> *restart*

> $Ecuacion := y'''' - 2\,y''' + 4\,y'' + 2\,y' - 5\,y = 0$

$$Ecuacion := \frac{d^4}{dx^4}\,y(x) - 2\left(\frac{d^3}{dx^3}\,y(x)\right) + 4\left(\frac{d^2}{dx^2}\,y(x)\right) + 2\left(\frac{d}{dx}\,y(x)\right) - 5\,y(x) = 0 \quad (1)$$

> $SolucionGeneral := dsolve(Ecuacion)$

$$SolucionGeneral := y(x) = \_C1\,e^{-x} + \_C2\,e^{x} + \_C3\,e^{x}\sin(2\,x) + \_C4\,e^{x}\cos(2\,x) \quad (2)$$

> $SolucionParticular := y(x) = 4\exp(x) + 2\exp(-x) - 6\exp(x)\cos(2\,x) + 8\exp(x)\sin(2\,x)$

$$SolucionParticular := y(x) = 4\,e^{x} + 2\,e^{-x} - 6\,e^{x}\cos(2\,x) + 8\,e^{x}\sin(2\,x) \quad (3)$$

> $Condicion_1 := eval(subs(x = 0, SolucionParticular))$

$$Condicion_1 := y(0) = 0 \quad (4)$$

> $Condicion_2 := D(y)(0) = eval(rhs(subs(x = 0, diff(SolucionParticular, x))))$

$$Condicion_2 := D(y)(0) = 12 \quad (5)$$

> $Condicion_3 := D(D(y))(0) = eval(rhs(subs(x = 0, diff(SolucionParticular, x\$2))))$

$$Condicion_3 := D^{(2)}(y)(0) = 56 \quad (6)$$

> $Condicion_4 := D(D(D(y)))(0) = eval(rhs(subs(x = 0, diff(SolucionParticular, x\$3))))$

$$Condicion_4 := D^{(3)}(y)(0) = 52 \quad (7)$$

> $CondicionesIniciales := Condicion_1, Condicion_2, Condicion_3, Condicion_4;$

$$CondicionesIniciales := y(0) = 0, D(y)(0) = 12, D^{(2)}(y)(0) = 56, D^{(3)}(y)(0) = 52 \quad (8)$$

> $SolPart := dsolve(\{Ecuacion, CondicionesIniciales\})$

$$SolPart := y(x) = 4\,e^{x} + 2\,e^{-x} - 6\,e^{x}\cos(2\,x) + 8\,e^{x}\sin(2\,x) \quad (9)$$

> $SolucionGeneral;$

$$y(x) = \_C1\,e^{-x} + \_C2\,e^{x} + \_C3\,e^{x}\sin(2\,x) + \_C4\,e^{x}\cos(2\,x) \quad (10)$$

> $Sistema := eval(subs(x = 0, rhs(SolucionGeneral))) = 0, eval(subs(x = 0,$
$rhs(diff(SolucionGeneral, x)))) = 12, eval(subs(x = 0, rhs(diff(SolucionGeneral, x$
$\$2)))) = 56, eval(subs(x = 0, rhs(diff(SolucionGeneral, x\$3)))) = 52 : Sistema_1;$
$Sistema_2; Sistema_3; Sistema_4;$

$$\_C1 + \_C2 + \_C4 = 0$$
$$-\_C1 + \_C2 + 2\,\_C3 + \_C4 = 12$$
$$\_C1 + \_C2 + 4\,\_C3 - 3\,\_C4 = 56$$
$$-\_C1 + \_C2 - 2\,\_C3 - 11\,\_C4 = 52 \quad (11)$$

> $Parametro := solve(\{Sistema\})$

$$Parametro := \{\_C1 = 2, \_C2 = 4, \_C3 = 8, \_C4 = -6\} \quad (12)$$

> *restart*

> $Ecuacion := y''' - y'' + 3\,y' + 5\,y = 32\exp(x)$

$$Ecuacion := \frac{d^3}{dx^3}\,y(x) - \left(\frac{d^2}{dx^2}\,y(x)\right) + 3\left(\frac{d}{dx}\,y(x)\right) + 5\,y(x) = 32\,e^{x} \quad (13)$$

> $SolGral := dsolve(Ecuacion)$

$$SolGral := y(x) = 4\,e^{x} + \_C1\,e^{-x} + \_C2\,e^{x}\cos(2\,x) + \_C3\,e^{x}\sin(2\,x) \quad (14)$$

> *restart*

> $SolucionGeneral := y(x) = C_1 \cdot \exp(-x) + C_2 \exp(x) \cos(2x) + 4 \cdot \exp(x) + 8 \exp(x) \cdot \sin(2x)$

$$SolucionGeneral := y(x) = C_1 e^{-x} + C_2 e^{x} \cos(2x) + 4 e^{x} + 8 e^{x} \sin(2x) \tag{15}$$

> $SolucionHomogenea := y(x) = C_1 \cdot \exp(-x) + C_2 \exp(x) \cos(2x);$

$$SolucionHomogenea := y(x) = C_1 e^{-x} + C_2 e^{x} \cos(2x) \tag{16}$$

> $SolucionParticular := y(x) = 4 e^{x} + 8 e^{x} \sin(2x);$

$$SolucionParticular := y(x) = 4 e^{x} + 8 e^{x} \sin(2x) \tag{17}$$

> $Sistema := diff(SolucionHomogenea, x), diff(SolucionHomogenea, x\$2) : Sistema_1; Sistema_2;$

$$\frac{d}{dx} y(x) = -C_1 e^{-x} + C_2 e^{x} \cos(2x) - 2 C_2 e^{x} \sin(2x)$$

$$\frac{d^2}{dx^2} y(x) = C_1 e^{-x} - 3 C_2 e^{x} \cos(2x) - 4 C_2 e^{x} \sin(2x) \tag{18}$$

> $Parametro := solve(\{Sistema\}, \{C_1, C_2\}) : Parametro_1; Parametro_2;$

$$C_1 = -\frac{1}{2} \frac{1}{e^{-x} (\cos(2x) + 3 \sin(2x))} \left(\cos(2x) \left(\frac{d^2}{dx^2} y(x)\right) + 3 \cos(2x) \left(\frac{d}{dx} y(x)\right) - 2 \sin(2x) \left(\frac{d^2}{dx^2} y(x)\right) + 4 \sin(2x) \left(\frac{d}{dx} y(x)\right)\right)$$

$$C_2 = -\frac{1}{2} \frac{\frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x)}{e^{x} (\cos(2x) + 3 \sin(2x))} \tag{19}$$

> $EcuacionIntermedia := simplify\left(subs\left(C_1 = rhs\left(Parametro_1\right), C_2 = rhs\left(Parametro_2\right), SolucionHomogenea\right)\right)$

$$EcuacionIntermedia := y(x) = -\frac{1}{\cos(2x) + 3 \sin(2x)} \left(\cos(2x) \left(\frac{d^2}{dx^2} y(x)\right) + 2 \cos(2x) \left(\frac{d}{dx} y(x)\right) - \sin(2x) \left(\frac{d^2}{dx^2} y(x)\right) + 2 \sin(2x) \left(\frac{d}{dx} y(x)\right)\right) \tag{20}$$

> $EcuacionSegunda := lhs(EcuacionIntermedia) \cdot (-(\cos(2x) + 3 \sin(2x))) = simplify(rhs(EcuacionIntermedia) \cdot (-(\cos(2x) + 3 \sin(2x))))$

$$EcuacionSegunda := -y(x) (\cos(2x) + 3 \sin(2x)) = \cos(2x) \left(\frac{d^2}{dx^2} y(x)\right) + 2 \cos(2x) \left(\frac{d}{dx} y(x)\right) - \sin(2x) \left(\frac{d^2}{dx^2} y(x)\right) + 2 \sin(2x) \left(\frac{d}{dx} y(x)\right) \tag{21}$$

> $EcuacionHomogenea := lhs(EcuacionSegunda) - rhs(EcuacionSegunda) = 0$

$$EcuacionHomogenea := -y(x) (\cos(2x) + 3 \sin(2x)) - \cos(2x) \left(\frac{d^2}{dx^2} y(x)\right) - 2 \cos(2x) \left(\frac{d}{dx} y(x)\right) + \sin(2x) \left(\frac{d^2}{dx^2} y(x)\right) - 2 \sin(2x) \left(\frac{d}{dx} y(x)\right) = 0 \tag{22}$$

> $Q(x) := simplify(eval(subs(y(x) = rhs(SolucionParticular), lhs(EcuacionHomogenea))))$

(23)

$$Q(x) := -16\, e^{x} (\cos(2x) + \sin(2x) + 4) \tag{23}$$

> $EcuacionFinal := lhs(EcuacionHomogenea) = Q(x)$

$$EcuacionFinal := -y(x)\,(\cos(2x) + 3\sin(2x)) - \cos(2x)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\cos(2x)\left(\frac{d}{dx}\,y(x)\right) + \sin(2x)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\sin(2x)\left(\frac{d}{dx}\,y(x)\right) = -16\, e^{x} (\cos(2x) + \sin(2x) + 4) \tag{24}$$

> restart

METODO DE PARAMETROS VARIABLES PARA RESOLVER NO HOMOGENEAS DE ORDEN SUPERIOR

> $EcuacionFinal := -y(x)\,(\cos(2x) + 3\sin(2x)) - \cos(2x)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\cos(2x)\left(\frac{d}{dx}\,y(x)\right) + \sin(2x)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\sin(2x)\left(\frac{d}{dx}\,y(x)\right) = -16\, e^{x} (\cos(2x) + \sin(2x) + 4)$

$$EcuacionFinal := -y(x)\,(\cos(2x) + 3\sin(2x)) - \cos(2x)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\cos(2x)\left(\frac{d}{dx}\,y(x)\right) + \sin(2x)\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\sin(2x)\left(\frac{d}{dx}\,y(x)\right) = -16\, e^{x} (\cos(2x) + \sin(2x) + 4) \tag{25}$$

> $SolucionHomogenea := y(x) = C_1 \cdot \exp(-x) + C_2 \exp(x) \cos(2x);$

$$SolucionHomogenea := y(x) = C_1\, e^{-x} + C_2\, e^{x} \cos(2x) \tag{26}$$

> $SolucionNoHomogenea := y(x) = A(x) \cdot \exp(-x) + B(x) \cdot \exp(x) \cdot \cos(2x)$

$$SolucionNoHomogenea := y(x) = A(x)\, e^{-x} + B(x)\, e^{x} \cos(2x) \tag{27}$$

> $Sol_1 := y(x) = \exp(-x);\ Sol_2 := y(x) = \exp(x) \cdot \cos(2x)$

$$Sol_1 := y(x) = e^{-x}$$

$$Sol_2 := y(x) = e^{x} \cos(2x) \tag{28}$$

> $with(linalg):$

> $WW := wronskian([rhs(Sol_1), rhs(Sol_2)], x)$

$$WW := \begin{bmatrix} e^{-x} & e^{x}\cos(2x) \\ -e^{-x} & e^{x}\cos(2x) - 2\, e^{x}\sin(2x) \end{bmatrix} \tag{29}$$

> $BB := array([0, rhs(EcuacionFinal)])$

$$BB := \begin{bmatrix} 0 & -16\, e^{x} (\cos(2x) + \sin(2x) + 4) \end{bmatrix} \tag{30}$$

> $SOLUCION := linsolve(WW, BB) : SOLUCION_1; SOLUCION_2;$

$$\frac{8\, e^{x} \cos(2x)\,(\cos(2x) + \sin(2x) + 4)}{e^{-x}\,(\cos(2x) - \sin(2x))}$$

(31)

$$-\frac{8\left(\cos(2x)+\sin(2x)+4\right)}{\cos(2x)-\sin(2x)} \tag{31}$$

> $A(x) := simplify(int(SOLUCION_1, x)) + C_1; B(x) := simplify(int(SOLUCION_2, x)) + C_2;$

$$A(x) := 8\,e^{2x} - 8\,I\,e^{2x} - e^{(2+2I)x} - I\,e^{(2+2I)x} - e^{(2-2I)x} + I\,e^{(2-2I)x} - 8\left(\int \frac{e^{(2+2I)x} + 2\,I\,e^{2x} + 2\,e^{2x}}{-e^{4Ix} + I}\,\mathrm{d}x\right) + 8\,I\left(\int \frac{e^{(2+2I)x} + 2\,I\,e^{2x} + 2\,e^{2x}}{-e^{4Ix} + I}\,\mathrm{d}x\right) + C_1$$

$$B(x) := 4\ln\left(\frac{-2\cos(x)^2 + 2\sin(x)\cos(x) + 1}{\cos(x)^2}\right) - 16\sqrt{2}\,\operatorname{arctanh}\left(\frac{1}{2}\,\frac{\sqrt{2}\,(\sin(x)+\cos(x))}{\cos(x)}\right) - 4\ln\left(\frac{1}{\cos(x)^2}\right) + C_2 \tag{32}$$

>

>