dy}{dx} + p(x)y = 0$$

$$e^{\int p(x)dx}\left(\frac{dy}{dx} + p(x)y\right) = 0$$

factor integrante.

$$\frac{d}{dx}\left(y e^{\int p(x)dx}\right) = 0$$

$$y e^{\int p(x)dx} = C_1$$

$$y = C_1 e^{-\int p(x)dx}$$

EDO(1)L CV NH.

$$\frac{dy}{dx} + p(x)\,y = q(x)$$

$$e^{\int p(x)dx}\left(\frac{dy}{dx} + p(x)\,y\right) = e^{\int p(x)dx} q(x)$$

$$\frac{d}{dx}\left(y e^{\int p(x)dx}\right) = e^{\int p(x)dx} \cdot q(x)$$

$$d\left(y e^{\int p(x)dx}\right) = e^{\int p(x)dx} q(x)\,dx$$

$$\int d\left(y e^{\int p(x)dx}\right) = \int e^{\int p(x)dx} q(x)\,dx$$

$$y e^{\int p(x)dx} + k_1 = \left[\int e^{\int p(x)dx} q(x)\,dx\right] + k_2$$

$$y e^{\int p(x)dx} = C_1 + \left[\int e^{\int p(x)dx} q(x)\,dx\right]$$

$$y_g = C e^{-\int p(x)dx} + e^{-\int p(x)dx}\left[\int e^{\int p(x)dx} q(x)\,dx\right]$$

$$\frac{dy}{dx} + p(x)\,y = q(x) \quad \text{EDO(1)L CV NH.}$$