

```
> restart
> Sistema := diff(x(t), t) = 3·x(t) + 4·y(t), diff(y(t), t) = 2·x(t) + 5·y(t) : Sistema1;
Sistema2;
```

$$\frac{d}{dt} x(t) = 3 x(t) + 4 y(t)$$

$$\frac{d}{dt} y(t) = 2 x(t) + 5 y(t)$$

(1)

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> Condiciones := x(0) = 5, y(0) = -5
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$$\text{Condiciones} := x(0) = 5, y(0) = -5$$

(2)

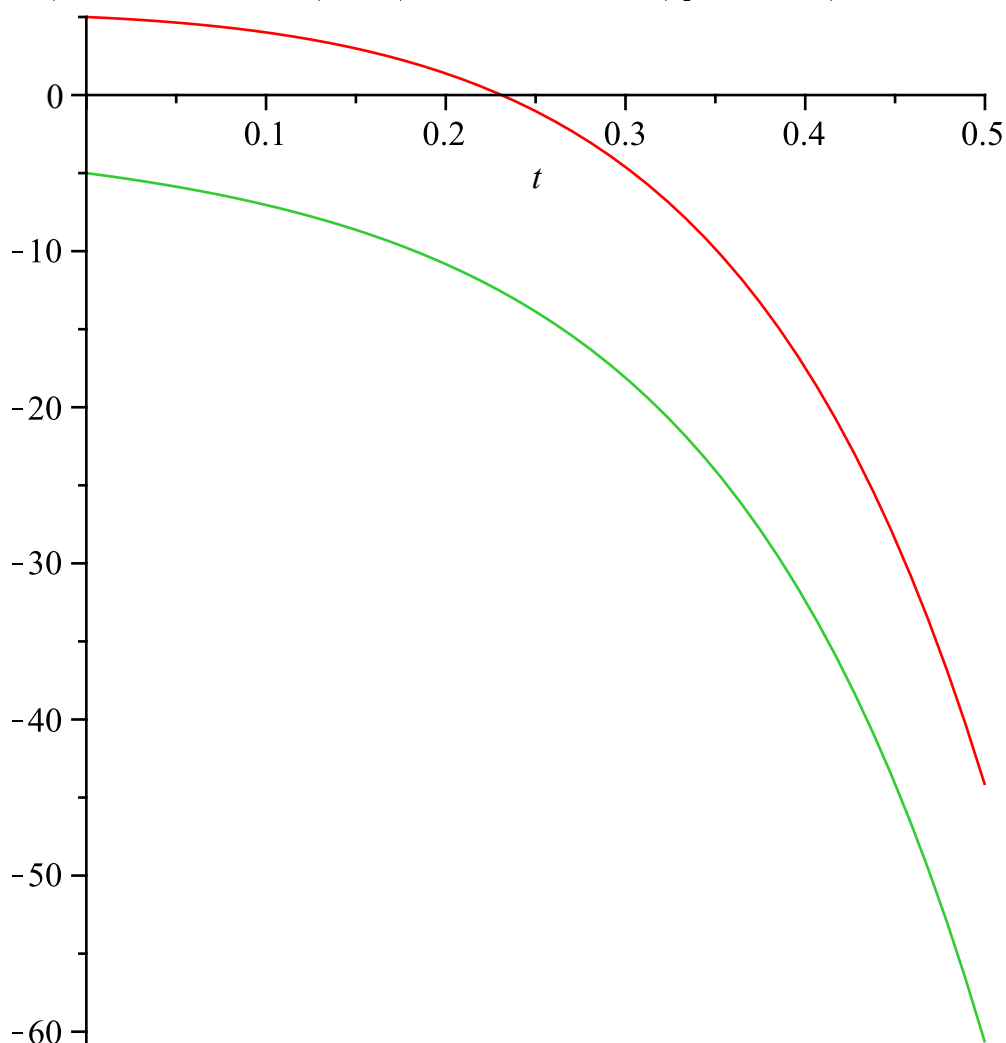
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> SolucionParticular := dsolve( {Sistema, Condiciones} ) : SolucionParticular1;
SolucionParticular2;
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$$x(t) = -\frac{5}{3} e^{7t} + \frac{20}{3} e^t$$

$$y(t) = -\frac{5}{3} e^{7t} - \frac{10}{3} e^t$$

(3)

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> plot( [ rhs(SolucionParticular1), rhs(SolucionParticular2) ], t = 0 .. 0.5)
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> AA := array( [[3, 4], [2, 5]])
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$$AA := \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad (4)$$

> with(linalg) :

> MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} \frac{2}{3} e^t + \frac{1}{3} e^{7t} & \frac{2}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{1}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{2}{3} e^{7t} \end{bmatrix} \quad (5)$$

> DerMatExp := map(diff, MatExp, t)

$$DerMatExp := \begin{bmatrix} \frac{2}{3} e^t + \frac{7}{3} e^{7t} & \frac{14}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{7}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{14}{3} e^{7t} \end{bmatrix} \quad (6)$$

> comprobacion₁ := simplify(evalm(DerMatExp - evalm(AA &* MatExp)))

$$comprobacion_1 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (7)$$

> Identidad := map(rcurry(eval, t=0), MatExp)

$$Identidad := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

> InvMatExp := map(rcurry(eval, t=-t), MatExp)

$$InvMatExp := \begin{bmatrix} \frac{2}{3} e^{-t} + \frac{1}{3} e^{-7t} & \frac{2}{3} e^{-7t} - \frac{2}{3} e^{-t} \\ \frac{1}{3} e^{-7t} - \frac{1}{3} e^{-t} & \frac{1}{3} e^{-t} + \frac{2}{3} e^{-7t} \end{bmatrix} \quad (9)$$

> Ident := simplify(evalm(InvMatExp &* MatExp))

$$Ident := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (10)$$

> evalm(MatExp)

$$\begin{bmatrix} \frac{2}{3} e^t + \frac{1}{3} e^{7t} & \frac{2}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{1}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{2}{3} e^{7t} \end{bmatrix} \quad (11)$$

> Xcero := array([5, -5])

$$Xcero := \begin{bmatrix} 5 & -5 \end{bmatrix} \quad (12)$$

> SOLUCION := evalm(MatExp &* Xcero) : Sol₁ := X(t) = SOLUCION₁; Sol₂ := Y(t) = SOLUCION₂;

$$\begin{aligned} Sol_1 &:= X(t) = -\frac{5}{3} e^{7t} + \frac{20}{3} e^t \\ Sol_2 &:= Y(t) = -\frac{5}{3} e^{7t} - \frac{10}{3} e^t \end{aligned} \quad (13)$$

> *SolucionParticular₁*; *SolucionParticular₂*;

$$\begin{aligned} x(t) &= -\frac{5}{3} e^{7t} + \frac{20}{3} e^t \\ y(t) &= -\frac{5}{3} e^{7t} - \frac{10}{3} e^t \end{aligned} \quad (14)$$

> *CondInic* := *x*(2) = 4, *y*(2) = -4;

$$CondInic := x(2) = 4, y(2) = -4 \quad (15)$$

> *MatExpDos* := *map*(*rcurry*(*eval*, *t* = *t* - 2'), *MatExp*)

$$MatExpDos := \begin{bmatrix} \frac{2}{3} e^{t-2} + \frac{1}{3} e^{7t-14} & \frac{2}{3} e^{7t-14} - \frac{2}{3} e^{t-2} \\ \frac{1}{3} e^{7t-14} - \frac{1}{3} e^{t-2} & \frac{1}{3} e^{t-2} + \frac{2}{3} e^{7t-14} \end{bmatrix} \quad (16)$$

> *Xdos* := *array*([4, -4])

$$Xdos := \begin{bmatrix} 4 & -4 \end{bmatrix} \quad (17)$$

> *SSOL* := *evalm*(*MatExpDos* &* *Xdos*) : *Ssol₁* := *XX*(*t*) = *SSOL₁*; *Ssol₂* := *YY*(*t*) = *SSOL₂*;

$$\begin{aligned} Ssol_1 &:= XX(t) = \frac{16}{3} e^{t-2} - \frac{4}{3} e^{7t-14} \\ Ssol_2 &:= YY(t) = -\frac{4}{3} e^{7t-14} - \frac{8}{3} e^{t-2} \end{aligned} \quad (18)$$

> *SSolPart* := *simplify*(*dsolve*({*Sistema*, *CondInic*}))

$$SSolPart := \left\{ x(t) = \frac{16}{3} e^{t-2} - \frac{4}{3} e^{7t-14}, y(t) = -\frac{4}{3} e^{7t-14} - \frac{8}{3} e^{t-2} \right\} \quad (19)$$

> *Xgral* := *array*([*C₁*, *C₂*])

$$Xgral := \begin{bmatrix} C_1 & C_2 \end{bmatrix} \quad (20)$$

> *SolGral* := *simplify*(*evalm*(*MatExp* &* *Xgral*)) : *x*(*t*) = *SolGral₁*; *y*(*t*) = *SolGral₂*;

$$\begin{aligned} x(t) &= \frac{2}{3} C_1 e^t + \frac{1}{3} C_1 e^{7t} + \frac{2}{3} C_2 e^{7t} - \frac{2}{3} C_2 e^t \\ y(t) &= \frac{1}{3} C_1 e^{7t} - \frac{1}{3} C_1 e^t + \frac{1}{3} C_2 e^t + \frac{2}{3} C_2 e^{7t} \end{aligned} \quad (21)$$

> *SolGralDos* := *x*(*t*) = -2 *C₁₀* · *exp*(*t*) + *C₂₀* · *exp*(7 *t*), *y*(*t*) = *C₁₀* *exp*(*t*) + *C₂₀* *exp*(7 *t*) :
SolGralDos₁; *SolGralDos₂*;

$$\begin{aligned} x(t) &= -2 C_{10} e^t + C_{20} e^{7t} \\ y(t) &= C_{10} e^t + C_{20} e^{7t} \end{aligned} \quad (22)$$

> *Comprobacion₁* := *simplify*(*eval*(*subs*(*x*(*t*) = *rhs*(*SolGralDos₁*), *y*(*t*) = *rhs*(*SolGralDos₂*),
lhs(*Sistema₁*) - *rhs*(*Sistema₁*) = 0)))

(23)

$$\text{Comprobacion}_1 := 0 = 0 \quad (23)$$

$$\text{Comprobacion}_2 := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolGralDos}_1), y(t) = \text{rhs}(\text{SolGralDos}_2), \text{lhs}(\text{Sistema}_2) - \text{rhs}(\text{Sistema}_2) = 0)))$$

$$\text{Comprobacion}_2 := 0 = 0 \quad (24)$$